Acknowledgments

Field Test:

America’s Choice®, Inc., wishes to acknowledge the following schools for their participation in the field testing of this material during the 2004–2005 school year:

america’s choice high school, sacramento, CA—Principal: Beate Martinez; Teacher: Nakia Edwards

Chattooga County High School, Summerville, GA—Principal: Roger Hibbs; Math Coaches: Claire Pierce, Terry Haney; Teachers: Renee Martin, David Whitfield

John Marshall High School, Rochester, NY—Principal: Joseph Munno; Math Coach: Tonette Graham; Teachers: Tonette Graham, Michael Emmerling

Woodland High School, Cartersville, GA—Principal: Nettie Holt; Math Coach: Connie Smith; Teachers: Leslie Nix, Kris Norris

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America’s Choice, Inc., developed and field-tested Ramp-Up to Algebra over a ten-year period. Initial development started with a grant from the Office of Educational Research and Improvement (OERI) of the U.S. Department of Education, and was based on the most recent research in mathematics. During that ten-year period many authors, reviewers, and math consultants—both from the United States and internationally—contributed to this course. The materials underwent three major revisions as we analyzed field-test results and consulted math content experts. During the entire time, Phil Daro guided the development of the entire Ramp-Up to Algebra curriculum. See the Getting Started Teacher Resource guide for a complete list of people involved in this effort.
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Lisa and Dwayne were shopping for a birthday present for Lisa’s little sister, Annie.

Lisa found a box of toy animals. “Annie would love these,” Lisa said.

Dwayne said, “You should buy three boxes for Annie—that’s more than fifty animals!”

“More than fifty?” Lisa asked.

“Sure,” answered Dwayne. “Four horses, six dogs, and seven cats makes seventeen animals in one box, times three makes fifty-one. Here, look at this,” he said. He sketched this on a piece of paper:

\[
\begin{align*}
\text{4 horses} & \quad + \quad \text{4 horses} \\
\text{6 dogs} & \quad + \quad \text{6 dogs} \\
\text{7 cats} & \quad + \quad \text{7 cats}
\end{align*}
\]

Total Number of Animals in Each Box

Number of Boxes

“But what I need to worry about is the kind of animals,” Lisa said. She took the piece of paper and wrote:

\[
3 \times 4 + 3 \times 6 + 3 \times 7 = 51
\]
“Interesting,” said Dwayne. “So what I did was this. And what you did was this.”

\[
\begin{align*}
4 + 6 + 7 &= 3 \times 4 + 6 + 7 = 51 \\
4 + 6 + 7 &\downarrow \\
3 \times 4 + 3 \times 6 + 3 \times 7 &= 51
\end{align*}
\]

Lisa said, “You added first and then multiplied.”
Dwayne said, “And you multiplied first and then added.”

**Work Time**

1. Dwayne looked at the diagrams he and Lisa sketched. He wondered if both ways of calculating the value would work for other numbers.

Is adding 5 to 1 then multiplying the result by 3 the same as multiplying 1 by 3, multiplying 5 by 3, and then adding the results?

   a. Calculate 1 + 5. Multiply the result by 3.

   b. Calculate 1 \times 3. Calculate 5 \times 3. Add the results together.

2. Sketch diagrams, using small circles to represent numbers, to show that adding 5 to 1 then multiplying the result by 3 is the same as multiplying 1 by 3, multiplying 5 by 3, and then adding the results.

   **Example**

   Adding 3 to 1 and multiplying the result by 2 is the same as multiplying 1 by 2, multiplying 3 by 2, and then adding the results.

   \[
   \begin{align*}
   \text{3 added to 1 multiplied by 2} &\quad \text{1 times 2 added to 3 times 2} \\
   \bigcirc + \bigcirc \bigcirc \bigcirc &\quad \bigcirc + \bigcirc \bigcirc \bigcirc \\
   \bigcirc + \bigcirc \bigcirc \bigcirc &\quad \bigcirc + \bigcirc \bigcirc \bigcirc
   \end{align*}
   \]

3. a. Multiply 2 by 3, multiply 4 by 3, and then add the results.

   b. Add 2 plus 4, and then multiply the result by 3.
4. Calculate.
   a. \[2 \times 3\] plus \[4 \times 3\]
   b. \[2 + 4\] times \[3\]

5. Lisa and Dwayne explained their conversation to their friend, Jamal.
   Jamal likes to use number lines to show the mathematics. He sketched these diagrams.

![Number Line Diagrams]

What do Jamal’s diagrams show you?

6. Lisa and Dwayne used diagrams to show that both of their solutions were the same. In your own words, explain what a “diagram” is.

**Preparing for the Closing**

When you use mathematical symbols and numbers to express an amount instead of using words or a diagram, the result is called an *expression*.

In future lessons, you will work with expressions that use letters to stand for numbers.

7. These expressions use parentheses to show the values from problems 3–5.
   \[(2 \times 3) + (4 \times 3)\]  \[3 \times (2 + 4)\]
   a. What do you think the parentheses mean?
   b. Dwayne said, “Hey, the two amounts are equal. I can write an equation.” He wrote the following showing that the two expressions are equal to each other:
      \[(3 \times 2) + (3 \times 4) = 3 \times (2 + 4)\]
      In your own words, write what an “equation” is.

8. Sketch a diagram to show that the expressions on each side of Dwayne’s equation in problem 7 have the same value.
Lesson 1

Skills

Solve.

a. \(48 + 10 = \)

b. \(48 + 100 = \)

c. \(48 + 1000 = \)

d. \(48 + 1 = \)

e. \(4810 - 1000 = \)

f. \(4810 - 100 = \)

g. \(4810 - 10 = \)

h. \(4800 - 1 = \)

Review and Consolidation

1. Each of these diagrams represents an expression using multiplication.
   Write the expression using only numbers and a symbol for multiplication.

   a.  
   
   b.  
   
   c.  
   
   d.  

2. Look at the diagram in problem 1a.
   a. Sketch a diagram, like the one in problem 1a, that represents \(6 \times 2\) instead of \(2 \times 6\).
   b. Say why your diagram and the one in problem 1a show different multiplication expressions that have the same value.
   c. Write an equation showing that the amounts in each diagram (yours and the one in problem 1a) are equal.

3. Look at the diagram in problem 1c.
   a. Sketch a diagram, like the one in problem 1c, that represents \(5 \times 2\) instead of \(2 \times 5\).
   b. Say why your diagram and the one in problem 1c show different multiplication expressions that have the same value.
   c. Write an equation showing that the amounts in each diagram (yours and the one in problem 1c) are equal.
4. A number multiplied by itself is called the square of the number.

Example

\[ 5 \times 5 = 25 \quad 5^2 = 25 \]

25 is the square of 5

Sketch a diagram that shows why. Use different numbers than those used in the example.

Homework

1. Sketch a diagram that shows each of these expressions.
   a. \[ 4 \times 2 \]
   b. \[ 4 \times 4 \]
   c. \[ 4 + 4 + 4 \]
   d. \[ 4 \times 4 \times 4 \]

2. Sketch a diagram that shows each of these expressions.
   a. \[ 3 + 3 + 3 + 3 \]
   b. \[ 4 + 4 + 4 + 4 \]
   c. \[ 8 \times 8 \]
   d. \[ 2 \times 4 \]

3. Each expression in problem 1 is equal to one expression in problem 2.

   Create equations using one expression from problem 1 and one expression from problem 2. Use your diagrams to help you determine which expressions are equal. For example, \[ 4 \times 2 = 2 \times 4 \]
To justify mathematical statements about odd and even numbers as always true, sometimes true, or never true.

In this lesson, you will decide whether statements about odd and even numbers are always true, sometimes true, or never true. Then you will say why your decision makes sense. Saying why something is true or false is a justification.

Here are some important definitions and properties you will need to know about odd and even numbers.

**Definitions**
- An even number is a whole number that is divisible by 2.
- An odd number is a whole number that is not divisible by 2.

**Properties**
- Every whole number is either odd or even.
- No number is both odd and even.

One way of representing odd and even is:

Any even number of items arranged in pairs has no unpaired item left over. Pairing is like dividing by two.

**Example**
Start with 14 marbles. Make pairs of marbles. There is no unpaired marble. So, 14 is an even number.

Any odd number of items arranged in pairs has exactly one unpaired item. The unpaired item is called a remainder. If this number of items has a remainder, it does not fit the definition of even, and must be odd.

**Example**
Start with 13 marbles. Make pairs of marbles. One marble is unpaired. So, 13 is an odd number.
Rosa Valdez and her brother, Carlos, help their father in the family’s restaurant. People arrive for dinner in groups of even numbers (2, 4, or 8) as well as groups of odd numbers (1, 3, or 7). One night, all of the groups had even numbers of people.

Rosa said, “Amazing, all of the groups of people were even numbers, so we must have had an even number of customers tonight.”

“Why do you think that is true?” asked her father. “Is it always true?”

Carlos said, “Well, I tried adding some even numbers in my head. 

2 + 12 = 14

2 + 20 + 4 = 26

“My answer was always an even number, so what Rosa said must always be true.”

“I’m not sure that a few examples are enough to prove my statement,” replied Rosa as she reached for a napkin. She sketched some of the groups on the napkin, arranging them in pairs.
Holding up her sketch, Rosa stated, “Every time I have an even number of people, they pair up without any remainder. That means each group is divisible by 2. “If I add them together, I am just joining the groups, so the sum can be grouped into twos and will be divisible by 2. “That’s why, when you add an even number to an even number, you always get an even number. That’s my justification.”

1. a. What definitions and properties of odd and even numbers did Rosa and Carlos use in their thinking and responses to their father’s question?
   b. Both Rosa and Carlos were correct in deciding that the total number of customers was an even number. Whose justification seems strongest to you? Say why.

2. Thinking about the problem, Carlos said, “If an odd number of people was added to an even number, you would get an odd number.”
   a. Decide whether his statement is always true, sometimes true, or never true.
   b. Write a justification for your decision, using Rosa’s justification as a model.

3. Mr. Valdez said, “If all the groups had odd numbers of people, the total number of people would be odd.”
   a. This statement is sometimes true. Say why.
   b. When is the statement true?
   c. When is it false?
Preparing for the Closing  

When you have to say why or justify why a statement is *always true* or *never true*, these are good tools to use in your justification:

- **Definitions**
- **Properties** of odd and even numbers
- A **diagram** of the situation that shows what the statement means—for example, Rosa’s diagram
- **Examples** that show the statement is true
- **Counterexamples** that show the statement is not true

*A counterexample* is an exception that you can find to a statement. Using examples and counterexamples alone, you can only justify statements that are *sometimes true*.

**Example**

Think about this statement: All basketball players are over six feet tall.
Since Jamal plays basketball and is not six feet tall, the statement cannot be true.
Jamal is a counterexample to “All basketball players are over six feet tall.”
The statement is *sometimes true*.

4. Look at Rosa’s justifications and your justifications in problems 2b and 3a.
   a. What tools did Rosa use in her justifications?
   b. What tools did you use in problem 2b?
   c. What tools did you use in problem 3a?

5. Only giving examples or counterexamples should never convince you that a statement is *always true*.
   Say why.

**Skills**

Solve.

- a. 525 + 100 =
- b. 738 + 100 =
- c. 1267 + 100 =
- d. 49 + 100 =
- e. 525 – 100 =
- f. 738 – 100 =
- g. 1267 – 100 =
- h. 49 – 10 =
Problems 1–3 each contain a statement that has been classified as *always true,* *sometimes true,* or *never true.* Each statement is followed by a justification for this classification.

For each statement, decide whether the justification is convincing. Rewrite the justifications if you feel they are not convincing.

1. When you subtract one number from another number, the result is an odd number.
   
   This statement is *sometimes true.*
   
   Justification: \(12 - 10 = 2.\) The number 2 is even, not odd.
   \(12 - 11 = 1.\) The number 1 is odd, not even.

   a. Is this justification convincing?
   b. If the justification is not convincing, rewrite it.
      Use the tools from page 9 in your justification.
   c. Can you explain when the statement is true?
      What types of numbers make this true?

2. When you multiply any odd number by 2, the result is an odd number.
   
   This statement is *never true.*
   
   Justification: When you multiply any number of items by 2, the result can be arranged so that each item is paired with another.

   \[
   2 \times \begin{array}{c} 
   \text{●} \text{●} \text{●} \text{●} \\
   \text{●} \text{●} \text{●} \text{●}
   \end{array} = \begin{array}{c} 
   \text{●} \text{●} \text{●} \text{●} \\
   \text{●} \text{●} \text{●} \text{●}
   \end{array}
   \]

   or:

   The resulting number of items will have no unpaired item, or remainder.

   A number that can be divided into pairs with no remainder is even. This is true because of the definition of *even:* a whole number that is divisible by 2.

   If it is even, it is not odd. This is true because of the properties of odd and even numbers: no number is both even and odd.

   a. Is this justification convincing?
   b. If the justification is not convincing, rewrite it.
      Use the tools from page 9 in your justification.
3. When you multiply any even number by 2, the result is an even number. This statement is always true.
   Justification: $6 \times 2 = 12$, $8 \times 2 = 16$, $10 \times 2 = 20$, and so on.
   a. Is this justification convincing?
   b. If the justification is not convincing, rewrite it. Use tools from page 9 in your justification.

4. Think about this statement: A number can be both even and odd.
   a. Is this statement always true, sometimes true, or never true?
   b. Justify your choice mathematically.

5. Think about this statement: When you add more than two odd numbers, the sum is odd.
   a. Is this statement always true, sometimes true, or never true?
   b. Justify your choice mathematically.

Homework

1. A justification that uses only counterexamples should not convince you that a statement is never true. Say why.

2. Think about this statement: When you subtract an odd number from an odd number, the result is an odd number.
   a. Is this statement always true, sometimes true, or never true?
   b. Justify your choice by using a diagram, definitions, and/or properties of odd and even numbers.

3. Think about this statement: When you add an odd number to an odd number, the result is an even number.
   a. Is this statement always true, sometimes true, or never true?
   b. Justify your choice by using a diagram, definitions, and/or properties of odd and even numbers.
In this lesson, the letter $a$ represents any number.

**Example**

- $a$ could be 1, 2, 3, 10, 100, or 1000
- $a$ could be $\frac{1}{2}$, or 2.5
- $a$ could be 0
- $a$ could be $-1$, or $-1000$, or $-\frac{1}{2}$, or $-2.5$

*(But you do not have to think about negative numbers today.)*

$a$ could be any number. Think of three different numbers that $a$ could be.

When you multiply two numbers, the product is another number.

When you multiply $a$ by 3, the product is written as $3a$ instead of $3 \times a$, $3 \cdot a$, $a \times 3$, $a \cdot 3$, or $3a$.

Since $a$ represents a number, $3a$ also represents a number.

When you add two numbers, the sum is another number.

When you add any number, $a$, to 3, the sum is $a + 3$.

Since $a$ represents a number, $a + 3$ also represents a number.

$3a$ and $a + 3$ are called expressions. An expression uses mathematical symbols, numbers, and/or letters to express an amount.

**Example**

$a + 4$, $3a$, and $4x + 5x - a$ are all expressions.

Comment:

Sometimes, expressions are also referred to as mathematical expressions.

In an expression like $a + 3$, the value of the expression depends on the value of $a$. 
1. $3a$ is any number $a$ multiplied by the number 3.

Calculate the value of $3a$ for each value of $a$ listed.

**Example**

If $a = 5$, then $3a = 3 \cdot 5 = 15$

In words, “If $a$ equals five, then the expression three times $a$ equals fifteen.”

a. $a = 1$  
   b. $a = 2$  
   c. $a = 3$  
   d. $a = 10$

2. $a + 3$ is the number 3 added to any number $a$.

Calculate the value of $a + 3$ for each value of $a$ listed.

**Example**

If $a = 5$, then $a + 3 = 5 + 3 = 8$

In words, “If $a$ equals five, then the expression $a$ plus three equals eight.”

a. $a = 1$  
   b. $a = 2$  
   c. $a = 3$  
   d. $a = 10$

3. Think about when $3a$ equals an odd number and when $3a$ equals an even number.

   a. Write five values for $a$ that make $3a$ an odd number.

   b. Write five values for $a$ that make $3a$ an even number.

   c. Write five values for $a$ that make $3a$ a number that is not a whole number.

   **Hint:** You could use fractions, decimals, or negative numbers.

4. Think about when $a + 3$ equals an odd number and when $a + 3$ equals an even number.

   a. Write five values for $a$ that make $a + 3$ an odd number.

   b. Write five values for $a$ that make $a + 3$ an even number.

   c. Write five values for $a$ that make $a + 3$ a number that is not a whole number.

**Remember**

The term “whole number” means the counting numbers, including 0.
Preparing for the Closing

5. Think about this statement: For any number \( a \), \( 3a \) is an odd number.
   a. Is this statement always true, sometimes true, or never true?
   b. Justify your choice mathematically.

6. Think about this statement: For any number \( a \), \( a + 3 \) is an odd number.
   a. Is this statement always true, sometimes true, or never true?
   b. Justify your choice mathematically.

7. Think about this statement: For any number \( a \), \( 3a \) is always divisible by 3.
   a. Is this statement always true, sometimes true, or never true?
   b. Justify your choice mathematically.

Skills

Solve.

<table>
<thead>
<tr>
<th></th>
<th>a. ( 10 + 10 + 10 + 10 = )</th>
<th>b. ( 10 \times 4 = )</th>
<th>c. ( 5 + 5 + 5 + 5 = )</th>
<th>d. ( 5 \times 4 = )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>e. ( 7 + 7 = )</td>
<td>f. ( 7 \times 2 = )</td>
<td>g. ( 4 + 4 + 4 + 4 = )</td>
<td>h. ( 4 \times 4 = )</td>
</tr>
</tbody>
</table>

Comment

To “justify mathematically,” refer to the list of tools in Lesson 2 on page 9.
A table is a good way to organize data. When you have more than one value, you can use a table to show which values go together.

1. This table shows some values for $a$ and $3a$.

<table>
<thead>
<tr>
<th>$a$</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>12</th>
<th>15</th>
<th>120</th>
</tr>
</thead>
<tbody>
<tr>
<td>$3a$</td>
<td>12</td>
<td>15</td>
<td>18</td>
<td>36</td>
<td>45</td>
<td>360</td>
</tr>
</tbody>
</table>

a. Make a table like the one above that shows other values for $a$ and $3a$.
b. Does your table include all of the possible values for $a$ and $3a$? If it does, say how you know. If it does not, would it be possible to make a table that included all the possible values for $a$ and $3a$?

2. This table shows some values for $a$ and $a + 3$.

<table>
<thead>
<tr>
<th>$a$</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>12</th>
<th>15</th>
<th>120</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a + 3$</td>
<td>7</td>
<td>8</td>
<td>9</td>
<td>15</td>
<td>18</td>
<td>123</td>
</tr>
</tbody>
</table>

a. Make a table like the one above that shows other values for $a$ and $a + 3$.
b. Does your table include all of the possible values for $a$ and $a + 3$? If it does, say how you know. If it does not, would it be possible to make a table that included all the possible values for $a$ and $a + 3$?

3. If any number, $a$, is multiplied by 3, and then 3 is added to the result, the amount can be written as the expression $3a + 3$.

Calculate the value of $3a + 3$ for each value of $a$.

a. $a = 1$
b. $a = 2$
c. $a = 3$

4. Make a table showing values for $a$ and $3a + 3$. Choose any five values for $a$, and then calculate the value of $3a + 3$ for each value of $a$ that you chose.
5. Think about this statement: If $a$ is an odd number, then $3a$ is an odd number.
   a. Is this statement always true, sometimes true, or never true?
   b. Justify your choice mathematically.

6. Think about this statement: If $a$ is an odd number, then $a + 3$ is an odd number.
   a. Is this statement always true, sometimes true, or never true?
   b. Justify your choice mathematically.

7. Think about this statement: If $a$ is an odd number, then $3a + 3$ is an odd number.
   a. Is this statement always true, sometimes true, or never true?
   b. Justify your choice mathematically.

Homework

1. $2a$ is any number $a$ multiplied by the number 2.
   Calculate the value of $2a$ for each value of $a$.
   a. $a = 1$  b. $a = 5$  c. $a = 10$  d. $a = 100$  e. $a = 0$

2. $a + 2$ is the number 2 added to any number $a$.
   Calculate the value of $a + 2$ for each value of $a$.
   a. $a = 1$  b. $a = 5$  c. $a = 10$  d. $a = 100$  e. $a = 0$
CONVENTIONS FOR USING NUMBERS AND LETTERS

Multiplication

Each dog has four legs. Can you calculate how many legs there are if you know how many dogs there are?

3 • 4 legs

<table>
<thead>
<tr>
<th>Number of Dogs</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>n</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Legs</td>
<td>1 • 4 = 4</td>
<td>2 • 4 = 8</td>
<td>3 • 4 = 12</td>
<td>4 • 4 = 16</td>
<td>5 • 4 = 20</td>
<td>n • 4 = 4n</td>
</tr>
</tbody>
</table>

The expression, 4n, represents the number of legs when there are n dogs.

Conventions are agreed upon ways of writing mathematics.

Conventions for Multiplication

• When multiplying numbers, a “•” is often used instead of the symbol “×”
• When you multiply a letter by 1, you just write the letter.

Example

When you multiply 1 by n, you just write n instead of 1 × n, 1 • n, or 1n.
• In Lesson 3, you learned that when you multiply a letter by a number or when you multiply a letter by another letter, no multiplication symbol is used.

<table>
<thead>
<tr>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>If you multiply 4 by a, you write 4a instead of $4 \times a$, $4 \cdot a$, or $a4$. You read this as “four a.”</td>
</tr>
</tbody>
</table>
| If you multiply a by b, you write $ab$ instead of $a \times b$ or $a \cdot b$. You read this as “$ab$."

• When you multiply a letter by a number, the number is written first.

<table>
<thead>
<tr>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>If you multiply n by 3, you write $3n$ instead of $n3$. You read this as “three n.”</td>
</tr>
</tbody>
</table>

**Division**

Dwayne has a rope 36 yards long. He wants to cut the rope into pieces that are equal in length. Can you calculate how long the pieces are if you know into how many pieces he cuts the rope?

<table>
<thead>
<tr>
<th>Pieces of Rope ($n$)</th>
<th>3</th>
<th>4</th>
<th>8</th>
<th>9</th>
<th>18</th>
<th>$n$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length of Each Piece (yds)</td>
<td>$36 \div 3 = 12$</td>
<td>$36 \div 4 = 9$</td>
<td>$36 \div 8 = 4.5$</td>
<td>$36 \div 9 = 4$</td>
<td>$36 \div 18 = 2$</td>
<td>$36 \div n = \frac{36}{n}$</td>
</tr>
</tbody>
</table>

The expression, $\frac{36}{n}$, represents the length of each piece when $n$ pieces are cut.

**Conventions for Division**

• When division uses letters, the fraction bar is often used instead of the symbol “÷”

<table>
<thead>
<tr>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>If you divide 8 by $n$, you can write $\frac{8}{n}$ instead of $8 \div n$. This is read as, “eight over $n$” or “eight divided by $n$.”</td>
</tr>
<tr>
<td>If you divide $m$ by $n$, you can write $\frac{m}{n}$ instead of $m \div n$. This is read as, “$m$ over $n$” or “$m$ divided by $n$.”</td>
</tr>
</tbody>
</table>
Work Time

1. Rewrite each expression using the conventions for multiplication and division.
   a. \( a \times 5 \)  
   b. \( a \div 5 \)  
   c. \( 3 \div n \)  
   d. \( a \times b \)  
   e. \( 3 \times 4 \times a \)

2. a. Calculate the value of \( \frac{2m}{6n} \) if \( m = 6 \) and \( n = 2 \).
   
   b. Write an explanation of how you found the value.
   
   c. Share your explanation with your partner.

3. Calculate the value of these expressions if \( x = 5 \) and \( y = 7 \).
   When you do this, you are substituting the given values, or numbers, for the letters.
   a. \( 5y - xy \)  
   b. \( \frac{6y + 2}{2x + 1} \)

4. Dwayne and Jamal manage the student store. Erasers are sold for 21 cents each.
   a. What is the cost of 4 erasers? Write an explanation of how you know.
   b. Using conventions correctly, write a mathematical expression for the cost of \( n \) erasers.

5. The school store sells pencils by the box. There are 15 pencils in each box.
   a. How many pencils are in 6 boxes?
   b. How many pencils are in \( n \) boxes?

Comment
"Substitution" means to replace one thing with another.
6. The supply room for the store is full of spiders, and each spider has 8 legs. Dwayne warned Jamal not to go in the supply room because there were 56 spider legs in there.
   a. How many spiders were there?
   b. If there were $n$ spider legs, how many spiders were there?
   c. How would you read your answer to part b out loud?

Preparing for the Closing

7. In mathematics, it is a convention to write $4n$ instead of $4 \times n$ or $4 \cdot n$. This convention is used to avoid confusion.
   What might be confusing if you used the $\times$ symbol to indicate multiplication of numbers and letters?

8. Another convention is to write $n$ instead of $1 \cdot n$.
   Explain why you think this convention makes sense.

9. What might be confusing if you did not use $\cdot$ or $\times$ to represent multiplication of two whole numbers?

10. In Work Time problems 2 and 3, you replaced letters with numbers in the expressions. Explain why you think doing this is called substitution.

Skills

Solve.

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>$7 + 7 + 7 + 7 =$</td>
<td>b</td>
<td>$7 \cdot 4 =$</td>
</tr>
<tr>
<td>e</td>
<td>$11 + 11 =$</td>
<td>f</td>
<td>$2 \cdot 11 =$</td>
</tr>
<tr>
<td>d</td>
<td>$3 \cdot 8 =$</td>
<td>h</td>
<td>$x + x + x =$</td>
</tr>
</tbody>
</table>
1. **Example**

   What does $8n$ mean?
   
   $8n$ means “eight multiplied by $n$.”

   Suppose $n$ is 5,
   
   then $8n$ is $8 \cdot 5$, or 40.
   
   So, if $n = 5$, then $8n = 40$.

   a. Write “nine multiplied by $x$” as a mathematical expression.
   b. If $x = 6$, what does $9x$ equal?
   c. Write “$a$ multiplied by twenty-four” as a mathematical expression.
   d. If $a = 3$, what does $24a$ equal?

2. **Example**

   What does $xy$ mean?
   
   $xy$ means $x$ multiplied by $y$.

   Suppose $x$ is 2 and $y$ is 3,
   
   then $xy$ is $2 \cdot 3$, or 6.
   
   So, if $x = 2$ and $y = 3$, then $xy = 6$.

   a. Write “$a$ multiplied by $b$” as a mathematical expression.
   b. If $a = 16$ and $b = 3$, what does $a$ multiplied by $b$ equal?

3. **Example**

   What does $\frac{8}{n}$ mean?
   
   $\frac{8}{n}$ means 8 divided by $n$.

   Suppose $n$ is 4,
   
   then $\frac{8}{n}$ is $8 \div 4$, or 2.
   
   So, if $n = 4$, then $\frac{8}{n} = 2$.

   a. Express “twenty-four divided by $a$” using conventions.
   b. If $a = 6$, what does “24 divided by $a$” equal?
   c. Write “$b$ divided by $a$” as a mathematical expression.
4. **Example**

What does $\frac{n}{8}$ mean? Suppose $n$ is 4, then $\frac{n}{8}$ is $4 \div 8$, or $\frac{1}{2}$.

So, if $n = 4$, then $\frac{n}{8} = \frac{1}{2}$.

a. Write “$a$ divided by twenty-four” as a mathematical expression.

b. If $a = 6$, what does “$a$ divided by 24” equal?

c. Write “$a$ divided by $b$” as a mathematical expression.

5. Let $m = 18$ and $n = 9$.

a. Calculate the value of “$m$ divided by $n$.”

b. Calculate the value of “$n$ divided by $m$.”

### Homework

Remember to use conventions when you write your answers.

1. Rosa bought some pencils from Jamal in the student store. Each pencil cost 11 cents.
   a. How much did it cost Rosa to buy 7 pencils?
   b. How much would it cost Rosa to buy $n$ pencils?
      Write your answer without using either the $\times$ or $\cdot$ signs for multiplication.

2. Calculate the value of $5n$ for each value of $n$.
   a. $n = 100$
   b. $n = 10$
   c. $n = 1$
   d. $n = 0$

3. Calculate the value of $\frac{n}{2}$ for each value of $n$.
   a. $n = 100$
   b. $n = 10$
   c. $n = 1$
   d. $n = 0$

4. Jamal sells calculators in the school store.
   a. If Ms. Reynolds spent $120 on calculators, and each calculator, including tax, cost $8, how many calculators did Jamal sell?
   b. If Ms. Reynolds spent $120 on calculators, and each calculator cost $x$ dollars, how many calculators did Jamal sell?
      Write an expression to represent this number without using $\times$, $\cdot$, or $\div$ signs.
Dwayne and Keesha were asked to calculate the value of the expression $3 \cdot 8 \div 4 \cdot 2$.

Dwayne wrote this: $3 \cdot 8 \div 4 \cdot 2 = 24 \div 8 = 3$

Keesha wrote this: $3 \cdot 8 \div 4 \cdot 2 = 3 \cdot 2 \cdot 2 = 12$

How did Dwayne get his answer? How did Keesha get her answer? Who is correct?

Mathematicians often use parentheses to make the meaning of an expression clearer. Parentheses are used to group parts of an expression so they can be understood and treated as a single quantity.

Using parentheses in an expression can make the meaning more clear in both writing and speaking.

When reading a mathematical expression with parentheses out loud, whatever is inside the parentheses is referred to as a quantity.

Example

$$(3 \cdot 8) \div (4 \cdot 2)$$

告诉我们将 $3 \cdot 8$ 作为一数量 (24) 和 $4 \cdot 2$ 作为一数量 (8) 处理。

它被读作，“The quantity three times eight divided by the quantity four times two.”

$$(3 \cdot 8) \div (4 \cdot 2) = 24 \div 8 = 3$$

the quantity three times eight divided by the quantity four times two

$3 \cdot (8 \div 4) \cdot 2$ 告诉我们将 $(8 \div 4)$ 作为一数量 (2) 处理。

它被读作，“Three times the quantity eight divided by four, times two.”

The comma, indicated by a pause in speech, signals the end of the parentheses.

Practice saying this expression out loud:

$3 \cdot (8 \div 4) \cdot 2 = 3 \cdot 2 \cdot 2 = 12$
There are conventions for parentheses.

A letter or a number written directly next to a set of parentheses is multiplied by the quantity inside the parentheses.

**Example**

\[ 5(4 + 7) \text{ means “5 multiplied by the quantity } (4 + 7),” \text{ or } 5 \times 11. \]

\[ 3(a + b) \text{ means “3 multiplied by the quantity } (a + b). \]

It can also be read as “3 times the quantity } a + b.”

### Work Time

1. Take turns with a partner saying each expression out loud. Then calculate the value for each. Refer to whatever is inside the parentheses as “the quantity.”
   
   - a. \[ 3 + (4 \times 5) \]
   - b. \[ 5(3 + 4) \]
   - c. \[ 8(4 ÷ 1) + 4 \]
   - d. \[ (8 \times 1) ÷ (4 + 4) \]
   - e. \[ (12 + 3) ÷ (6 – 3) \]
   - f. \[ 12 + (6 ÷ 3) – 3 \]
   - g. \[ [(12 + 3) ÷ 6] – 3 \]

2. Use parentheses to rewrite each expression to give two different results. Calculate the value of each new expression.
   
   - a. \[ 3 + 5 \times 4 \]
   - b. \[ 2 \times 5 ÷ 2 + 4 \]

3. a. Use parentheses to rewrite this expression to give two different results.
   
   - a + b ÷ 5

   b. Substitute the values \( a = 10 \) and \( b = 5 \) into each expression you wrote. Calculate the value of each expression.
4. Use the conventions to write each of these as a mathematical expression.

Example

“The quantity eight divided by \( k \), minus five” can be written as: \( \frac{8}{k} - 5 \)

a. The quantity \( a \) divided by \( k \), minus \( b \).
b. The quantity eight minus five, divided by \( k \).
c. The quantity \( a \) minus two, divided by three.
d. The quantity \( a \) minus \( b \), divided by \( k \).
e. The quantity \( a \) multiplied by 3, plus the quantity \( c \) multiplied by 3.
f. The quantity \( a \) multiplied by \( b \), plus the quantity \( a \) multiplied by \( c \).

Preparing for the Closing

5. Compare the expressions you wrote in problem 2 with those written by other students.
   a. Did you group your expressions using parentheses in exactly the same way?
   b. Did you get the same values for your expressions?
   c. Did the parentheses make a difference in the values you calculated?

6. Think about this statement: For any numbers \( a \), \( b \), and \( c \), \((a + b)c = a + bc\).
   a. Is this statement always true, sometimes true, or never true?
   b. Give examples with numbers.
   c. Justify your choice mathematically.

   **Hint:** Reread the tools listed on page 9 to help with your justifications.

7. Think about this statement: For any numbers \( a \), \( b \), and \( c \), \((a + b) + c = a + (b + c)\).
   a. Is this statement always true, sometimes true, or never true?
   b. Give examples with numbers.
   c. Justify your choice mathematically.

8. Think about this statement: For any numbers \( a \), \( b \), and \( c \), \((a ÷ b) + c = a ÷ (b + c)\).
   a. Is this statement always true, sometimes true, or never true?
   b. Give examples with numbers.
   c. Justify your choice mathematically.
Lesson 5

Skills

Solve.

a. \( 6 + 8 = \)  

b. \( 8 + 6 = \)  

c. \( 125 + 13 = \)

d. \( 13 + 125 = \)

e. \( 6 \cdot 8 = \)  

f. \( 8 \cdot 6 = \)  

g. \( 12 \cdot 10 = \)  

h. \( 10 \cdot 12 = \)

How does problem a relate to problem b?

Review and Consolidation

1. Calculate the value of each expression.

   a. \((3 \cdot 6) ÷ (6 – 3)\)

   b. \(5(3 + 2) ÷ (2 \cdot 5)\)

2. a. Use parentheses and the • and ÷ symbols to rewrite this expression.

   \[
   \frac{5 \times 9}{3 \times 6}
   \]

   b. Calculate the value of the expression.

   c. Why is it appropriate to use the • symbol in this problem but not in \(\frac{5n}{3m}\)?

3. Write an expression for each statement. If necessary, use parentheses to make the meaning of the expression clear.

   a. An unknown number is multiplied by 5, and 3 is added to the result.

   b. The number 3 is added to an unknown number, and the result is multiplied by 5.

4. a. Using parentheses and a fraction bar, rewrite this expression in two ways that each give a different result.

   \(a ÷ b – 2\)

   b. Substitute the values \(a = 10\) and \(b = 5\) into each expression you wrote.

   Calculate the value of each expression.

5. A student using a calculator multiplied a number, \(n\), by 4 and then divided the result by 10. The number 400 appeared in the calculator display.

   Suppose the student had first divided the number, \(n\), by 10 and then multiplied the result by 4. What number would appear in the calculator display? Say how you know.
6. Lisa and Dwayne were throwing darts. Lisa got \( n \) points on her first throw. On her second throw, she got an additional three points. On her third throw, she hit the bull’s eye, which doubled her existing score.

Write an expression to show Lisa’s score after she hit the bull’s eye.

7. Rosa said that the perimeter of this rectangle can be represented by the expression \( 2h + 14 \).

Jamal said the perimeter is \( 2(h + 7) \).

Who is correct?
Justify your answer.
Lesson 5

Homework

1. Calculate the value of each expression.
   a. $5 \cdot 2 \div 10$
   b. $5(2 \div 10)$
   c. $(5 \cdot 2) \div (10 \cdot 2)$
   d. $\frac{5 + 3}{5 - 3}$
   e. $(5 + 3) \div (5 - 3)$

2. Using parentheses and a fraction bar, rewrite these expressions in two ways so that each way gives a different result. Then, substitute the values $a = 6$ and $b = 2$ into each expression you wrote. Calculate the value of each expression.
   a. $a \cdot b \div 4 + 8$
   b. $a \div 3 \cdot b - 1$
THE NUMBER PROPERTIES

CONCEPT BOOK

GOAL
To learn the number properties.

Arithmetic includes four operations: addition, subtraction, multiplication, and division.

These operations follow certain properties that are always true.

These properties will help you prepare for algebra.

Although you may not have learned their names, you have probably been using these properties for some time.

Example

Suppose you need to calculate the sum $12 + 23 + 8 + 7$.

You might begin by thinking, “$12 + 23$ is 35 and $35 + 8$ is 43,” and so on.

But there is an easier way to add these numbers.

You could rearrange the numbers: “$12 + 8$ is 20, and $23 + 7$ is 30

$30 + 20$ is 50”

The calculations would look like this:

$$12 + 23 + 8 + 7 = (12 + 8) + (23 + 7) = 20 + 30 = 50$$

By changing the order and grouping numbers that add easily, you have made the addition easier.

You have also used two of the number properties.

In order to study the number properties, you need to understand how equations work.

An equation can be true or false.
Lesson 6

Example
This is an equation: \(4 + 3 = 2 + 5\).
The equals sign acts like a balance.

An equation is a true statement when the left side is equal to the right side.

“\(4 + 3 = 2 + 5\)” is an equation because the left side and the right side both have a value of 7.

\[
\begin{align*}
4 + 3 &= 2 + 5 \\
7 &= 7
\end{align*}
\]

“\(4 + 3 = 7 + 3\)” is not true.
The left side has a value of 7.
The right side has a value of 10.

7 is not equal to 10.
You write \(7 \neq 10\).

Comment
The symbol “\(\neq\)" means “not equal.”

During Work Time, you will begin to identify the number properties.
In Preparing for the Closing, you will name and summarize these properties.

Work Time

1. Say whether each equation is true or false.
   a. \(8 + 0 = 8\)
   b. \(8 + 0 = 80\)
   c. \(8 + 0 = 0\)
   d. \(8 = 8 + 0\)
   e. \(24 + 0 = 24\)
   f. \(24 + 0 = 240\)
   g. \(a + 0 = a\)

2. When you add 0 to a number, what is the result?
3. Say whether each equation is true or false.
   a. 5 • 1 = 5
   b. 5 • 1 = 50
   c. 5 • 1 = 15
   d. 5 • 1 = 1
   e. 25 • 1 = 25
   f. 25 • 1 = 1
   g. a • 1 = a

4. When you multiply a number by 1, what is the result?

5. Say whether each equation is true or false.
   a. 5 + 1 = 1 + 5
   b. 9 + 3 = 3 + 9
   c. 5 – 1 = 1 – 5
   d. 9 – 3 = 3 – 9
   e. a + b = b + a
   f. a – b = b – a

6. When you add two numbers together, what happens to the sum if you change the order of the numbers?

7. When you subtract one number from another number, what happens to the difference if you change the order of the numbers?
8. Say whether each equation is true or false.
   a. $5 \cdot 1 = 1 \cdot 5$
   b. $9 \cdot 3 = 3 \cdot 9$
   c. $5 \div 1 = 1 \div 5$
   d. $9 \div 3 = 3 \div 9$
   e. $ab = ba$
   f. $a \div b = b \div a$

9. When you multiply two numbers together, what happens to the product if you change the order of the numbers?

10. When you divide one number by another, what happens to the quotient if you change the order of the numbers?

11. Say whether each equation is true or false.
   a. $(4 + 3) + 2 = 4 + (3 + 2)$
   b. $(5 + 3) + 7 = 5 + (3 + 7)$
   c. $(4 \cdot 3) \cdot 2 = 4 \cdot (3 \cdot 2)$
   d. $(5 \cdot 3) \cdot 7 = 5 \cdot (3 \cdot 7)$
   e. $(a + b) + c = a + (b + c)$
   f. $(a \cdot b) \cdot c = a \cdot (b \cdot c)$

12. When you add more than two numbers together, what happens to the sum if you group the numbers differently using parentheses?

13. When you multiply more than two numbers together, what happens to the product if you group the numbers differently using parentheses?
Preparing for the Closing

14. In this table are the first six number properties. These properties are always true for all numbers.

   In each property, the letters represent any numbers. You can also describe these properties in words. For example, the identity property of addition can be described as, “When you add zero to any number, you get that number.”

   a. Say each number property aloud to your partner using letters, as shown.
   b. Say each number property in words.

<table>
<thead>
<tr>
<th>Properties of Addition</th>
<th>Properties of Multiplication</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>The Identity Property of Addition</strong></td>
<td>a + 0 = a</td>
</tr>
<tr>
<td><strong>The Identity Property of Multiplication</strong></td>
<td>a • 1 = a</td>
</tr>
<tr>
<td><strong>The Commutative Property of Addition</strong></td>
<td>a + b = b + a</td>
</tr>
<tr>
<td><strong>The Commutative Property of Multiplication</strong></td>
<td>ab = ba</td>
</tr>
<tr>
<td><strong>The Associative Property of Addition</strong></td>
<td>(a + b) + c = a + (b + c)</td>
</tr>
<tr>
<td><strong>The Associative Property of Multiplication</strong></td>
<td>(ab)c = a(bc)</td>
</tr>
</tbody>
</table>

15. Addition and multiplication are commutative. Subtraction and division are not commutative. Say why. Give examples with numbers.

16. 0 is often called the additive identity. Why do you think this is true?

17. 1 is often called the multiplicative identity. Why do you think this is true?

Skills

Solve.

a. 20 • 10 =
   b. 2000 • 10 =
   c. 2000 • 100 =
   d. 2000 ÷ 100 =
   e. 20,000 ÷ 10 =

How does problem a relate to problem b?
Lesson 6

Review and Consolidation

1. Write an equation with numbers (no letters) that shows each property.
   a. Commutative Property of Multiplication
   b. Associative Property of Addition
   c. Identity Property of Addition
   d. Commutative Property of Addition
   e. Associative Property of Multiplication
   f. Identity Property of Multiplication

   Example
   Commutative Property of Multiplication
   \[ 2 \cdot 3 = 3 \cdot 2 \]

2. Dwayne said that the equation \(10 \div 20 = 20 \div 10\) is true because \(10 \div 20\) has the same value as \(20 \div 10\).
   Rosa said that the equation is false because \(10 \div 20\) does not have the same value as \(20 \div 10\).
   Who is correct? Say why.

3. Say whether each equation is true or false.
   For each false equation, explain why it is false.
   a. \(\frac{15}{3 + 2} = 15 \div (3 + 2)\)
   b. \(\frac{15}{3 + 2} = 15 \div 3 + 2\)
   c. \(\frac{15}{3} + 2 = 15 \div 3 + 2\)
   d. \(\frac{15}{3} + 2 = 15 \div (3 + 2)\)

Homework

1. Decide whether each equation is true or false.
   For each true equation, write the number property shown by the equation.
   a. \(0 + 6 = 6\)
   b. \(6 + 0 = 60\)
   c. \(3 \cdot 7 = 7 \cdot 3\)
   d. \(\frac{3}{7} = \frac{7}{3}\)
   e. \((3 + 9) + 8 = 3 + (9 + 8)\)
   f. \(8 \cdot 1 = 8\)
   g. \((2 \cdot 3) \cdot 4 = 2 \cdot (3 \cdot 4)\)
   h. \(5 + 9 = 9 + 5\)
   i. \(5 - 9 = 9 - 5\)
During Work Time, you will work in groups. Each group will make a poster for one topic that you have studied so far. Your group will be assigned one of these topics:

- Conventions for using numbers and letters
- The number properties

Follow these four steps to create your poster. Read all the steps before you start.

1. Discuss what should be included on the poster and how it should be presented.

2. Make a rough draft of the poster.

3. Present the draft to another group and get feedback about two aspects of your work:
   - Content
     - Does the poster include all the important information on the topic?
     - Does the poster include examples to show the important information?
   - Presentation
     - Does the poster present the information in a way that is organized as a useful reference for future work?
     - Is the poster easy to read and understand?

4. Use the feedback from the other group to make a final draft of the poster.

**Work Time**

1. Follow steps 1–4 above to create a poster to present to the class.
   Be sure to pay attention to the content and presentation.
Preparing for the Closing

2. You can use the number properties and conventions to justify statements. When you justify a statement, you are “saying why” the statement is true. In earlier lessons, you justified statements that were always true, sometimes true, and never true.

The tools you have so far for justifying statements include:

- Definitions
- The number properties, for example, the commutative property of multiplication
- A diagram of the situation that shows what a statement means
- Examples that show the statement is true
- Counterexamples that show the statement is not true

Which of the tools are you most likely to use to justify each of these choices?

a. The statement is always true.
b. The statement is sometimes true.
c. The statement is never true.

Skills

Solve each side of the equation, and then state whether the equation is true.

a. \(4 + 8 = 8 + 4\)  
b. \(8 - 4 = 4 - 8\)  
c. \(8 \cdot 4 = 4 \cdot 8\)  
d. \(8 \div 4 = 4 \div 8\)

How does problem a relate to problem b?

Review and Consolidation

1. Say whether each equation is true or false. For each equation that is true, write the number property shown by the equation.

a. \(2 + 6 = 3 + 4\)  
b. \(0 + 4 = 4\)  
c. \(1 \cdot 6 = 6\)

d. \(0 \cdot 8 = 8\)  
e. \(3 \cdot 8 = 8 \cdot 3\)  
f. \(7 - 2 = 2 - 7\)

g. \((3 + 7) + 12 = 3 + (7 + 12)\)  
h. \(8 \div 2 = 2 \div 8\)  
i. \((2 \cdot 8) \cdot 5 = 2 \cdot (8 \cdot 5)\)

2. Let \(x = 1\). What is the value of \(y\) if:

a. \(y = 2x\)  
b. \(y = 3x\)  
c. \(y = 8x\)
**3.** Let \( y = 2x \). What is the value of \( y \) if:

a. \( x = 1 \)

b. \( x = 5 \)

c. \( x = 15 \)

**4.** The area of a rectangle is defined as \( \text{Area} = \text{base} \cdot \text{height} \).

You can write this in letters as \( A = bh \).

Suppose you know that the area of a rectangle is 15 square centimeters, and its base is 3 centimeters.

You can write this: \( 15 = 3h \)

\((A = bh)\)

How can you find what \( h \) stands for?

Comment

The convention for writing square centimeters is cm\(^2\).

How can you find what \( h \) stands for? Explain how you know.

**5.** This table gives the area and base of several different rectangles.

Notice that all of the rectangles have an area of 12 square centimeters, but each has a different base.

<table>
<thead>
<tr>
<th>Area (cm(^2))</th>
<th>12</th>
<th>12</th>
<th>12</th>
<th>12</th>
<th>12</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td>Base (cm)</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>(n)</td>
</tr>
<tr>
<td>Height (cm)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

a. Copy the table and fill in the height for each rectangle.

b. What is the rule for finding the height of a rectangle if the area is 12 cm\(^2\) and the base is known? Give the rule in words or write an equation.

**6.** Lisa wanted to put 144 baseball cards into a book.

a. If each page held 16 cards, how many pages did Lisa need to buy?

b. How many pages did Lisa need to buy if each page held \( x \) cards?

Each page holds 16 cards.
Lesson 7

7. For each of these equations, write a set of values for \( q \) and \( r \) that make the equation false. Try fractions, decimals, negative numbers, and the numbers 0 and 1. If you find some values that make a statement true, write the specific conditions. (Which values make the statement true?)

   a. \( q \div r = r \div q \)
   b. \( q \div r < q \)
   c. \( qr = rq \)

Comment

The symbol “<” means “is less than.”

8. One of the statements in problem 7 is always true because it is an expression of a number property. Which number property is it?

Homework

1. Which number property guarantees that this equation is true for any number \( n \)?
   \[ 5n = n5 \]

2. In the equation given in problem 1, which expression uses mathematical conventions correctly, the one on the right side or the one on the left side of the equation?

3. Give an example with numbers to show why there is no number property that justifies this equation.
   \[ \frac{5}{n} = n \div 5 \]

4. In the equation given in problem 3, both expressions (left side and right side) use mathematical conventions correctly. Rewrite each expression using a division symbol instead of a fraction bar.
The base and the height are two important measurements of a rectangle. Sometimes, these measurements are called the length and the width.

The area of a rectangle is the product of its base and its height.

Example

A rectangle with a base of 3 cm and a height of 2 cm has an area of $3 \cdot 2 = 6$ square cm.

A rectangle with a base of 6 units and a height of 4 units has an area of $6 \cdot 4 = 24$ square units.

(Sometimes, the word “unit” is used if units are not indicated on the diagram.)

When a rectangle has a height of 2 and a base of 3, you can say this rectangle is “2 by 3.”

To find the area of a rectangle, a formula is used. A formula is a mathematical way of writing a rule for computing a value.

$$A = bh$$ is the formula for computing the area of a rectangle.

$b$ represents the base
$h$ represents the height
$A$ represents the area

How would you compute the area of a rectangle when $b = 4$ cm and $h = 2$ cm?

How would you compute the base of a rectangle when $A = 72$ square cm and $h = 6$ cm?
The *perimeter* of a figure, represented by $P$, is the distance around its outer edge. In other words, the perimeter is the sum of the lengths of all of the sides of a figure.

$$P = 2b + 2h$$ is the formula for computing the perimeter of a rectangle.

How would you compute the perimeter of a rectangle when $b = 4$ cm and $h = 2$ cm? How would you compute the base of a rectangle when $P = 36$ cm and $h = 6$ cm?

**Work Time**

1. **a.** Write a formula for the area of a rectangle with a base of 4 units and a height of $y$ units.
   
   **b.** Write a formula for the perimeter of the rectangle described in part a.

2. A rectangle measures $y$ centimeters by 4 centimeters.
   
   **a.** If $y = 7$ cm, what is the area of the rectangle?
   
   **b.** If $y = 7$ cm, what is the perimeter of the rectangle?

3. Calculate the area of a square that has side length, $s$, for each value of $s$.
   
   **a.** $s = 2$ units
   
   **b.** $s = 5$ units
   
   **c.** $s = 1$ unit
   
   **d.** Write a formula in which $A$ is the area of a square and $s$ is the measure of the side of a square.

4. Calculate the perimeter of a square that has side length, $s$, for each value of $s$.
   
   **a.** $s = 2$ units
   
   **b.** $s = 5$ units
   
   **c.** $s = 1$ unit
   
   **d.** Write a formula in which $P$ is the perimeter of a square and $s$ is the measure of the side of a square.

5. **a.** Sketch two rectangles that each have a height of 2 cm but different bases. Your sketches do not have to be perfect.
   
   **b.** Calculate the area of each rectangle using the formula $A = bh$.
   
   **c.** Calculate the sum of the areas of the two rectangles.
Using Letters in Formulas

6. a. Sketch two rectangles that have a base of 3 cm but different heights. Your sketches do not have to be perfect.
   
b. Calculate the area of each rectangle using the formula $A = bh$.
   
c. Calculate the sum of the areas of the two rectangles.

7. This table shows sets of values for three letters representing the measurements of a rectangle: $b$ represents the base, $h$ represents the height, and $A$ represents the area.

<table>
<thead>
<tr>
<th>$b$ (in)</th>
<th>3</th>
<th>6</th>
<th>4</th>
<th>12</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$h$ (in)</td>
<td>2</td>
<td>4</td>
<td>2</td>
<td>6</td>
<td>4</td>
</tr>
<tr>
<td>$A$ (in$^2$)</td>
<td>24</td>
<td></td>
<td></td>
<td>40</td>
<td>30</td>
</tr>
</tbody>
</table>

Calculate the missing values in the table. For each column with a missing value, list the set of three measurements that go together. For example, for the first column you would write $b = 3$, $h = 2$, $A = 6$.

Preparing for the Closing

8. Choose three different rectangles from the table above and sketch a diagram of each one. The diagrams do not need to be perfect.

9. Calculate the perimeters for the three rectangles you chose in problem 8.

10. Explain the difference between perimeter and area.

Skills

Solve.

a. $47 + 5 = $  
b. $47 + 10 = $  
c. $47 + 15 = $  
d. $47 + 20 = $  
e. $47 + 25 = $  

Do you see a pattern? Can you solve $47 + 30$ by following the pattern?

Review and Consolidation

1. Calculate the height of a rectangle with area $A = 28$ square units for each base, $b$.

a. $b = 4$ units  
b. $b = 14$ units  
c. $b = x$ units

2. The rule for calculating the area of a rectangle is $Area = base \cdot height$.

You can write this rule using the formula $A = bh$.

Explain in words how to calculate the length of the base if you are given the area and height of a rectangle.
3. Each of these rectangles has an area of 24 square units.

<table>
<thead>
<tr>
<th>1</th>
<th>24</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>8</td>
</tr>
<tr>
<td>2</td>
<td>12</td>
</tr>
</tbody>
</table>

a. Use the information given in these diagrams to make a table showing the height, base, and area of these four rectangles.

b. Write a general formula that will allow you to calculate the base of a rectangle, $b$, if you know the height, $h$, and that the area is 24 (square units).

4. The perimeter of a figure is the distance around its outer edge.

**Example**

For a rectangle with a base of 3 units and height of 2 units, the perimeter is $3 + 3 + 2 + 2 = 10$ units.

For a rectangle with a base of 6 units and a height of 4 units, the perimeter is $6 + 6 + 4 + 4 = 20$ units.

a. Write a description of how you would calculate the perimeter of a rectangle when $b = 4$ cm and $h = 2$ cm.

b. Calculate the perimeter of a rectangle when $b = 12$ cm and $h = 6$ cm.

c. Write a formula for the perimeter of a rectangle. Let $b$ stand for base and $h$ for height.

5. A rectangle measures $d$ centimeters by 6 centimeters.

a. Write a formula to represent the area of the rectangle.

b. Write a formula to represent the perimeter of the rectangle.
Using Letters in Formulas

1. Calculate the area of each of these rectangles. The dimensions are given in centimeters. When using numbers and letters together, follow the appropriate conventions.

   a. $2 \times 3$
   b. $2 \times 5$
   c. $2 \times 8$
   d. $h \times 10$
   e. $h \times 2$
   f. $h \times 12$

2. Calculate the length of the base for each of these rectangles. The dimensions are given in feet and square feet. Follow the conventions for using numbers and letters in mathematical expressions.

   a. $2 \times 6$
   b. $2 \times 10$
   c. $2 \times 16$
   d. $2 \times 24$
   e. $12h \times 24$
In Lesson 6, you learned three properties of addition and three properties of multiplication. In this lesson, you will use a property that involves both multiplication and addition.

It is called the distributive property of multiplication over addition, or simply the distributive property:

$$a(b + c) = ab + ac$$

The left side of the equation is the product of two quantities: $a$ and $(b + c)$.

The right side of the equation is $a$ multiplied by $b$ plus $a$ multiplied by $c$.

The distributive property tells you to “distribute” the multiplication by $a$ to each term inside the parentheses.

Example

$$3(5 + 7) = (3 \cdot 5) + (3 \cdot 7)$$

Is the equation true?

You can find out by calculating the value of each side of the equation separately to see if the sides are equal.

$$a(b + c) = ab + ac$$

$$3(5 + 7) = (3 \cdot 5) + (3 \cdot 7)$$

$$3 \cdot 12 = 15 + 21$$

$$36 = 36$$

The equation is true.
The distributive property is easy to illustrate with an area model.

Suppose you want to calculate the area of this rectangle.

\[
a \times (b + c)
\]

In this case, the height is \(a\) and the base is \(b + c\), so the area is \(a(b + c)\).

You can also calculate the area of the two smaller rectangles that together form the large one. The area of the rectangle on the left is \(ab\). The area of the rectangle on the right is \(ac\).

The area of the large rectangle must be equal to the sum of the areas of the smaller rectangles. So:

\[
a(b + c) = ab + ac
\]

This is the distributive property.
1. **a.** What is the height of this rectangle?

**b.** What are the bases of each of the smaller rectangles (that together form the large rectangle)?

**c.** Use the distributive property to calculate the area of the large rectangle in two different ways:

- Calculate the area of each smaller rectangle, then add those areas together.
- Calculate the area of the large rectangle using \((2 + 5)\) as the base.

2. The equation \(8(5 + 10) = (8 \cdot 5) + (8 \cdot 10)\) shows the distributive property.

**a.** Calculate the value of each side separately to prove that the equation is true.

**b.** Sketch an area model to show the equation by sketching a rectangle with a height of 8 units and a base of 5 + 10 units.

Number lines can also be used to show the distributive property.

**Example**

Look at each side of the equation: \(3(2 + 4) = (3 \cdot 2) + (3 \cdot 4)\)
3. Sketch number lines to represent this equation.

\[ 4(3 + 2) = (4 \cdot 3) + (4 \cdot 2) \]

In Lesson 2: Reasoning with Numbers, you used examples to help you decide if statements were true. You can substitute numbers for \( a, b, \) and \( c \) in the equation \( ab + ac \).

**Example**

If \( a = 2, b = 5, \) and \( c = 3 \):

\[
\begin{align*}
    &a(b + c) = ab + ac \\
    &2(5 + 3) = (2 \cdot 5) + (2 \cdot 3) \\
    &2 \cdot 8 = 10 + 6 \\
    &16 = 16
\end{align*}
\]

4. Substitute each set of numbers in the equation \( a(b + c) = ab + ac \), and calculate the value of each side separately to be sure that the equation is true.

   a. \( a = 1, b = 4, c = 10 \)  
   b. \( a = 2, b = 4, c = 10 \)  
   c. \( a = 3, b = 4, c = 10 \)

Preparing for the Closing

5. Do you think that you can distribute multiplication using subtraction? Say why or why not.

**Hint:** You can use examples with numbers to help you think about this question, such as:

\[
\begin{align*}
    &3(5 - 2) = (3 \cdot 5) - (3 \cdot 2) \\
    &8(10 - 5) = (8 \cdot 10) - (8 \cdot 5)
\end{align*}
\]

You can also use area models to represent these examples.

**Skills**

Solve.

a. \( 10 - 4 = \)  
   b. \( 6 + 4 = \)  
   c. \( 9 - 5 = \)  
   d. \( \_ + 5 = 9 \)

   e. \( 21 + 12 = \)  
   f. \( 33 - \_ = 12 \)  
   g. \( 36 - 14 = \)  
   h. \( 22 + 14 = \)

Do you see a pattern?
Lesson 9

Review and Consolidation

1. Substitute each set of numbers in the equation \( a(b + c) = ab + ac \), and calculate the value of each side separately to be sure that the equation is true.
   
   a. \( a = 5, \ b = 9, \ c = 4 \)   
   b. \( a = 5, \ b = 6, \ c = 8 \)   
   c. \( a = 8, \ b = 5, \ c = 6 \)

2. Sketch number lines that represent both sides of the following equation:
   \( 3(1 + 4) = (3 \cdot 1) + (3 \cdot 4) \)

3. Sketch a diagram of each rectangle, using the height and base given. Show the distributive property by calculating the area of each rectangle in two different ways.
   
   a. height = 5   
   base = 3 + 4   
   
   b. height = 4   
   base = x + 2

4. Use the distributive property to rewrite these expressions.
   
   a. 3(9 – 5)   
   b. 2(x + 8)

5. Use the distributive property to rewrite each expression as the product of a number or letter and a quantity in parentheses, as in this example.
   
   Example
   
   Start with the expression \( 2a + 2b \).
   
   The number 2 is multiplied by both \( a \) and \( b \).
   
   Using the distributive property, you can rewrite \( 2a + 2b \) as \( 2(a + b) \).
   
   a. \( 5a + 5b \)   
   b. \( (4 \cdot 2) + (4 \cdot 7) \)   
   c. \( 3x + 4x \)

6. In the next lesson, you will use the distributive property to help you multiply two-digit numbers.
   
   a. How could the distributive property help you multiply \( 9 \cdot 27 \)?
      Explain your thinking.
   
   b. How could the distributive property help you multiply \( 24 \cdot 13 \)?
      Explain your thinking.
1. Substitute each set of numbers in the equation \(a(b + c) = ab + ac\), and calculate the value of each side separately to be sure that the equation is true.
   a. \(a = 2, b = 5, c = 20\)  
   b. \(a = 7, b = 4, c = 1\)  
   c. \(a = 6, b = 12, c = 10\)

2. Use the distributive property to calculate the area of each rectangle in two different ways. Use conventions for using numbers and letters in part b. Remember that area is given in square units.
   a. \(\text{Area} = 10 \times (11 + 23)\)  
   b. \(\text{Area} = 10 \times (3 + x)\)
The distributive property can make it easier to multiply by a single-digit number.

**Example**

\[
7 \cdot 34 = 7(30 + 4) \quad \rightarrow \quad \text{Write 34 as 30 + 4. This is called } \text{expanded form}. \\
= (7 \cdot 30) + (7 \cdot 4) \quad \rightarrow \quad \text{Use the distributive property}. \\
= 210 + 28 \quad \rightarrow \quad \text{Multiply the products in your head}. \\
= 238 \quad \rightarrow \quad \text{Add.}
\]

With an area model, \(7 \cdot 34\) can be represented like this:

You can also use the distributive property to multiply a pair of two-digit numbers.

**Example**

\[
12 \cdot 13 = 12(10 + 3) \quad \rightarrow \quad \text{Write 13 in expanded form}. \\
= (12 \cdot 10) + (12 \cdot 3) \quad \rightarrow \quad \text{Use the distributive property}. \\
= (10 + 2)10 + (10 + 2)3 \quad \rightarrow \quad \text{Write 12 in expanded form}. \\
= (10 \cdot 10) + (2 \cdot 10) + (10 \cdot 3) + (2 \cdot 3) \quad \rightarrow \quad \text{Use the distributive property}. \\
= 100 + 20 + 30 + 6 \quad \rightarrow \quad \text{Multiply the products}. \\
= 156 \quad \rightarrow \quad \text{Add.}
\]
This area model shows how to use expanded form and the distributive property to multiply the pair of two-digit numbers, 12 • 13. The area of the large rectangle is equal to the sum of the areas of the four smaller rectangles.

\[
\begin{align*}
13 & \quad 12 \\
10 & \quad 10 \\
3 & \quad 2
\end{align*}
\]

156 square units

10 • 10 = 100
3 • 10 = 30
10 • 2 = 20
3 • 2 = 6

**Work Time**

1. Rosa used the distributive property to multiply 53 by 97.

   Use the number properties you have learned to justify each of Rosa’s steps. Four steps can be justified by simple arithmetic; in these cases, name the operation that justifies each step. For example, the final step is justified by addition.

   \[
   53 \cdot 97 = 53(90 + 7) \\
   = (53 \cdot 90) + (53 \cdot 7) \\
   = (90 \cdot 53) + (7 \cdot 53) \\
   = 90(50 + 3) + 7(50 + 3) \\
   = (90 \cdot 50) + (90 \cdot 3) + (7 \cdot 50) + (7 \cdot 3) \\
   = 4500 + 270 + 350 + 21 \\
   = 5141
   \]

   **Hint:** Look at the examples on the previous page.

2. Use the distributive property to calculate these products.
   
   a. 83 • 97 
   b. 53 • 38 
   c. 37 • 37

3. In problem 2, you solved 83 • 97 using the distributive property. Represent that problem using an area model.
4. Jamal was taught to multiply two-digit numbers, such as 48 • 37, using the model shown here.

Remember that in the number 48, 4 is in the tens place and has a value of 40, while 8 is in the ones place and has a value of 8.

Explain how this model is an application of the distributive property.

5. The distributive property is essential in doing algebra. This problem will help you see why.

Remember that \(a\) and \(b\) each represent any number.

a. Use this area model and what you have learned about the distributive property to help you multiply \((a + 3)(b + 4)\).

b. Write a description of what happens when you multiply \((a + 3)(b + 4)\).

Skills

Solve.

a. \(2 \cdot \square = 18\)  
   b. \(18 \div \square = 2\)  
   c. \(7 \cdot \square = 49\)  
   d. \(49 \div \square = 7\)  
   e. \(7 \cdot 9 = \square\)  
   f. \(54 \div 6 = \square\)  
   g. \(81 \div 9 = \square\)  
   h. \(54 \div 9 = \square\)  
   i. \(9 \cdot 9 = \square\)  
   j. \(63 \div \square = 7\)

Do you see a pattern?
Review and Consolidation

1. A student wrote: \( a(b + c) = ab + c \).
   a. What mistake did the student make?
   b. Sketch an area model to show why the student’s equation is not true.

2. Use the number properties you have learned to justify each step. Four steps can be justified by simple arithmetic; in these cases, name the operation that justifies each step. For example, the final step is justified by addition.

   \[
   19 \cdot 23 = 19(20 + 3) \\
   = (19 \cdot 20) + (19 \cdot 3) \\
   = (20 \cdot 19) + (3 \cdot 19) \\
   = 20(10 + 9) + 3(10 + 9) \\
   = (20 \cdot 10) + (20 \cdot 9) + (3 \cdot 10) + (3 \cdot 9) \\
   = 200 + 180 + 30 + 27 \\
   = 437
   \]

3. Use the distributive property to multiply \( 39 \cdot 64 \).

4. Look at the expression \( 9a + 9b \). The number 9 is multiplied by both \( a \) and \( b \). Using parentheses, you can write \( 9a + 9b \) as \( 9(a + b) \).

   Use parentheses to rewrite each expression in the same way as in the example above.
   a. \( 2x + 2y \)       b. \( 7a + 7b \)       c. \( 6x + 6 \)       d. \( 5y + 9y \)       e. \( ab + bc \)

5. Suppose you cannot remember the product of 8 and 9. How can you use the distributive property to help you multiply \( 8 \cdot 9 \)?

6. Sketch an area model to support your multiplication of \( 39 \cdot 64 \) in problem 3.

7. Multiply \((x + 6)(y + 7)\). Sketch an area model that represents this calculation.

Homework

1. Multiply using the distributive property.
   a. \( 29 \cdot 78 \)       b. \( 58 \cdot 31 \)

2. Sketch an area model to support your work in part a of problem 1.
The idea of an inverse is important when working with equations and the number properties.

**An inverse is an action or operation that gets you back to where you started.**

The inverse of an action undoes that action.

**Example**

Suppose you got out of bed. The inverse would be to go to bed.

Suppose you put on your shoes. The inverse would be to take off your shoes.

Suppose you deposited $5 in your savings account. The inverse would be to withdraw $5.

Not everything has a practical inverse.

**Example**

Suppose you spill some milk, which your cat then drinks.

You could not get back to the starting point.

The inverse of an operation gets you back to where you started.

**Example**

Suppose you add 3. The inverse would be to subtract 3.

Suppose you subtract 3. The inverse would be to add 3.

Suppose you divide by 3. The inverse would be to multiply by 3.

Suppose you multiply by 3. The inverse would be to divide by 3.

For example, suppose you begin with 2.

If you multiply 2 by 3, you get 6. (2 \* 3 = 6)

If you divide 6 by 3, you get 2, the number with which you started. (6 \div 3 = 2)

You can show this with a diagram.

Think about multiplying by 0. What is the inverse?
THE INVERSES OF ADDITION AND MULTIPLICATION

Work Time

1. a. What is the inverse of division?
   b. What is the inverse of addition?
   c. What is the inverse of multiplication?
   d. What is the inverse of subtraction?

2. Write the inverse, if it exists, of each of these actions or operations.
   a. Multiplying by 7
   b. Opening a door
   c. Subtracting $\frac{2}{3}$
   d. Dividing by 2
   e. Burning a log

3. What is the inverse of dividing by $\frac{1}{2}$?

4. What is the inverse of subtracting $\frac{1}{2}$?

5. $-a$ is called the additive inverse of $a$.
   $\frac{1}{a}$ is called the multiplicative inverse of $a$.

   a. What is the additive inverse of 2?
   b. What is the multiplicative inverse of 2?
   c. What is the additive inverse of $\frac{1}{2}$?
   d. What is the multiplicative inverse of $\frac{1}{2}$?
6. As you learned in Lesson 6: The Number Properties,
   • 0 is the additive identity.
   • 1 is the multiplicative identity.

The inverse properties of addition and multiplication allow you to get from any number, $a$, back to either the additive identity, 0, or the multiplicative identity, 1.

<table>
<thead>
<tr>
<th>The Inverse Property of Addition</th>
<th>The Inverse Property of Multiplication</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a - a = 0$ or $a + (-a) = 0$</td>
<td>$a \div a = 1$ or $a \cdot \frac{1}{a} = 1$</td>
</tr>
</tbody>
</table>

Write a description of the inverse properties of addition and multiplication.

7. Calculate the value of $a$ that makes each of these equations true.
   a. $5 + a = 0$
   b. $6 + a = 0$
   c. $7 + a = 0$
   d. $5 + a = 5$
   e. $6 + a = 6$
   f. $7 + a = 7$

8. Calculate the value of $a$ that makes each of these equations true.
   a. $5a = 1$
   b. $6a = 1$
   c. $7a = 1$
   d. $5a = 5$
   e. $6a = 6$
   f. $7a = 7$

Preparing for the Closing

9. Look at the posters your class created in Lesson 7 to present the number properties.
   Choose someone to add the inverse property of addition and the inverse property of multiplication to these posters.

10. How are the identity properties and the inverse properties related to each other?

Skills

Solve.

<table>
<thead>
<tr>
<th>a. 14 • 1 = □ • 7</th>
<th>b. 14 • 2 = □ • 7</th>
<th>c. 14 • 3 = □ • 7</th>
<th>d. 28 • 3 = □ • 7</th>
</tr>
</thead>
</table>

Do you see a pattern?
Review and Consolidation

1. What is the inverse of each of these operations?
   a. Dividing by 12
   b. Multiplying by $\frac{1}{5}$
   c. Adding $\frac{3}{4}$

2. What is the multiplicative inverse of 5?

3. What is the multiplicative inverse of $\frac{1}{5}$?

4. What is the additive inverse of 5?

5. What is the additive inverse of −5?

6. Write the value of $a$ that makes each of these equations true.
   a. $\frac{1}{5}a = 1$
   b. $\frac{1}{5}a = 5$
   c. $\frac{1}{5}a = 10$
   d. $\frac{1}{5}a = 20$
   e. $\frac{1}{5}a = 100$
   f. $5a = 100$

7. Write a description of how to calculate the values of $a$ in problem 6 using inverses.

Homework

1. Calculate the value of $x$ that makes each of these equations true.
   a. $2 + x = 0$
   b. $2 + x = 2$
   c. $\frac{1}{2} + x = 0$
   d. $\frac{1}{2} + x = \frac{1}{2}$
   e. $2 \cdot x = 2$
   f. $2 \cdot x = 1$
   g. $\frac{1}{2} \cdot x = \frac{1}{2}$
   h. $\frac{1}{2} \cdot x = 1$

2. What number is called the additive identity?

3. What number is called the multiplicative identity?

4. Look at the different equations in problem 1. In which of the equations was $x$:
   a. Equal to the additive identity?
   b. Equal to the multiplicative identity?
   c. An additive inverse?
   d. A multiplicative inverse?
PROGRESS CHECK

GOAL
To review definitions, conventions, the number properties, and how they help with mathematical reasoning.

CONCEPT BOOK
See pages 1–21 in your Concept Book.

Work Time

1. On a recent trip to the movies, Rosa paid $x$ dollars for her ticket. Then she bought some food. The food cost $3$ more than her ticket.
   a. Sketch a diagram to represent this situation. Label your diagram correctly with letters and numbers.
   b. Express the amount of money Rosa spent on food in terms of $x$.
   c. Express the total amount of money Rosa spent on food and her ticket in terms of $x$.

2. Write a formula for the area of each of these rectangles.
   a. \[
   \text{Area} = ab
   \]
   b. \[
   \text{Area} = a \times 7.2 \text{ cm}
   \]
   c. \[
   \text{Area} = ab + 7.2 \text{ cm}
   \]

3. Calculate the value of each expression when $n = 4$.
   a. \[
   3(5n - 2)
   \]
   b. \[
   \frac{n + 6}{2}
   \]
   c. \[
   \frac{n}{2} + 6
   \]

4. For problem 3, did your answer for part b equal your answer for part c? Say why or why not.

5. Write each of the expressions in problem 3 in words.
### PROGRESS CHECK

#### 6. Decide which of these equations are true
For each true equation, write the name of the number property you used to justify the true equations.

<table>
<thead>
<tr>
<th>Equation</th>
<th>property</th>
</tr>
</thead>
<tbody>
<tr>
<td>a. $5x \cdot 1 = 5x$</td>
<td>( b. x + 4 = 4 + x )</td>
</tr>
<tr>
<td>d. $3(x + y) = (x + y)3$</td>
<td>e. $5\left(\frac{1}{5}\right) = 1$</td>
</tr>
<tr>
<td>g. $x - x = 1$</td>
<td>h. $x - x = 0$</td>
</tr>
<tr>
<td>j. $12 + 0 = 12$</td>
<td>k. $5(3x) = (5 \cdot 3)x$</td>
</tr>
</tbody>
</table>

#### 7. Sketch an area model to show the equation $3(x + y) = 3x + 3y$.
The height of your rectangle should be 3 and the base should be $x + y$.

#### 8. Show that the equation $3(x + y) = 3x + 3y$ is true for each pair of values of $x$ and $y$ by substituting the values into the right and left sides of the equation and solving.

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
<th>a. $x = 10$, $y = 7$</th>
<th>b. $x = 21$, $y = 4$</th>
<th>c. $x = \frac{1}{3}$, $y = 3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>a. $x = 10$, $y = 7$</td>
<td>b. $x = 21$, $y = 4$</td>
<td>c. $x = \frac{1}{3}$, $y = 3$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### Skills

#### Solve.

<table>
<thead>
<tr>
<th>Equation</th>
<th>Answer</th>
</tr>
</thead>
<tbody>
<tr>
<td>a. $14 \cdot 12 =$</td>
<td>b. $38 \cdot 12 =$</td>
</tr>
</tbody>
</table>

**Hint:** Try expanding one of the numbers and using the distributive property.
Lesson 12

Review and Consolidation

Refer to equations a–h for problems 1–3.

a. \(2(a + b) = 2a + 2b\)  
b. \(xy = y + x\)  

c. \(3 + (4 + x) = x(3 + 4)\)  
d. \(a(3 + b) = 3a + ab\)  

e. \(2(xy) = 2x \cdot y\)  
f. \(10 - 10 = 1\)  

g. \(\frac{1}{a} \cdot a = 1\)  
h. \(x \cdot 1 = x\)  

1. For each true equation, state which number property justifies the equation.

2. Look at each false equation. Modify each false equation to make it true.

   Comment
   There is more than one way to modify the equations to make them true.

3. For each false equation you modified in problem 2, state which number property justifies the true equation you wrote.

Homework

1. a. Review the problems in this lesson and find three that were difficult for you.

   b. Use your Concept Book and previous lessons to work through these three problems more carefully until you understand them.

   If you still have questions about the problems, be prepared to ask them during the next lesson.

2. Write a summary page of the concepts you have learned in Lessons 1 through 11.
1. Translate each of these eight expressions into words.

**Example**

\[
\begin{align*}
2n + 12 & \quad \text{add 12} \\
\overbrace{2n} \quad \text{multiply } n \text{ by 2} \\
\overbrace{2n + 12} &
\end{align*}
\]

In words: “Multiply \( n \) by 2 and then add 12”

a. \( \frac{n + 4}{5} \)  
   b. \( \left( \frac{n}{5} \right) + 4 \)  
   c. \( 5n + 20 \)  
   d. \( 5(n + 4) \)

e. \( 3(5n - 2) \)  
   f. \( n(m + 3) \)  
   g. \( m - \left( \frac{n}{2} \right) \)  
   h. \( m - 2n \)

2. Calculate the value of each expression in problem 1 when \( n = 5 \).

**Example**

\[
\begin{align*}
n(m + 3) = 5(m + 3) = 5m + 15
\end{align*}
\]

3. Write an expression for each of these:
   a. “double \( n \)”
   b. “one half of \( n \)”
   c. “3 less than \( n \)”

4. For any number \( x \), \( \frac{x}{3} \) is the same as \( \frac{1}{3}x \). What number properties and conventions could you use to say why this statement is true?
5. The perimeter of a figure is the total length around its outer edge. In other words, it is the sum of the lengths of the sides.

Use the conventions to write an expression for the perimeter of this hexagon.

6. Make a table like this.

<table>
<thead>
<tr>
<th></th>
<th>Rectangle 1</th>
<th>Rectangle 2</th>
<th>Rectangle 3</th>
<th>Rectangle 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>( h ) (height)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( b ) (base)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( A ) (area)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Fill in the columns of the table by choosing your own values for the base and height of four rectangles. Determine the area of each rectangle for the values you chose.

7. The length of a rope is \( n \) meters. A pipe is 3 times as long as the rope.

\[ n \]

\[ \text{Pipe length} = 3n \]

a. Use the conventions to write the length of the pipe in terms of the length of the rope.

b. If the pipe is 27 meters long, how long is the rope?

8. Write an expression for the price, in dollars, of a movie ticket that is $5 off, if the original price is represented by \( y \) dollars.

Preparing for the Closing

9. Dwayne said that \( \frac{n + 8}{4} \) is the same as \( \frac{n}{4} + 8 \). Keesha said they are different. Who is correct?

Explain your thinking.
10. Is this equation true for any number $n$?

$$\frac{1}{4} (n + 8) = \frac{n}{4} + 2$$

Test the equation using several different values of $n$. Then say why the equation is always true, sometimes true, or never true. Use the number properties in your justification.

11. a. Are any expressions in problem 1 equivalent to each other when $n = 5$? Which ones?

b. Explain why you think this is true.

**Definition**

Equivalent means they have the same value.

---

**Skills**

Solve by rewriting each equation with the missing sign ($+, -, \cdot, \div$)

a. $60 \cdot 12 = 72$

b. $60 \cdot 12 = 720$

c. $60 \cdot 12 = 5$

d. $60 \cdot 12 = 48$

e. $450 \cdot 10 = 440$

f. $550 \cdot 10 = 55$

g. $2500 \cdot 100 = 25$

h. $250 \cdot 10 = 2500$

---

**Review and Consolidation**

1. For each of these equations:

   - Determine which equations are not true.
   - Correct the equations that are not true.
   - Give the number property that each equation represents.

a. $9 + 0 = 0$

   b. $9 - 9 = 9$

c. $9 \cdot \frac{1}{9} = 9$

   d. $9 \cdot 1 = 1$

e. $9 + (3 + 10) = (9 + 3) + (9 + 10)$

   f. $9(3 \cdot 10) = (9 \cdot 3) + (9 \cdot 10)$

g. $9 + 3 = 3 - 9$

   h. $9 \cdot 3 = 3 \div 9$

**Hint:** There is more than one way to correct each equation. Try not to use the same number property twice.
Lesson 13

2. Substitute each set of values for $a$, $b$, and $c$ in the equation $a(b + c) = ab + ac$. Calculate the value of each side to prove that each equation is true.

a. $a = 10$, $b = \frac{1}{2}$, $c = 1$

b. $a = 2$, $b = x$, $c = 6$

3. Sketch an area model that represents the distributive property for each equation in problem 2.

Your area models do not need to be perfect. Just sketch the rectangles with the dimensions given. Be sure to label the base and the height of each rectangle.

4. This rectangle has an area of $36$ cm$^2$.

This means that the product of the base, $b$, and the height, $h$, is $36$ square centimeters.

In other words, $bh = 36$ cm$^2$.

Make a table like this one and fill in four more possible pairs of values for the base, $b$, and height, $h$, of this rectangle. Remember that the area in each case is $36$ cm$^2$.

<table>
<thead>
<tr>
<th>$b$ (cm)</th>
<th>1</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$h$ (cm)</td>
<td>36</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

5. Write a formula for the base length of the rectangle in problem 4 in terms of $h$.

6. Write a formula for the height of the rectangle in problem 4 in terms of $b$. 

64  Foundations of Algebra
Homework

Use this rectangle for problems 1–3.

1. Write a formula for the area of the rectangle in terms of $n$.

2. Calculate the area of the rectangle for each value of $n$.
   a. $n = 1$ meter
   b. $n = 5$ meters
   c. $n = 10$ meters
   d. $n = 0.5$ meter
   e. $n = (5 + m)$ meters

3. Create a table with a row for the height, $n$, and a row for the area, $A$, of the rectangle. Fill in your table with the values from problem 2. Remember, the base in each case is 6 meters.

<table>
<thead>
<tr>
<th>$n$ (meters)</th>
<th>1</th>
<th>5</th>
<th>10</th>
<th>0.5</th>
<th>$m + 5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A$ (meters$^2$)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
A formula defines a relationship between specific quantities according to a rule. A quantity is an amount that can be counted or measured.

**Example**

- \( A = bh \) defines the relationship between the area, base, and height of a rectangle.
- \( P = 2(b + h) \) defines the relationship between the perimeter, base, and height of a rectangle.

Think about the relationship between the quantities in this situation:

Suppose you know that Lisa is 63 inches tall. With shoes on, Lisa will be taller; but how much taller? That depends on how tall her shoes are.

In this situation, there are two quantities that vary (or change) in relation to each other:

Quantity 1: Lisa’s overall height (with shoes on)

Quantity 2: The height of Lisa’s shoes

As the height of Lisa’s shoes (Quantity 2) increases, Lisa’s height (Quantity 1) increases.

As the height of Lisa’s shoes (Quantity 2) decreases, Lisa’s height (Quantity 1) decreases.

As shoe height (Quantity 2) changes, so does Lisa’s height (Quantity 1).
Using \( h \) to stand for Lisa’s overall height and using \( s \) to stand for the height of her shoes, you can write a formula that defines the relationship between these two quantities: \( h = 63 + s \)

If you know that Lisa’s shoes are 1 inch tall, then her overall height must be 64 inches.

What is her overall height if she wears shoes that are 2 inches tall?

Notice that Lisa’s overall height, \( h \), varies (or changes) in relation to the height of her shoes, \( s \), which also varies. What does it mean for a quantity to vary?

A quantity that varies is called a variable quantity.

**Work Time**

1. This table shows values for some of the pairs of variable quantities in the situation involving Lisa and her shoes. Copy the table and fill in the missing values.

<table>
<thead>
<tr>
<th>Variable Quantities (represented by letters)</th>
<th>Different Values for Each Quantity</th>
</tr>
</thead>
<tbody>
<tr>
<td>( s )</td>
<td>0</td>
</tr>
<tr>
<td>( h )</td>
<td>63</td>
</tr>
</tbody>
</table>

These diagrams show cups. Refer to these diagrams for problems 2–4.

The diagram on the left shows a cup with a 1-cm lip. The rest of the cup is 7 cm tall. The diagram on the right shows five cups stacked together.
2. There are two quantities that vary in this situation:
   • Quantity 1: the height of the stack of cups
   • Quantity 2: the number of cups in the stack

   a. Do these quantities vary in relation to each other?
      In other words, as the value of one increases or decreases, does the value of the other also increase or decrease?

   b. How is the relationship between these quantities similar to the relationship between Lisa’s overall height and the height of her shoes discussed at the beginning of the lesson?

3. This table shows some pairs of values for the two quantities that vary in this situation.

<table>
<thead>
<tr>
<th>Number of Cups in Stack</th>
<th>1</th>
<th>5</th>
<th>9</th>
<th>10</th>
<th>25</th>
<th>50</th>
</tr>
</thead>
<tbody>
<tr>
<td>Height of Stack (cm)</td>
<td>8</td>
<td>12</td>
<td>16</td>
<td>17</td>
<td>32</td>
<td>57</td>
</tr>
</tbody>
</table>

   a. What is the overall height if there are 5 cups in the stack?
   b. What is the overall height if there are 10 cups in the stack?
   c. How much does the overall height of the stack increase as each cup is added?
   d. How many cups are in the stack if the overall height of the stack is 57 cm?
   e. How many cups are in the stack if the overall height of the stack is 8 cm?
   f. Choose three more values for the number of cups in the stack and give the corresponding values for the height of the stack.

4. a. Look at the diagram of cups on the previous page and the table in problem 3. Describe in words the rule for finding the height of the stack.
   b. Choose letters to represent the height of the stack and the number of cups.
   c. Use the letters you chose in part b to translate your written description from part a into a formula. This formula will define the relationship between these two quantities.
Preparation for the Closing

5. How does a table help you represent a relationship between quantities?

6. Compare the values you came up with in problem 3f that could be included in the table with those of your classmates.
   a. Could all of the values your class came up with be included in the same table?
   b. Is it possible to make a table with all of the possible values for each quantity in this situation?

7. How does a formula help you represent a relationship between quantities?

Skills

Solve.

a. \(47 - 5 = \)
   b. \(47 - 10 = \)
   c. \(47 - 15 = \)
   d. \(47 - 20 = \)
   e. \(47 - 25 = \)

Do you see a pattern? Can you solve \(47 - 30\) by following the pattern?

Review and Consolidation

1. Each dog has four legs.

<table>
<thead>
<tr>
<th>Number of Dogs</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Legs</td>
<td>4</td>
<td>8</td>
<td>12</td>
<td>16</td>
<td>20</td>
</tr>
</tbody>
</table>

a. What are the two quantities in this situation?

b. Do the quantities vary in relation to each other? In other words, as one quantity increases or decreases, does the other quantity also increase or decrease?

c. Think of three more values for the number of dogs that could be included in the table. For each value for the number of dogs, identify the corresponding value for the number of legs.
d. Which of these descriptions accurately describes the relationship between the two quantities in this situation?

A. The number of dogs is equal to the number of legs.
B. The number of dogs is equal to twice the number of legs.
C. The number of dogs is equal to four times the number of legs.
D. The number of legs is equal to four times the number of dogs.
E. The number of legs is equal to the number of dogs plus three.

e. Which of these formulas (where \( d \) stands for the number of dogs and \( l \) stands for the number of legs) gives the number of legs in terms of the number of dogs?

A. \( d = l \)  
B. \( d = 2l \)  
C. \( d = 4l \)  
D. \( l = 4d \)  
E. \( l = d + 3 \)

2. Rosa’s mother sent a copy of Rosa’s school picture to her friends and relatives. School pictures are ordered in sheets with 8 pictures on 1 sheet.

a. How many sheets are shown in the diagram above?

b. How many pictures are shown in the diagram above?

c. Make a table showing four possible pairs of values for the number of sheets and the number of pictures.

d. Write a formula for the relationship between the number of sheets and the number of pictures.
3. The length of a shoelace needed for a shoe or boot depends on the number of eyelets that the shoe has. (Eyelets are the little holes that the laces go through.) For any shoe, you need a total of 7 inches on each end of the lace to tie a bow plus 1 extra inch for each eyelet. Sketch a diagram to help you understand this situation better.

a. What are the two quantities that vary in this situation?

b. What is the length of the shoelace if the shoe has 2 eyelets?

c. What is the length of the shoelace if the shoe has 6 eyelets?

d. What is the length of the shoelace if the shoe has 16 eyelets?

e. How many eyelets does the shoe have if the shoelace is 20 inches long?

f. How many eyelets does the shoe have if the shoelace is 36 inches long?

g. Make a table showing the values for each quantity that you found in parts b–f.

h. Write a formula for the length of shoelace needed in terms of the number of eyelets.

**Homework**

In Lesson 4: *Conventions for Using Numbers and Letters*, Dwayne cut 36 yards of rope into equal lengths? Look at page 18 to review the situation.

The length of each piece of rope depends on the number of pieces into which Dwayne cut the original 36-yard rope.

The formula that defines the relationship between these quantities is: \( l = \frac{36}{n} \)

In words, the length of each piece of rope, \( l \), is equal to 36 divided by the number of pieces, \( n \).

<table>
<thead>
<tr>
<th>Number of Rope Pieces (( n ))</th>
<th>3</th>
<th>4</th>
<th>8</th>
<th>9</th>
<th>18</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length of Each Piece (( l ), in yards)</td>
<td>12</td>
<td>9</td>
<td>4.5</td>
<td>4</td>
<td>2</td>
</tr>
</tbody>
</table>

1. Make a table like the one above, but without any values filled in.

2. Fill the top row of your table with different values for the number of pieces cut.

3. Fill the bottom row of your table with the corresponding values for the length of each piece of rope.
Lesson 15
Using Graphs to Represent Relationships

**Goal**
To represent the relationship between two quantities as a graph.

**Concept Book**
See pages 199–209 in your Concept Book.

To make a graph, you need a vertical axis and a horizontal axis. These axes intersect to form a right angle.

Each axis is like a number line.

Intervals of equal length are marked off by tick marks.

A graph is an excellent way to represent data from a table.

Look at the table in this example. It shows the relationship between two quantities.

### Example
Quantity 1: Number of Cans of Cat Food
Quantity 2: Total Cost of the Cans in Dollars

<table>
<thead>
<tr>
<th>Number of Cans</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Total Cost</strong> (dollars)</td>
<td>$0.00</td>
<td>$0.40</td>
<td>$0.80</td>
<td>$1.20</td>
<td>$1.60</td>
</tr>
</tbody>
</table>

Data from this table can be represented by a graph. The pairs of values from the table that are the two quantities are plotted, or placed, as points on the graph. Each pair of values that defines a point is called an ordered pair.

### Example
Here are some ordered pairs:

(0, 0) (1, 0.40) (2, 0.80) (3, 1.20) (4, 1.60)

When plotting, the first value is graphed using numbers on the horizontal axis. The second value is graphed using numbers on the vertical axis.

The graph “Cost of Cat Food” shows that the total cost increases 40 cents for each additional can of cat food.
Work Time

In the previous lesson, *Relationships between Quantities*, you solved problems involving a stack of cups. Use the table from that situation (repeated below) for problems 1 and 2.

<table>
<thead>
<tr>
<th>Number of Cups in Stack</th>
<th>1</th>
<th>5</th>
<th>9</th>
<th>10</th>
<th>25</th>
<th>50</th>
</tr>
</thead>
<tbody>
<tr>
<td>Height of Stack (cm)</td>
<td>8</td>
<td>12</td>
<td>16</td>
<td>17</td>
<td>32</td>
<td>57</td>
</tr>
</tbody>
</table>

When you represent a pair of values from a table as a point on a graph, there is a convention for giving the coordinates of the point in the form of ordered pairs \((x, y)\). Using this convention,

- \(x\) stands for the value that gives the horizontal position of the point.
- \(y\) stands for the value that gives the vertical position of the point.
- \((\text{Number of Cups in Stack}, \text{Height of Stack}) = (x, y)\)

**Example**
The pair of values 9 cups and 16 cm high is represented like this: \((9, 16)\).

1. Rewrite each pair of values from the table into ordered pairs in the form \((x, y)\).

2. This graph represents the same stack-of-cups situation as the table. The graph displays points that represent three pairs of values from the table used for problem 1.

   Give the ordered pair of values that corresponds to each point on the graph.
3. Use Handout 1: *Graph Paper*, which has several sets of horizontal and vertical axes, or make your own graphs like the one given in problem 2. Plot the points from these tables on two separate sets of axes.

<table>
<thead>
<tr>
<th>Number of Dogs</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Legs</td>
<td>4</td>
<td>8</td>
<td>12</td>
<td>16</td>
<td>20</td>
</tr>
</tbody>
</table>

b.  

<table>
<thead>
<tr>
<th>Number of Pieces of Rope Cut ((n))</th>
<th>3</th>
<th>4</th>
<th>8</th>
<th>9</th>
<th>18</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length of Each Piece of Rope ((l, \text{ in yards}))</td>
<td>12</td>
<td>9</td>
<td>4.5</td>
<td>4</td>
<td>2</td>
</tr>
</tbody>
</table>

Preparing for the Closing

4. Look at the graphs you made in problem 3.
   a. Label the two axes for each graph with the appropriate quantity. Follow the examples from the beginning of this lesson and from problem 2. Use the description for each quantity given in the tables as your labels.
   b. Make sure that each point on both graphs is correct by checking that each pair of values in the tables corresponds to one point plotted on the graph. Check that each point is in the correct position.
   c. Label each point with ordered pairs. Plot one additional point on each graph that represents another possible pair of values for the quantities in each table. Label each with an ordered pair.
   d. In one of the relationships represented by your graphs, as one quantity increases, the other quantity also increases. In the other relationship, as one quantity increases, the other quantity decreases. Say how this is shown on the graphs.

5. Think about the values in the tables and on the graphs you have seen so far.
   a. In which situations do decimals or fractions make sense for the quantities represented?
   b. In which situations do decimals and fractions not make sense for the quantities represented? Why not?
   c. Where would pieces of rope with lengths between 9 and 10 yards be on the graph?
   d. Where would pieces of rope with lengths between 0 and 1 yard be on the graph?
**Using Graphs to Represent Relationships**

**Skills**

Solve.

a. \(239 - 5 = \)  
   b. \(239 - 10 = \)  
   c. \(239 - 15 = \)  
   d. \(239 - 20 = \)  
   e. \(239 - 25 = \)

Do you see a pattern? Can you solve \(239 - 30\) by following the pattern?

**Review and Consolidation**

Think about a stack of cups with dimensions different from those described on page 67. The lip is 1 cm, as before, but the rest of each cup is 5 cm (instead of 7 cm) tall.

1. a. Sketch a diagram of one cup with these new dimensions.
   b. Sketch a diagram of a stack of three of these cups.

2. The two quantities that vary in relation to each other in this situation are:
   - The number of cups in the stack
   - The height of the stack of cups

   a. What is the height of a stack of 2 of these cups?
   b. What is the height of a stack of 5 of these cups?
   c. What is the height of a stack of 10 of these cups?

3. Make a table showing five pairs of values for the two quantities that vary in this situation. You may include the values you found in problem 2 or find all new values to include in your table.

4. Represent each pair of values shown in your table using the convention \((x, y)\).
   - Let \(x\) stand for the numbers of cups.
   - Let \(y\) stand for the heights of the stacks.

Remember that \(x\) stands for the value that gives the horizontal position of the point, and \(y\) stands for the value of the vertical position of the point.
5. Label the x- and y-axes of your graph with the appropriate quantity. Plot and label the points from your table on your graph. (You can use Handout 1 from Work Time.)

6. Compare your graph to the graph given for Work Time problem 2.
   a. How are the two graphs similar?
   b. How are the two graphs different?
   c. How are the cups in Work Time problem 2 like the cups used here?
   d. How are the cups in Work Time problem 2 different from the cups used here?

**Homework**

1. Look at the graph and write the ordered pairs \((x, y)\) for each point.

2. Which relationship between the quantities could be represented by the graph in problem 1?
   - A Each dog has four legs.
   - B Each spider has eight legs.
   - C A rope is cut into four parts.
   - D A shoelace is 4 inches long.

3. Say why your choice makes sense.
A word problem describes a situation and then asks questions about the situation. The first step in solving a word problem is to understand the situation.

You will not solve problems in this lesson. Instead, you will focus on understanding a situation described in a problem.

Dwayne needs to buy a phone. There are two plans he is thinking about:

**PLAN A**
$25.00 a month plus 10 cents a minute
Two-year minimum contract

**PLAN B**
$30.00 a month plus 5 cents a minute
One-year minimum contract

There are many numbers in this situation. Each number is part of a quantity. Remember, a quantity is an amount that can be counted or measured.

Dwayne identified these quantities in Plan A:

25 dollars, 10 cents, and 2 years.

Notice that each quantity has a number and a word.

The word tells you what is being counted or measured (dollars, cents, or years). In other situations, the words might be apples, inches, or liters. This word is called a *unit*. Sometimes units are abbreviated.

**Examples**

cm (centimeter), mL (milliliter), $ (dollar), and lb (pound)
1. Identify the quantities in Plan B.
   Be sure to give the numbers and units, as Dwayne did for Plan A.

2. Dwayne sketched this diagram to represent Plan A:

   \[
   \begin{align*}
   \$25 & \quad + \quad 10\text{¢} + 10\text{¢} + 10\text{¢} + 10\text{¢} \quad \cdots \quad 10\text{¢} \\
   \text{per month} & \quad \text{per minute}
   \end{align*}
   \]

   Jamal sketched this diagram:

   Lisa sketched this diagram:

   a. Notice that not all of the quantities are shown in all of the diagrams.
      Which quantities are shown in each diagram?

   b. Which quantities are not shown in each diagram?

   c. Use one of the diagrams as a model to sketch a diagram to represent Plan B.

   d. Find a student who sketched a different type of diagram from yours.
      Say how each quantity is shown in the two different diagrams.

3. Keesha decided to make a table representing Plan A.

<table>
<thead>
<tr>
<th>Number of Minutes</th>
<th>0</th>
<th>1</th>
<th>10</th>
<th>100</th>
<th>300</th>
<th>600</th>
<th>1000</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total Cost per Month</td>
<td>$25.00</td>
<td>$25.10</td>
<td>$26.00</td>
<td>$35.00</td>
<td>$55.00</td>
<td>$85.00</td>
<td>$125.00</td>
</tr>
</tbody>
</table>

   Make a table like Keesha’s showing the relationship between the number of minutes and the total cost for one month for Plan B. Include at least three pairs of values.
Preparing for the Closing

4. Look at the quantities you identified for Plan B in problem 1. Are there any other quantities to add to your list? Think about the quantities you represented in your table for problem 3.

5. Dwayne looked at the diagrams and tables and decided to express the relationship between quantities for Plan A in words:

   “The total cost is (twenty-five dollars times the number of months) plus (ten cents times the total number of minutes).”

Write the relationship between quantities for Plan B in words.

   Simplify what he wrote in words as a mathematical expression using any mathematical tools you know:
   • Letters that stand for numbers
   • Conventions for using numbers and letters
   • Equations
     (Remember, an equation is a statement that two amounts are equal)
   • The number properties

7. How would a graph be helpful for representing the relationship between quantities in Plans A and B?

Skills

Solve.

  a. 2724 – 5 =  
  b. 2724 – 10 = 
  c. 2724 – 15 =  
  d. 2724 – 20 = 
  e. 2724 – 25 = 

Do you see a pattern? Can you solve 2724 – 30 by following the pattern?
1. Dwayne made some notes about the quantities in Plan A.

He wrote:

1. The charge per month for Plan A is $25.00
2. The charge per minute is 10 cents, so you have to multiply the number of minutes by $0.10
3. The cost for a month depends on the number of minutes.

a. Say why Dwayne’s second note makes sense.

b. Does Dwayne’s last note make sense to you? If not, say why not.

2. Make notes about the quantities in the description of Plan B.
Use Dwayne’s notes as a model.

3. Rosa wrote questions that could be asked about the quantities in this situation.

- How much would it cost for one year with Plan A if I use the phone 500 minutes per month?
- How much would it cost for one year with Plan B if I use the phone 500 minutes per month?
- How much would it cost for eight months with Plan A if I use the phone 1000 minutes per month?
- How much would it cost for eight months with Plan B if I use the phone 1000 minutes per month?
- Which is a better plan?

Write three more questions that would help Rosa decide which plan is better.
4. Here is Rosa’s table for Plan A:

<table>
<thead>
<tr>
<th>Number of Minutes</th>
<th>1</th>
<th>10</th>
<th>100</th>
<th>300</th>
<th>600</th>
<th>1000</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total Cost per Month (after the first month)</td>
<td>$25.10</td>
<td>$26.00</td>
<td>$35.00</td>
<td>$55.00</td>
<td>$85.00</td>
<td>$125.00</td>
</tr>
</tbody>
</table>

a. Use Handout 2: Phone Plan Comparison, to sketch a graph of this relationship.

The handout looks like this coordinate grid.

b. Label the line Plan A.

5. Look at the table you made for Work Time problem 3.

a. Sketch a graph of this relationship on the same handout you used for problem 4.

b. Label the line Plan B.

---

**Homework**

1. Mr. Jackson has to be at work by 9:00 in the morning. It takes him 10 minutes to wake up, 15 minutes to get himself ready, 20 minutes to get his children ready, and 15 minutes to walk to work.

For what time should Mr. Jackson set his alarm so that he can get to work on time?

a. Identify the quantities in this situation.

b. Sketch a diagram that shows the relationship between quantities.
Carlos has a dog named Bea. He wanted to know how much Bea weighed, but Bea did not want to stay on the scale.

Rosa suggested, “Why don’t you weigh Bea in your arms?”

There are three quantities in this situation. Assigning a letter to each one will make it easier to represent the quantities in diagrams, tables, and formulas.

\[
\begin{align*}
C &= \text{Carlos’s weight} \\
B &= \text{Bea’s weight} \\
T &= \text{Total weight on the scale with Bea in Carlos’s arms}
\end{align*}
\]

Remember that a quantity is something that can be measured or counted. What is being measured or counted?

A quantity has a number and a unit. What might the units be for these quantities?

Here are some points about the quantities in this situation:

• The weight shown on the scale is the sum of Carlos’s and Bea’s weights.
• The total weight depends on both Carlos’s weight and Bea’s weight.
• Carlos’s weight is equal to Bea’s weight subtracted from the total weight.
• Bea’s weight is equal to Carlos’s weight subtracted from the total weight.
• Both Bea’s weight and Carlos’s weight are positive numbers.
• Both Bea’s weight and Carlos’s weight could be numbers that are not whole numbers.

Rosa sketched this diagram:
Rosa used her diagram, as well as her understanding of the situation, to write these three formulas:

\[ C + B = T \]  \hspace{1cm}  \[ B = T - C \]  \hspace{1cm}  \[ C = T - B \]

It is easier to use letters that stand for quantities when you write formulas.

**Work Time**

1. Look at each of Rosa’s formulas. All three formulas represent the same situation, but the formulas look different. Say how the formulas are different.

2. If Carlos weighs 173 pounds, and the total weight is 177.5 pounds, which of Rosa’s formulas should you use to find Bea’s weight?

3. If the total weight is 177.5 pounds, and Bea weighs 4.5 pounds, which of Rosa’s formulas should you use to find Carlos’s weight?

4. If Carlos weighs 173 pounds, and Bea weighs 4.5 pounds, which of Rosa’s formulas should you use to find the total weight?

Rosa sketched this diagram:

---

Lisa sketched this diagram:

---

Jamal sketched this diagram:

---

5. Compare Rosa’s, Lisa’s, and Jamal’s diagrams.
   a. How is Bea’s weight shown in each diagram?
   b. How is Carlos’s weight shown in each diagram?
   c. How is the total weight shown in each diagram?
6. Carlos volunteers at the local animal shelter. When animals are brought to the shelter, they must be weighed, but it is hard to get the animals to stand still on the scale. To solve this problem, Carlos used the same method Rosa had suggested to find out how much Bea weighed. Compare this new situation at the animal shelter to the situation with just Carlos and Bea.

a. Are the quantities the same? (Think about the values and the units of each quantity.)

b. Sketch a diagram to show the relationship between quantities.

c. Label your diagram with letters for quantities that are different from the earlier situation.

d. Compare your diagram with those of other students. Say how each quantity is shown in different diagrams.

e. Which two quantities would you represent in a table?

Skills

Solve.

a. \(215 \times 4 = \)  
b. \(215 \times 5 = \)  
c. \(215 \times 6 = \)  
d. \(215 \times 7 = \)  
e. \(215 \times 8 = \)

Do you see a pattern? What could you add to “215 \(\times 8\)” to get “215 \(\times 9\)”?
What could you subtract from “215 \(\times 10\)” to get “215 \(\times 9\)”?

Review and Consolidation

For problems 1–5, use this situation:

Carlos volunteers at the local animal shelter. When animals are brought to the shelter, they must be weighed, but it is hard to get the animals to stand still on the scale. To solve this problem, Carlos weighs each animal in his arms. Carlos weighs 173 pounds.
1. In this situation, two quantities vary in relation to each other:
   - Weight of each animal, in pounds
   - Total weight shown on the scale, in pounds
   a. Assign letters to these quantities.
   b. There is another quantity in this situation that does not vary. What is it?

2. Make a table showing at least four values for each of the quantities that vary in this situation. Use the letters you assigned in problem 1 to label the rows in your table, and be sure to clearly show which values correspond to each other. Your values should reflect realistic weights for cats and dogs.

3. Which graph best represents the relationship between the quantities in this situation?
4. Look at the graph you chose in problem 3.

   Every point on the graph represents the weight of an animal and the corresponding total weight shown on the scale.

   **Example**

   A cat that weighs 3 pounds is represented by the point with coordinates (3, 176).

   Find the point (3, 176) on the graph you chose in problem 3.

   Write the coordinates of at least four other points on this graph as ordered pairs, \((x, y)\), as in the example.

   Choose at least one point that has a weight that is not a whole number.

5. Write a formula that represents the relationship between the weight of each animal, \(W\), and the total weight shown on the scale, \(T\).

---

**Homework**

A scale at the animal shelter is broken.
The scale adds 5.2 pounds to anything that is weighed.

1. Identify the quantities in this situation. Assign a letter to each one.

2. Sketch a diagram that shows the relationship between quantities.

3. Make a table that shows the relationship between quantities.

4. Using letters, write a formula that represents the relationship between the quantities in this situation.
Lisa wanted to buy a bicycle that cost $120. Her grandfather promised to give her $2 for every $1 Lisa saved to buy the bike. Lisa told her friends about her grandfather’s promise.

Dwayne wrote:  
\[ 120 = \text{cost of the bike} \]
\[ L = \text{amount Lisa saves} \]
\[ G = \text{amount Grandpa contributes} \]

Keesha said, “Lisa’s grandfather promised to give Lisa $2 for every $1 she saved. That means there is a relationship between the two quantities, L and G. G is twice as much as L.”

Dwayne said, “You’re right! And we can sketch a diagram to show that.”

\[ L + G \]
1. Use words to describe what Dwayne’s diagram shows about the relationship between the quantities.

2. Write three different formulas that represent Dwayne’s diagram. Use Dwayne’s letters in your formulas.

3. Keesha used words to describe the relationship between the quantities \( G \) and \( L \). She said, “\( G \) is twice as much as \( L \).” Write a formula that represents this relationship.

4. Work with a partner to make notes about the quantities in this situation. Include statements about the relationships between the quantities.

5. Look at the formulas you wrote for problems 2 and 3. Is the formula based on Dwayne’s diagram equivalent to the formula based on Keesha’s observation? If so, write the number properties that justify your answer.

6. Two of the quantities in this situation vary:
   
   \[ L, \text{ the amount Lisa saves} \]
   \[ G, \text{ the amount Grandpa contributes} \]
   
   a. Make a table that shows the relationship between how much Lisa saves and how much her grandpa contributes. Include four pairs of corresponding values, starting with \( G = 2 \) when \( L = 1 \).
   
   b. Does it make sense for Lisa to save $50 to buy the bike? Say why or why not.
   
   c. At a certain point in the bike problem, these two variable quantities will stop varying and become fixed. When will this happen, and why?

Preparing for the Closing

7. The situation about the bike has quantities that vary up to a point, and then become fixed.
   
   a. How did the table help you understand the relationship between the quantities?
   
   b. How would a graph be helpful for representing the relationship between the quantities?
   
   c. Did Dwayne’s diagram represent the quantities varying or the value of the quantities after they were fixed?
Skills

Solve.

a. $15 \cdot 5 =$

b. $15 \cdot 10 =$

c. $15 \cdot 15 =$

d. $15 \cdot 20 =$

e. $15 \cdot 25 =$

Do you see a pattern?

Review and Consolidation

On Saturday, Rosa, Jamal, Dwayne, Lisa, and Lisa’s sister, Annie, decide to go to the park where they can rent bicycles.

On weekends, there is a $5 initial fee, plus a rental price of $5 per hour.

1. What are the quantities in this situation? Assign a letter to each one.

2. Write several questions about the quantities in this situation that would be helpful to people renting bicycles.

3. In this situation, the two quantities that vary in relation to each other are:
   - The total cost to rent a bike in the park
   - The number of hours the bike is rented

   a. How much would it cost to rent a bike for 1 hour?
   b. How much would it cost to rent a bike for 2 hours?
   c. How much would it cost to rent a bike for 10 hours?
   d. Jamal’s total rental cost is $15. For how many hours did he rent a bike?
   e. Rosa’s total rental cost is $35. For how many hours did she rent a bike?

4. This table shows values of the two quantities that vary in this situation.

<table>
<thead>
<tr>
<th>Hours Rented</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total Rental Cost ($)</td>
<td>20</td>
<td>25</td>
<td>30</td>
<td>55</td>
</tr>
</tbody>
</table>

Give three more pairs of values that could be included in this table.
5. Sketch a graph to represent the relationship between the total rental cost and the hours rented. Graph three points defined in the table for problem 4.

6. Write a formula that tells you how much it costs to rent a bike for \( h \) hours.

7. Use the formula you wrote in problem 6 to calculate the total cost when \( h = 8 \).

8. Use the formula you wrote in problem 6 to calculate how many hours a bike was rented if the total cost was $40.

9. How would you need to change the formula you wrote in problem 6 if you wanted to know the total cost for five people to rent bikes for \( h \) hours? Use the distributive property to write a new formula.

10. Use the formula you wrote in problem 9 to calculate the total cost for five people to rent bikes for:
    a. 3 hours
    b. 5 hours
    c. 10 hours

**Homework**

The bike rental shop has a weekday special. The cost is $5 per hour with no initial fee.

1. Identify the quantities in this situation.
2. Assign letters to the quantities.
3. Sketch a diagram that shows the relationship between the quantities.
4. Make a table that represents the relationship between the quantities that vary.
5. What would the total cost be to rent a bike for 4 hours?
6. Write another question that could be asked about the quantities in this situation.
Work Time

1. The definition of “divisible by 3” states: If a number is divisible by 3, the number can be divided into three equal groups with no remainder.

   Think about this statement: For any number \(a\), \(3a\) is always divisible by 3.

   a. Is this statement always true, sometimes true, or never true?

   b. Justify your choice mathematically using any or all of the following:

      • The definition given above of “divisible by 3”
      • A diagram
      • Conventions for using numbers and letters
      • The number properties

2. a. Which value is equal to the expression \(8(5 + 11)\)?
   
   \[
   \begin{align*}
   51 & \quad \text{A} \\
   24 & \quad \text{B} \\
   128 & \quad \text{C} \\
   48 & \quad \text{D} \\
   101 & \quad \text{E}
   \end{align*}
   \]

   b. Which number property can help you make the calculation in part a easier?

   c. Sketch an area model to show that \(8(5 + 11) = (8 \times 5) + (8 \times 11)\).

3. Write these expressions in words.
   Use the word “quantity” if it makes your answer clearer.

   a. \((n \div 2) + 1\)  
   b. \(\frac{n + 1}{2}\)  
   c. \(\frac{n}{2} + 1\)  
   d. \(2n + 1\)  
   e. \(n + 2 + 1\)  
   f. \(\frac{1}{2}n + 1\)

4. Calculate the value of each expression in problem 3 when \(n = 8\).

   Do any of the expressions have the same value when \(n = 8\)? If so, say why.
5. Which statements are true for any numbers \( x, y, \) and \( z? \)

For each true statement, name the number property or convention that justifies it.
Use the posters you made with your classmates to help you identify the number properties and conventions.

a. \( x + 0 = 0 \)  
b. \( \frac{1}{x} \cdot x = 1 \)  
c. \( \frac{1}{2} x = \frac{x}{2} \)

d. \( \frac{1}{2} x = 2x \)  
e. \( x + y = y + x \)  
f. \( x - y = y - x \)

g. \( x + (y + z) = (x + y) + z \)  
h. \( xy = yx \)  
i. \( \frac{x}{y} = \frac{y}{x} \)

j. \( x(yz) = (xy)z \)  
k. \( x(y + z) = xy + xz \)  
l. \( x(y - z) = xy - xz \)

6. Substitute a number for each letter to show that these statements are true.

a. \( a + (b + c) = (a + b) + c \)

b. \( a(b - c) = ab - ac \)

c. \( \frac{1}{2} a = \frac{a}{2} \)

d. \( \frac{1}{a} \cdot a = 1 \)

7. Write counterexamples with a number substituted for each letter to show that these statements can be false:

a. \( a - b = b - a \)  
b. \( \frac{1}{2} a = 2a \)  
c. \( a \div b = b \div a \)

8. This table gives the area and base of several different rectangles.

Notice that all of the rectangles have an area of 24 square centimeters, but each has a different base.

<table>
<thead>
<tr>
<th>Area (cm²)</th>
<th>24</th>
<th>24</th>
<th>24</th>
<th>24</th>
<th>24</th>
<th>24</th>
</tr>
</thead>
<tbody>
<tr>
<td>Base (cm)</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>( n )</td>
</tr>
<tr>
<td>Height (cm)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

a. Copy the table and fill in the height for each rectangle.

b. What is the rule for calculating the height of a rectangle if the area is 24 cm² and the base is known? Write the rule in words or with a formula.
9. Dwayne needed to buy onions to make soup. The price of onions is $0.50 per pound.
If Dwayne buys one pound of onions, the cost will be $0.50
If he buys 2 pounds of onions, the cost will be $1.00
The two quantities that vary in relation to each other in this situation are:

- The cost, \( c \), in dollars
- The amount of onions, \( a \), in pounds that Dwayne buys

a. Which formula represents the relationship between the quantities in this situation?
   - \( c = a + 0.5 \)
   - \( a = 0.5c \)
   - \( c = 2a \)
   - \( c = 0.5a \)
   - \( a = 5c \)

b. Which table represents the relationship between the quantities in this situation?

<table>
<thead>
<tr>
<th></th>
<th>( a )</th>
<th>( c )</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>$0.50</td>
<td>$0.50</td>
</tr>
<tr>
<td></td>
<td>$1.00</td>
<td>$1.00</td>
</tr>
<tr>
<td></td>
<td>$2.00</td>
<td>$2.00</td>
</tr>
<tr>
<td></td>
<td>$5.00</td>
<td>$5.00</td>
</tr>
<tr>
<td>B</td>
<td>1</td>
<td>$0.50</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>$1.00</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>$1.50</td>
</tr>
<tr>
<td></td>
<td>10</td>
<td>$2.00</td>
</tr>
<tr>
<td>C</td>
<td>$0.50</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>$1.00</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>$2.00</td>
<td>4</td>
</tr>
<tr>
<td></td>
<td>$5.00</td>
<td>10</td>
</tr>
<tr>
<td>D</td>
<td>$0.50</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>$1.00</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>$2.00</td>
<td>4</td>
</tr>
<tr>
<td></td>
<td>$5.00</td>
<td>10</td>
</tr>
<tr>
<td>E</td>
<td>$1.00</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>$2.00</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>$4.00</td>
<td>4</td>
</tr>
<tr>
<td></td>
<td>$10.00</td>
<td>10</td>
</tr>
</tbody>
</table>

C. Which three pairs of values could also be included in the table representing the relationship between quantities in this situation?

- \( a = 1.5 \)  \( c = 1 \)
- \( a = 3 \)  \( c = 1.5 \)
- \( a = 18 \)  \( c = 9 \)
- \( a = 1.5 \)  \( c = 0.75 \)
- \( a = 6 \)  \( c = 12 \)

D. How much will it cost to buy \( n \) pounds of onions?

- \( n + 0.50 \)
- \$0.50
- \( 5n \)
- \$0.50\( n \)
- \( 2n \)
e. Which of the graphs represents the relationship between the quantities?
10. Jamal climbed a ladder to pin a number properties poster up in his classroom. For each step he went up the ladder, he gained 6 inches in height.

**Example**

- By stepping up 2 steps, Jamal gained 12 inches in height.
- By stepping up 3 steps, Jamal gained 18 inches in height.

a. How many inches did Jamal gain in height by stepping up 5 steps on the ladder?

b. Name the two quantities that vary in relation to each other in this situation, and assign letters to these quantities.

c. Make a table that includes four possible values for each quantity. Be sure that your first column has the letters you assigned to each quantity, and that you clearly show which values in your table correspond to each other.

d. Write a formula to represent the relationship between the quantities in this situation, using the given values and the letters you assigned.

**Skills**

Solve.

- a. $25 \cdot 10 =$  
- b. $25 \cdot 100 =$  
- c. $25 \cdot 1000 =$  
- d. $25 \cdot 10,000 =$  
- e. $25 \cdot 100,000 =$

Do you see a pattern?
Review and Consolidation

Assessing Your Work

1. Review your work in this unit, and select one piece that you believe is the best example of your understanding of the mathematics presented in the unit. The piece of work that you select could show your use of letters, your understanding of conventions and the number properties, your ability to justify, or your ability to solve word problems. The piece of work could show more than one concept.

When choosing your piece of work, show that you have:

- Used the concept accurately to solve the problem
- Represented the concept in multiple ways (numbers, graphs, symbols, diagrams, or words)
- Explained your solution and the concept well

2. Mark the piece in the way described by your teacher.

3. Write a brief explanation of why you chose this piece and how it demonstrates your understanding of the concept.

Homework

1. a. Review the problems in this lesson and find three that were difficult for you.
   
   b. Use your Concept Book and previous lessons to work through these three problems more carefully until you understand them.

2. Write a summary page of the concepts you have learned in the unit.
1. Sketch a diagram that shows the expression $4 \cdot 3$.

2. Calculate the value of $7x$ for each value of $x$.
   a. $x = 1$
   b. $x = 5$
   c. $x = 10$
   d. $x = 100$
   e. $x = 0$

3. Without doing any calculations, use what you have learned about equations, inverses, and the rules of arithmetic to find the value of the letter that will make each number sentence true.
   a. $96 + 67 = b + 67$
   b. $27 + 47 - 47 = p$
   c. $a + 568 = 568$
   d. $39 - k = 39 - 40$

4. Use the distributive property to calculate this rectangle’s area in two different ways.

5. Multiple $24 \cdot 31$ using the distributive property.
6. What is the inverse of each of these operations?
   a. Adding $\frac{1}{9}$
   b. Multiplying by $\frac{1}{2}$
   c. Dividing by 3

7. a. Copy and complete this table of values.

<table>
<thead>
<tr>
<th>Value A</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>10</th>
<th>x</th>
</tr>
</thead>
<tbody>
<tr>
<td>Value B</td>
<td>3</td>
<td>5</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>2x + 1</td>
</tr>
</tbody>
</table>

b. Sketch a graph to represent the relationship between Value A and Value B.

8. Tickets to a concert are $30 each. The ticket agency charges a $3 convenience fee per ticket. Shipping and handling is free if tickets are printed on-line, $2 if shipped by regular mail, and $16 if overnight delivery is needed.
   a. How much would it cost for 2 tickets printed on-line?
   b. How much would it cost for 4 tickets shipped by regular mail?
   c. How much would it cost for 6 tickets delivered overnight?
### Number Sense

Gr. 2 NS: 2.1  
Understand and use the inverse relationship between addition and subtraction (e.g., an opposite number sentence for $8 + 6 = 14$ is $14 - 6 = 8$) to solve problems and check solutions. 54–57; 15–16, 21

Gr. 3 NS: 2.3  
Use the inverse relationship of multiplication and division to compute and check results. 54–57; 15–16, 21, 150–151

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### Algebra and Functions

Gr. 2 AF: 1.1  
Use the commutative and associative rules to simplify mental calculations and to check results. 29–38; 15, 20, 88

Gr. 3 AF: 1.5  
Recognize and use the commutative and associative properties of multiplication. (e.g., if $5 \cdot 7 = 35$, then what is $7 \cdot 5$? and if $5 \cdot 7 \cdot 3 = 105$, then what is $7 \cdot 3 \cdot 5$?). 29–38; 15, 20

Gr. 3 AF: 2.2  
Extend and recognize a linear pattern by its rules (e.g., the number of legs on a given number of horses may be calculated by counting by 4s or by multiplying the number of horses by 4). 17–22, 66–71; 13, 204–206

Gr. 4 AF: 1.0  
Students use and interpret variables, mathematical symbols, and properties to write and simplify expressions and sentences. 12–22; 9–13

Gr. 4 AF: 1.2  
Interpret and evaluate mathematical expressions that now use parentheses. 23–28; 12–13

Gr. 4 AF: 1.3  
Use parentheses to indicate which operation to perform first when writing expressions containing more than two terms and different operations. 1–5, 23–28, 91–97; 12–13, 26, 29–30, 90–91

Gr. 4 AF: 1.4  
Use and interpret formulas (e.g., $area = length \cdot width$ or $A = lw$) to answer questions about quantities and their relationships. 39–43, 58–60; 10, 236–238, 242–243

Gr. 5 AF: 1.0  
Students use variables in simple expressions, compute the value of the expression for specific values of the variable, and plot and interpret the results. 12–22, 39–43, 72–97; 10, 27–31, 202–207, 236–238, 242–243

Gr. 5 AF: 1.1  
Use information taken from a graph or equation to answer questions about a problem situation. 72–76, 82–86, 91–97; 26–31, 199–209

Gr. 5 AF: 1.2  
Use a letter to represent an unknown number; write and evaluate simple algebraic expressions in one variable by substitution. 12–22, 54–65; 9–13, 15–16, 21, 150–151

Gr. 5 AF: 1.3  
Know and use the distributive property in equations and expressions with variables. 1–5, 44–53, 58–65, 91–97; 15–18, 21, 105–107
Algebra and Functions (continued)

Gr. 5 AF: 1.5
Solve problems involving linear functions with integer values; write the equations; and graph the resulting ordered pairs of integers on a grid. 66–76, 87–97; 23–31, 199–209

Gr. 6 AF: 1.0
Students write verbal expressions and sentences as algebraic expressions, solve simple linear equations, and graph and interpret their results. 87–97; 23–31, 199–209

Gr. 6 AF: 1.3
Apply algebraic order of operations and the commutative, associative, and distributive properties to evaluate expressions; and justify each step in the process. 29–38, 44–53, 58–65; 1–21, 88, 105–107

Measurement and Geometry

Gr. 4 MG: 2.1
Draw the points corresponding to linear relationships on graph paper (e.g., draw 10 points on the graph of the equation $y = 3x$ and connect them by using a straight line). 72–76; 199–209

Mathematical Reasoning

Gr. 4 MR: 2.3
Use a variety of methods, such as words, numbers, symbols, charts, graphs, tables, diagrams, and models, to explain mathematical reasoning. 1–5, 39–53, 58–97; 1–31, 90–91, 105–107, 199–209, 236–238, 242–243

Gr. 4 AF: 1.3
Gr. 4 AF: 1.4

Gr. 5 MR: 1.0
Students make decisions about how to approach problems. 77–90; 23–31

Gr. 5 AF: 1.0

Gr. 5 MR: 1.1
Analyze problems by identifying relationships, distinguishing relevant information, sequencing and prioritizing information, and observing patterns. 77–90; 23–31

Gr. 5 AF: 1.0

Gr. 6 MR: 1.2
Formulate and justify mathematical conjectures based on a general description of the mathematical question or problem posed. 6–11; 2–7
Symbols

<, less than, 38; 51–53, 67–68
≠, not equal to, 30

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