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# Table of Contents

## LESSON

1. **Multiples and Factors** 1–5
   - Work Time ................................................................. 2
   - Skills ........................................................................... 4
   - Review and Consolidation ......................................... 4
   - Homework .................................................................... 5

2. **Prime and Composite Numbers** 6–9
   - Work Time ................................................................. 6
   - Skills ........................................................................... 8
   - Review and Consolidation ......................................... 8
   - Homework .................................................................... 9

3. **Prime Factorization** 10–14
   - Work Time ................................................................. 12
   - Skills .......................................................................... 13
   - Review and Consolidation ......................................... 14
   - Homework .................................................................... 14

4. **Common Multiples** 15–18
   - Work Time ................................................................. 16
   - Skills .......................................................................... 17
   - Review and Consolidation ......................................... 17
   - Homework .................................................................... 18

5. **GCF and LCM** 19–22
   - Work Time ................................................................. 20
   - Skills .......................................................................... 21
   - Review and Consolidation ......................................... 21
   - Homework .................................................................... 22

6. **Reviewing Multiples and Factors** 23–25
   - Work Time ................................................................. 24
   - Skills .......................................................................... 24
   - Review and Consolidation ......................................... 24
   - Homework .................................................................... 25
# Table of Contents

## Lesson

<table>
<thead>
<tr>
<th>Lesson</th>
<th>Pages</th>
<th>Work Time</th>
<th>Skills</th>
<th>Review and Consolidation</th>
<th>Homework</th>
</tr>
</thead>
<tbody>
<tr>
<td>7. Making Sizes that Fit</td>
<td>26–28</td>
<td>26</td>
<td>27</td>
<td>28</td>
<td>28</td>
</tr>
<tr>
<td>8. Adding and Subtracting Fractions</td>
<td>29–34</td>
<td>30</td>
<td>32</td>
<td>32</td>
<td>34</td>
</tr>
<tr>
<td>9. Adding with Different Denominators</td>
<td>35–38</td>
<td>36</td>
<td>37</td>
<td>37</td>
<td>38</td>
</tr>
<tr>
<td>10. Multiplying by a Whole Number</td>
<td>39–43</td>
<td>40</td>
<td>41</td>
<td>42</td>
<td>43</td>
</tr>
<tr>
<td>11. Multiplying Fractions</td>
<td>44–47</td>
<td>45</td>
<td>46</td>
<td>46</td>
<td>47</td>
</tr>
<tr>
<td>12. Progress Check</td>
<td>48–51</td>
<td>49</td>
<td>50</td>
<td>50</td>
<td>51</td>
</tr>
</tbody>
</table>
# Table of Contents

<table>
<thead>
<tr>
<th>LESSON</th>
<th>Pages</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>13. Finding Differences</strong></td>
<td>52–55</td>
</tr>
<tr>
<td>Work Time</td>
<td>53</td>
</tr>
<tr>
<td>Skills</td>
<td>54</td>
</tr>
<tr>
<td>Review and Consolidation</td>
<td>54</td>
</tr>
<tr>
<td>Homework</td>
<td>55</td>
</tr>
<tr>
<td><strong>14. Addition and Subtraction as Inverses</strong></td>
<td>56–58</td>
</tr>
<tr>
<td>Work Time</td>
<td>57</td>
</tr>
<tr>
<td>Skills</td>
<td>57</td>
</tr>
<tr>
<td>Review and Consolidation</td>
<td>58</td>
</tr>
<tr>
<td>Homework</td>
<td>58</td>
</tr>
<tr>
<td><strong>15. Shortest Distance</strong></td>
<td>59–61</td>
</tr>
<tr>
<td>Work Time</td>
<td>59</td>
</tr>
<tr>
<td>Skills</td>
<td>60</td>
</tr>
<tr>
<td>Review and Consolidation</td>
<td>61</td>
</tr>
<tr>
<td>Homework</td>
<td>61</td>
</tr>
<tr>
<td><strong>16. Dividing Fractions</strong></td>
<td>62–65</td>
</tr>
<tr>
<td>Work Time</td>
<td>63</td>
</tr>
<tr>
<td>Skills</td>
<td>65</td>
</tr>
<tr>
<td>Review and Consolidation</td>
<td>65</td>
</tr>
<tr>
<td>Homework</td>
<td>65</td>
</tr>
<tr>
<td><strong>17. Mixed Operations</strong></td>
<td>66–69</td>
</tr>
<tr>
<td>Work Time</td>
<td>67</td>
</tr>
<tr>
<td>Skills</td>
<td>68</td>
</tr>
<tr>
<td>Review and Consolidation</td>
<td>68</td>
</tr>
<tr>
<td>Homework</td>
<td>69</td>
</tr>
<tr>
<td><strong>18. Progress Check</strong></td>
<td>70–73</td>
</tr>
<tr>
<td>Work Time</td>
<td>70</td>
</tr>
<tr>
<td>Skills</td>
<td>72</td>
</tr>
<tr>
<td>Review and Consolidation</td>
<td>72</td>
</tr>
<tr>
<td>Homework</td>
<td>73</td>
</tr>
</tbody>
</table>
# Table of Contents

## Lesson 19.
### Learning from the Progress Check 74–77
- Work Time ................................................................. 74
- Skills ................................................................. 76
- Review and Consolidation ........................................ 76
- Homework .......................................................... 77

## Lesson 20.
### Shortest Time 78–80
- Work Time ................................................................. 78
- Skills ................................................................. 79
- Review and Consolidation ........................................ 79
- Homework .......................................................... 80

## Lesson 21.
### The Unit in Review 81–84
- Work Time ................................................................. 81
- Skills ................................................................. 83
- Review and Consolidation ........................................ 83
- Homework .......................................................... 84

### Comprehensive Review (Units 1–3) 85–86

### California Mathematics Content Standards 87–88

### Index 89–90
A \textit{multiple} is the product of two natural numbers.

\begin{center}
\textbf{Example}

\begin{align*}
3 \cdot 8 &= 24 \\
\text{This means that } 24 \text{ is a multiple of both } 3 \text{ and } 8.
\end{align*}
\end{center}

24 is a multiple of other numbers too. $2 \cdot 12 = 24$, so 24 is a multiple of 2 and 12.
Of what other numbers is 24 a multiple?

A natural number is always a multiple of itself and 1.
Many natural numbers, like 24, are also multiples of other natural numbers.

When you multiply a natural number by a natural number, the product is a multiple of both numbers. The natural numbers you multiply are factors of the product.

\begin{center}
\textbf{Example}

\begin{align*}
3 \cdot 8 &= 24 \\
\text{factor} & \quad \text{factor} \\
\text{multiple}
\end{align*}

The equation $3 \cdot 8 = 24$ allows you to make two statements:

\begin{center}
3 and 8 are factors of 24. \quad 24 \text{ is a multiple of 3 and 8.}
\end{center}

\end{center}

A \textit{factor} is any natural number that exactly divides another.

\begin{center}
\textbf{Example}

Since $24 \div 3 = 8$ with no remainder, 3 is a factor of 24.
Since $24 \div 8 = 3$ with no remainder, 8 is also a factor of 24.
\end{center}

The multiplication $3 \cdot 8$ is a \textit{factorization} of 24.
The multiples of 2 are 2, 4, 6, 8, 10,…, 2n (where n represents any natural number).
This pattern of counting by twos goes on forever. It can be shown on the number line:

20 is a multiple of 1, 2, 4, 5, 10, and 20. You can also say that 1, 2, 4, 5, 10, and 20 are
the factors of 20.
20 is not a multiple of 3, 6, 7, 8, 9, or 11, so the numbers 3, 6, 7, 8, 9, and 11 are
not factors of 20.

Work Time

Use Handout 1: Multiplication Table for the Work Time problems.
The handout has a multiplication table like this one.
1. Look at the top row (→), 1 to 12, and the \( n \) column (↑) of the handout. The top row can go beyond 12. The letter \( n \) represents any natural number.
   a. Complete the “3” column.
   b. Complete the “3” row through \( n \).

2. a. Complete the “7” column and the “7” row.
   b. Write a description of these numbers.

3. a. Complete the “12” column and the “12” row.
   b. Write the multiples of 12 that are given in the table.
   c. Write three other multiples of 12 that are not given in the table.

4. a. Draw a circle around every 12 in the multiplication table. (Do not include the numbers in the shaded boxes.)
   b. Using your circles, write the factors of 12.
   c. Are there other factors of 12 not shown in the table? Say why or why not.

5. a. Cross out every 11 in the table. (Do not include the numbers in the shaded boxes.)
   b. Write the factors of 11.
   c. Are there any factors of 11 not shown in the table? Say why or why not.

6. a. Draw a box around every 24 in the table.
   b. Write the factors of 24.
   c. Are there any factors of 24 not shown in the table? Say why or why not.

7. Sketch five number lines to show each of these types of numbers.
   a. The multiples of 3 up to 40
   b. The multiples of 4 up to 40
   c. The multiples of 5 up to 40
   d. The multiples of 6 up to 40
   e. The multiples of 7 up to 40
Preparation for the Closing

8. a. In the multiplication table, what does the column labeled $n$ mean?
   b. What numbers could $n$ represent?

9. Decide whether the following statement is always true, sometimes true, or never true. Justify your answer.
   Given any two natural numbers, the greater the number, the more factors it has.

10. The number 1 is a factor of every natural number. Say why.

11. The number 0 is a not a factor of any natural number. Say why.

12. Compare the multiplication table with the number lines you made. What do you notice?

Skills

Copy and complete the following table.

<table>
<thead>
<tr>
<th>Number</th>
<th>$\times 10$</th>
<th>$\times 100$</th>
<th>$\times 1000$</th>
</tr>
</thead>
<tbody>
<tr>
<td>19.5</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.326</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.745</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5.007</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5.16</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Review and Consolidation

1. Write the first 20 multiples of 8.

2. Divide 64 by each natural number from 1 through 10. Which of the divisions have no remainder?

3. In the multiplication table on Handout 1, look at the column that contains the multiples of 12. One of the numbers in that column has 15 factors.
   a. Which number is it?
   b. Write all 15 factors of that number.
**Homework**

1. What are the multiples of 7 between 20 and 60?

2. a. Write all the factors of 12.
   b. Write all the factors of 36.
   c. Your list for part b should contain an odd number of factors. Say why.

3. A teacher asks her students to write all the factors and multiples of 50 for homework.
   a. Which part of this assignment is possible? Say why.
   b. Which part of this assignment is not possible? Say why.
   c. What number is both a factor and a multiple of 50?

4. Which of these is the best description of the numbers 3, 6, 9, 12, 15, 18, 21, 24, 27, and 30?
   A 3 and the remaining factors of 30
   B All the multiples of 3
   C All the factors of 3
   D The first nine multiples of 3
   E The first ten multiples of 3
A prime number is a natural number that has exactly two distinct factors, itself and 1. A composite number is a natural number that has more than two distinct factors.

<table>
<thead>
<tr>
<th>Prime Number</th>
<th>Composite Number</th>
</tr>
</thead>
<tbody>
<tr>
<td>Has exactly two distinct factors, itself and 1.</td>
<td>Has more than two distinct factors.</td>
</tr>
</tbody>
</table>
| **Example** 13  
Factors: 13 and 1 | **Example** 30  
Factors: 30, 15, 10, 6, 5, 3, 2, and 1 |

The number 1 does not fit the definition of a prime number or a composite number. It does not have two distinct factors. This means that 1 is neither prime nor composite.

**Work Time**

1. a. How can you tell that a number is prime?

   b. How can you tell that a number is composite?

2. A hallway in Monroe High School has lockers numbered from 1 to 50.
   Chen is putting flyers on the lockers about the basketball game, and Chen’s cousin, Taylor, is putting flyers on the lockers about the pep rally. Chen puts his flyer on **every other locker**, starting with locker number 2. Taylor puts his flyer on **every third locker**, starting with locker number 3.

   a. Rosa walks by the first thirty lockers (lockers 1–30). Which of these lockers does not have any flyers on it?

   b. Rosa points to the locker that has the smallest composite number of the lockers with no flyers. Which locker is it?
c. As they move to a different hallway, which also has lockers numbered 1–50, Chen turns to Rosa and says, “Here, we will not put flyers on lockers with prime numbers. We will only put flyers on lockers with composite numbers. Rosa, can you help us use multiples to do this?”

What instructions does Rosa give to Taylor and Chen?

Preparing for the Closing

3. For each of the four statements, decide whether it is always true, sometimes true, or never true. Support your reasoning with examples chosen from the first 20 natural numbers.

a. The product of two prime numbers is a prime number.

   \[ \text{prime number } \times \text{prime number} = \text{prime number} \]

b. The sum of two prime numbers is a prime number.

   \[ \text{prime number} + \text{prime number} = \text{prime number} \]

c. The product of two composite numbers is a composite number.

   \[ \text{composite number} \times \text{composite number} = \text{composite number} \]

d. The sum of two composite numbers is a composite number.

   \[ \text{composite number} + \text{composite number} = \text{composite number} \]
Lesson 2

Skills

Solve for the missing numbers.

a. $8.56 \div \underline{} = 0.856$

b. $21.85 \div \underline{} = 2.185$

c. $3050 \div \underline{} = 30.5$

d. $3007 \div \underline{} = 3.007$

e. $3007 \div \underline{} = 30.07$

f. $8.06 \div \underline{} = 0.0806$

g. $900.8 \div \underline{} = 9.008$

h. $648.7 \div \underline{} = 64.87$

i. $648.7a \div \underline{} = 64.87$

c. $648.7a \div \underline{} = 64.87a$

Review and Consolidation

1. Use Handout 2: *Prime Numbers from 1 to 100* to identify all of the primes from 1 to 100.

   a. Circle all the prime numbers.

   b. For each number that is not a prime number, write at least one factor other than itself and 1.

   c. Write a description of the strategies you used to do this task.

2. Write a description of a different strategy you could use to find prime numbers.
Homework

Lisa and Keesha rode on a bus to the basketball game. To pass the time, they decided to play “Guess the Number.” One person described a number and the other person had to identify which number it is.

Read the descriptions, review the summary of information given, and identify the numbers that each problem describes.

1. Lisa said, “This is a prime number between 10 and 20. It is a factor of 26. What is the number?”

   **Hint:** You know the number is:
   • prime
   • between 10 and 20
   • a factor of 26

2. Keesha challenged Lisa with, “This number is prime. One of the factors of the number is 29. What is the number?”

3. Lisa said, “This number is divisible by 4 and 7. The number is between 50 and 60.”
   a. Is it a prime number?
   b. What is the number?
To write any natural number greater than 1 as a product of its prime factors.

36 is a composite number. You can write 36 as the product of factors in various ways.

**Example**

<table>
<thead>
<tr>
<th>Factorizations</th>
<th>36 = 1 • 36</th>
<th>36 = 2 • 18</th>
<th>36 = 3 • 12</th>
<th>36 = 4 • 9</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>36 = 1 • 4 • 9</td>
<td>36 = 6 • 6</td>
<td>36 = 3 • 3 • 4</td>
<td>36 = 2 • 2 • 3 • 3</td>
</tr>
</tbody>
</table>

The order in which the factors are shown does not affect the factorization. These are the same factorizations: $2 \cdot 2 \cdot 9$ and $9 \cdot 2 \cdot 2$ (remember, $ab = ba$).

The factorization $36 = 2 \cdot 2 \cdot 3 \cdot 3$ has only prime factors.

It is called the prime factorization of 36.

The convention for writing prime factorizations is to write the smaller prime factors first: $2 \cdot 2 \cdot 3 \cdot 3$.

This could also be written using exponents: $2^2 \cdot 3^2$.

The Fundamental Theorem of Arithmetic:

*For every composite number, there is one, and only one, prime factorization.*

The prime factorization of 36 is $2 \cdot 2 \cdot 3 \cdot 3$. The prime factors of 36 are 2 and 3.

The factors of 36 are 1, 2, 3, 4, 6, 9, 12, 18, and 36. One way to find the factors of a number is to start with the prime factorization of the number and then calculate the other factors using the prime factors.
The prime factorization of 36 is $2 \cdot 2 \cdot 3 \cdot 3$.

Other factors are products of combinations of these four prime factors.

$2 \cdot 2 = 4$, so 4 is a factor of 36. $2 \cdot 3 = 6$, so 6 is a factor of 36.

The other factors of 36 are 9, 12, 18, and 36.

$3 \cdot 3 = 9$ \hspace{1cm} $2 \cdot 2 \cdot 3 = 12$ \hspace{1cm} $2 \cdot 3 \cdot 3 = 18$ \hspace{1cm} $2 \cdot 2 \cdot 3 \cdot 3 = 36$

Summary: 36 (composite number)

Factors: 1, 2, 3, 4, 6, 9, 12, 18, and 36
Prime Factorization: $2 \cdot 2 \cdot 3 \cdot 3$, or $2^2 \cdot 3^2$

How can you find the prime factors of any large number?

One way is to start with a factor pair of any number and continue to factor the factors until all the factors are prime numbers.

This example uses a factor tree to find the prime factors of 525, beginning with the factor pair 5 and 105.

The prime factorization of 525 is $3 \cdot 5 \cdot 5 \cdot 7$. It can also be written with exponents: $3 \cdot 5^2 \cdot 7$. 

Comment

A factor pair is any two factors whose product equals a number.
Four students were asked to find the prime factorization of 306. Their division calculations and factor trees are shown.

<table>
<thead>
<tr>
<th>Student A</th>
<th>Student B</th>
<th>Student C</th>
<th>Student D</th>
</tr>
</thead>
</table>
| \[ \begin{array}{c}
9 \\
3 \mid 3 \overline{0} \overline{6} \\
3 \mid 1 \underline{2} \\
3 \mid 1 \underline{2} \\
2 \mid 4 \\
\end{array} \] | \[ \begin{array}{c}
10 \underline{2} \\
3 \mid 3 \overline{0} \overline{6} \\
5 \mid 1 \underline{0} \underline{2} \\
3 \mid 1 \underline{7} \\
5, \text{rem} 2 \\
3 \mid 1 \underline{7} \\
\end{array} \] | \[ \begin{array}{c}
2 \mid 3 \overline{0} \overline{6} \\
3 \mid 1 \underline{5} \underline{3} \\
2 \mid 1 \underline{5} \underline{3} \\
\end{array} \] | \[ \begin{array}{c}
17 \\
3 \mid 1 \underline{5} \underline{3} \\
3 \mid 1 \underline{5} \underline{3} \\
2 \mid 3 \overline{0} \overline{6} \\
\end{array} \] |

| 306 = 2 \cdot 2 \cdot 3 \cdot 3 | 306 = 3 \cdot 2 \cdot 3 \cdot 3 \cdot 5 | 306 = 2 \cdot 3 \cdot 51 | 306 = 2 \cdot 3 \cdot 17 |

1. Each student’s answer is incorrect. Write an explanation of what each student did wrong.

2. Write advice for the four students about how to avoid making the mistakes that each has made.

3. Calculate the prime factorization of 306, using three correct factor trees.
Preparing for the Closing

Work with a partner to discuss and improve your answers to problems 1 through 3.

4. How can you be sure a factorization is the prime factorization?

5. a. Dwayne claimed that there is only one prime factorization of 84.
   Rosa claimed that $3 \cdot 2 \cdot 7 \cdot 2$ is different from $2^2 \cdot 3 \cdot 7$, and that both are prime factorizations of 84.
   Who is correct?

   b. Dwayne said that he used $2^2 \cdot 3 \cdot 7$ to check that he had found all the factors of 84.
   How did he do this?

6. What is it about the numbers 18 and 24 that allows you to write the following equivalent fractions?

\[
\begin{align*}
\frac{18}{24} &= \frac{9}{12} = \frac{6}{8} = \frac{3}{4}
\end{align*}
\]

Skills

Solve for the missing numbers.

\[
\begin{align*}
a. \ 8.56 \times &\ _{\ \ } = 856 \\
b. \ 21.85 \times &\ _{\ \ } = 2185 \\
c. \ 3050 \times &\ _{\ \ } = 30,500 \\
d. \ 3007 \times &\ _{\ \ } = 300,700 \\
e. \ 30.07 \times &\ _{\ \ } = 3007 \\
f. \ 8.06 \times &\ _{\ \ } = 806 \\
g. \ 0.9008 \times &\ _{\ \ } = 9008 \\
h. \ 0.9008 \times &\ _{\ \ } = 9008a \\
i. \ 0.6487 \times &\ _{\ \ } = 64.87 \\
j. \ 0.6487a \times &\ _{\ \ } = 64.87a
\end{align*}
\]
1. You and your partner will make a set of three prime factorization cards, like the card in this example. Each set of cards is to be solved by another pair of students.

Here are the steps for making the cards:

- For each of the three cards, choose a number between 50 and 500.
- Write the problem on the front of the card.
- Write the answer—a factor tree and the prime factorization—on the back.
- Label each card. One card should be easy to solve, one card should be of medium difficulty, and one card should be hard to solve.

2. When your cards are finished, exchange cards with another pair of students.

Solve the problems given on the cards you get, using a separate piece of paper.

After you solve each problem, check the solution on the back. Your solutions may have different factor trees, but they should have the same prime factorizations.

When you have solved all three problems, decide whether you agree with the easy, medium, and hard classifications. Talk to the students who created the cards and suggest any revisions you and your partner think are needed.

Homework

1. Write the prime factorization for each number.
   a. 54       b. 72       c. 88       d. 225

2. a. The prime factorization of 120 is not $2 \cdot 3 \cdot 4 \cdot 5$. Say why.
    b. Write the correct prime factorization.

3. What are the factors (prime or composite) that 54 and 72 have in common? Use your answers to problem 1 to help you.
Lisa, Keesha, Jamal, and Chen were sitting at a table in the Valdez Restaurant. Dwayne came in and said that he missed class today and needed help with common multiples.

Keesha showed him a list of the first twenty-five multiples of 3, and a list of the first ten multiples of 8.

Dwayne noticed that 24 was on both lists. He pointed out that 24 is the result of 3 times 8, which is why it is on both lists.

“You’re on the right track,” Keesha said. She pointed to 24, 48, and 72. “These are the first three numbers you get by multiplying 3 times 8 times something else. So they are common multiples of 3 and 8.” She wrote $3 \cdot 8 \cdot n = 24n$.

When a number is a multiple of two different natural numbers, it is a common multiple of the two numbers.

The easiest way to find a common multiple of two numbers is to multiply the numbers together. The product of two natural numbers is always a multiple of both.

Example

24 is the first common multiple of 3 and 8.

$3 \cdot 8 = 24$

The first twenty-five multiples of 3 are in the set of numbers represented by the expression $3n$.

$3, 6, 9, 12, 15, 18, 21, 24, 27, 30, 33, 36, 39, 42, 45, 48, 51, 54, 57, 60, 63, 66, 69, 72, 75, …$

The first ten multiples of 8 are in the set of numbers represented by the expression $8n$.

$8, 16, 24, 32, 40, 48, 56, 64, 72, 80, …$

The first three common multiples of 3 and 8 are 24, 48, and 72.
1. Find some common multiples of 12 and 15.

2. What is the smallest common multiple of 12 and 15?

3. Two rectangles \( A \) and \( B \) have the same area. Rectangle \( A \) has a base of 18 cm, and rectangle \( B \) has a base of 24 cm.
   - a. What could be the height of each rectangle?
   - b. There are many pairs of heights that will work. Find another pair.

4. Two speed skaters are practicing on an oval ice rink. The champion goes around every 35 seconds. The slower skater goes around every 40 seconds. If they start at the same time, how long will it take for the champ to be ahead of the slower skater by a full lap?

Preparing for the Closing

5. Explain the solutions to problems 3 and 4 in terms of common multiples.

6. Multiplying two or more natural numbers always results in a common multiple of those numbers. Say why.

7. The multiples of 3 are represented as \( 3n \). The multiples of 8 are represented as \( 8n \). The common multiples of 3 and 8 are represented as \( 3 \cdot 8 \cdot n \).
   - a. Say why \( 3 \cdot 8 \cdot n \) gives the common multiples of 3 and 8.
   - b. Say why \( 6 \cdot 8 \cdot n \) does not give all the common multiples of 6 and 8.
Skills

A single digit (shown as □) is missing from a number in each of these equations. Write the range of answers that is possible for each.

Example

34 + 5□ could be 34 + 50 to 34 + 59. So, the range of answers is 84 to 93.

a. 2 + 5□ =

b. 12 + 6□ =

c. □ + 24 =

d. □ + 456 =

e. 345 – □ =

f. 345 – 4□ =

g. 250 – □ =

h. 987,987 – 10□ =

What do you notice about each range of answers?

Review and Consolidation

Use Handout 1: Multiplication Table from Lesson 1 for these problems.

1. Explain how the rows in the multiplication table can be used to find the first three common multiples of 8 and 12.

2. Use the table to find common multiples of each of these pairs of numbers.
   a. 3 and 5
   b. 3 and 9
   c. 6 and 9

3. Explain why common multiples of 3 and 5 are also multiples of 15, but common multiples of 3 and 9 are not necessarily multiples of 27.

4. Which of these expressions gives all of the common multiples of 6 and 9?
   A. 6n  
   B. 9n  
   C. 18n  
   D. 54n
Lesson 4

5. What is the least (or first, or smallest) common multiple of each of these pairs of numbers?
   a. 2 and 5
   b. 3 and 9
   c. 6 and 9
   d. 8 and 12

Homework

1. In the school store, Jamal and Dwayne sell pencils in packages of 8 and pens in packages of 10.
   a. If Chen wants to buy the exact same number of pencils as pens, what is the smallest number of packages of each that he needs to buy?
   b. How many pens would he get?

2. In the grocery store, hot dogs are sold in packages of 8, but hot dog buns are sold in packages of 12.
   How many packages of each do you have to buy to get the same number of hot dogs and buns?

3. Find three common multiples for each set of numbers.
   a. 9 and 12
   b. 6, 8, and 9

4. Write the prime factorization of each number shown in problem 3.

5. Write the prime factorization of each multiple you found in problem 3.
The prime factorizations of two numbers can be used to obtain, in factor form:

- The product of the two numbers
- The greatest common factor (GCF)
- The least common multiple (LCM)

**Example**

Consider these two numbers: 30 and 75

<table>
<thead>
<tr>
<th>Prime Factorization</th>
<th>Greatest Common Factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>30</td>
<td>$2 \cdot 3 \cdot 5$</td>
</tr>
<tr>
<td>75</td>
<td>$3 \cdot 5 \cdot 5$</td>
</tr>
</tbody>
</table>

- The product of 30 • 75 is: $(2 \cdot 3 \cdot 5) \cdot (3 \cdot 5 \cdot 5) = 2250$.
- The greatest common factor (GCF) of 30 and 75 is 15: the $3 \cdot 5$ that occurs in both prime factorizations.
- The least common multiple (LCM) of 30 and 75 is $2 \cdot (3 \cdot 5) \cdot 5 = 150$ (or $5 \cdot 30$ or $2 \cdot 75$).

The LCM contains all of the same factors as 2250, except that the GCF is included only once.

In the example, the product $\text{GCF} \cdot \text{LCM} = (3 \cdot 5) \cdot (2 \cdot 3 \cdot 5 \cdot 5) = 15 \cdot 150 = 2250$.

Can you explain why the GCF • LCM of two numbers is the same as the product of those two numbers?
1. What is the least common factor of these numbers?
   a. 30 and 75
   b. Any two numbers

2. Chen thought that 2250 was the greatest common multiple of 30 and 75. Explain why he was wrong.

3. 13 and 17 are both prime numbers.
   a. What is the greatest common factor of 13 and 17?
   b. What is the least common multiple of 13 and 17?
   c. What is the greatest common factor of any two prime numbers?
   d. What is the least common multiple of any two prime numbers?

4. Two natural numbers, both less than 10, have a greatest common factor of 2. One of these numbers is 8. Let n represent the other number.
   a. What are the possible values of n?
   b. What is the least common multiple of 8 and n?

5. You are given \( p = 2 \cdot 3 \cdot 3 \cdot 5 \) and \( q = 2 \cdot 2 \cdot 5 \cdot 5 \).
   Without doing the multiplication, write these numbers as a prime factorization.
   a. The GCF of \( p \) and \( q \).
   b. The LCM of \( p \) and \( q \).
   c. The product GCF \( \cdot \) LCM.
   d. The product \( pq \).
   e. Your answers to parts c and d should be the same. Say why.
Preparing for the Closing

Work on the following problems with a partner.

6. The product of two numbers is 896 and the GCF of the two numbers is 8.
   \[ \square \times \square = 896 \quad \text{GCF} = 8 \]
   a. If one of the numbers is 8, what is the other number?
   b. If one of the numbers is 16, what is the other number?
   c. Explain why there are no other possible pairs of factors.

7. In your own words, explain why, for any two natural numbers \( m \) and \( n \):
   \[ \text{GCF} \times \text{LCM} = m \times n \]

8. Lisa was given two numbers and she calculated that their GCF was 7 and their LCM was 96.
   Say why she must be wrong.

Skills

A single digit (shown as a \( \square \)) is missing from a number in each of these equations. Write the range of answers that is possible for each.

\[ \begin{align*}
   a. \quad 12 \times \square &= \quad b. \quad 150 \times \square &= \\
   c. \quad 45 \times \square &= \quad d. \quad 15 \times \square &= 
\end{align*} \]

Review and Consolidation

1. For the numbers 48 and 66:
   a. Find the prime factorizations.
   b. Calculate the GCF.
   c. Calculate the LCM.
   d. Perform the calculations to show that \( 48 \times 66 = \text{GCF} \times \text{LCM} \)—the product of the two numbers is equal to the product of the GCF and the LCM.
   e. Calculate \( \text{LCM} \div \text{GCF} \).

2. Explain why, for any two numbers, \( \text{LCM} \div \text{GCF} \) is a whole number.

3. Repeat problem 1 with two numbers of your own choosing. Give this new problem to your partner to solve.
Homework

1. a. Perform the calculations to show that \(2^3 \cdot 3 \cdot 5^2 = 600\) and \(2 \cdot 3^2 \cdot 5 = 90\).
   b. Write the prime factorization of 600 \(\cdot\) 90.
   c. Write the prime factorization of the greatest common factor (GCF) of 600 and 90.
   d. Write the prime factorization of the least common multiple (LCM) of 600 and 90.
   e. Perform the calculations to check that GCF \(\cdot\) LCM = 600 \(\cdot\) 90.
   f. Calculate LCM \(\div\) GCF.

2. Numbers \(p\) and \(q\) are both greater than 10. They have a GCF of 2 and an LCM of 126.
   a. Calculate the value of the product \(pq\).
   b. Explain why \(p\) and \(q\) are both even numbers.
   c. Find possible values for \(p\) and \(q\).
Jamal made this poster to show all that he knew about the numbers 15 and 36.

**15 and 36**

These are all the factors of 36:
- 1, 2, 3, 4, 6, 9, 12, 18, 36

These are all the factors of 15:
- 1, 3, 5, 15

The greatest common factor of 15 and 36 is 3.

These are some multiples of 15:
- 15, 30, 45, 60, 75, 90...
  Notice that they increase by 15.

These are some multiples of 36:
- 36, 72, 108, 144, 180, 216...
  Notice that they increase by 36.

We can find the least common multiple of 15 and 36 from their prime factorizations:
- $15 = 5 \times 3$
- $36 = 2 \times 2 \times 3 \times 3$
- $LCM = 2 \times 2 \times 3 \times 3 \times 5$

**There is only one factor tree for 15**

- $\sqrt{15} = 5 \times 3$

**This is the prime factorization of 36:**

- $36 = 3^2 \times 2^2$

If you write the smallest factors first then $2^2 \times 3^2$ or $2 \times 2 \times 3 \times 3$ is the only way of writing 36 as a product of its prime factors.

The numbers 15 and 36 are composite. Why? Because each can be written as a product of two factors, neither of which is 1. (see the arrays below)

<table>
<thead>
<tr>
<th>15 = 1 x 15</th>
</tr>
</thead>
<tbody>
<tr>
<td>15 = 3 x 5</td>
</tr>
<tr>
<td>36 = 1 x 36</td>
</tr>
<tr>
<td>36 = 2 x 18</td>
</tr>
<tr>
<td>36 = 3 x 12</td>
</tr>
<tr>
<td>36 = 4 x 9</td>
</tr>
<tr>
<td>36 = 6 x 6</td>
</tr>
</tbody>
</table>
1. Make a poster that shows the concepts listed below. In Jamal’s poster, he picked the numbers 15 and 36 to use in examples. You should use two different numbers.

Choose your numbers carefully—just any pair will not show all the concepts. Illustrate the concepts when possible with factor trees, diagrams, or number lines.

- multiples
- factors
- composite numbers
- prime factorization
- greatest common factor
- least common multiple

Preventing for the Closing

2. Write a word problem that involves finding the least common multiple of your two chosen numbers.

3. Compare your poster with those of other students in the class, and make a list of possible improvements to your poster. Make the changes to your poster.

Skills

A single digit (shown as a □) is missing from a number in each of these equations. Write the range of answers that is possible for each.

a. $5 \times 7 □ = $  

b. $4 \times 2 □ = $  

c. $3 \times 9 □ = $  

d. $5 \times 5 □ = $  

Review and Consolidation

1. Look at other students’ posters. Are they mathematically correct?

2. Exchange word problems with your partner, and solve your partner’s word problem.
Homework

1. a. Use a factor tree to find the prime factorization of 72.
   b. List all of the factors of 72.
   c. List all of the factor pairs of 72.
   d. There is one number that is both a factor and a multiple of 72. What number is it?

2. a. Which common multiples of 36 and 72 are less than 200?
   b. Which common factors of 36 and 72 are less than 200?
Lesson 7
Making Sizes that Fit

Goal
To apply the concepts of multiples, factors, prime factorization, greatest common factor, and least common multiple.

Concept Book
See pages 117–138 in your Concept Book.

Work Time

1. Keesha and her mom want to make a 54 × 108-inch quilt out of equal-sized, square pieces of cloth. They have to decide what size squares to use. They do not want to use fractions of squares.

a. What is the largest square that they can use to create a 54 × 108-inch quilt? Sketch and label a diagram of the quilt made with the largest squares.

b. What is the smallest square Keesha and her mom can use? How many of the smallest squares would they need?

c. List the dimensions of each square that can be used to create a 54 × 108-inch quilt.

d. Sketch two of the quilts from your list.
2. This diagram shows the gear wheels of a particular machine.
   a. If the gear wheels start turning, how many full rotations will each wheel make before the wheels are again in the same positions that are shown in the diagram?
   b. If Gear Wheel 1 rotates at a speed of 20 revolutions per second, at what speed does Gear Wheel 2 rotate?
   c. How many times will $X$, $Y$, and $Z$ be in the positions shown in the diagram if the wheels turn for 1 minute at the speeds given in part b?

Preparing for the Closing

3. Choosing from the following set of terms, write a full explanation of your solution to each of the Work Time problems listed below.

   - factor
   - common factor
   - greatest common factor
   - multiple
   - common multiple
   - least common multiple

   a. Problem 1c
   b. Problem 2a
   c. Problem 2c

Skills

Solve.

   a. $32,370 + 4950 = $
   b. $323.70 + 4950 = $
   c. $32,370 + 49,500 = $
   d. $32,370 - 4950 = $
   e. $323.70 - 49.50 = $
   f. $323,700 - 49,500 = $
**Review and Consolidation**

1. **a.** Chen is making a poster with \(9 \times 9\)-inch square pieces of cardboard. He wants to try all of the possible combinations for a rectangular poster that can be made using 20 of these \(9 \times 9\)-inch squares. What are the possible rectangular poster sizes? Give dimensions in inches.

   b. Chen decides to make a giant poster using the \(9 \times 9\)-inch squares. He makes a poster that is 54 inches by 126 inches. How many squares does he use?

   c. Rosa says, “You can make the same-sized poster using larger squares.” What is the largest square Chen can use?

**Homework**

1. In the school store, Jamal is stacking boxes that are 5 inches tall. Dwayne is stacking boxes that are 4 inches tall. Each boy has 24 boxes. At what height will both stacks be the same?

   a. List three heights at which the stacks will be the same.

   b. How many times will the heights of the stacks be the same?

   c. Make a coordinate graph showing both stacks.

2. It is not possible to make a stack of 5"-tall boxes and a stack of 4"-tall boxes that are both 30 inches. Say why.
Adding and subtracting fractions on the number line is like adding and subtracting whole numbers, except that the units on the number line are divided into smaller units.

Example

With a number line marked in quarters, you can add or subtract fractions with denominators of 4.

This number line shows the addition $\frac{7}{4} + \frac{3}{4} = \frac{10}{4} = 2 + \frac{1}{4} = 2\frac{1}{4}$

You can also show subtraction of fractions on a number line.

Example

For instance, $\frac{10}{4} - \frac{3}{4} = \frac{7}{4}$ can be shown as the difference between $\frac{10}{4}$ and $\frac{3}{4}$. 
With a number line marked in sixteenths, you can add or subtract fractions with denominators of 16.

**Example**

This number line shows the addition \( \frac{7}{16} + \frac{13}{16} = \frac{20}{16} = 1 \frac{4}{16} = 1 \frac{1}{4} \).

Fractions can also be represented as equal-sized parts of a square that has an area of 1 square unit. In this diagram, the unit square is divided into twentieths. Putting the two shaded parts together represents the addition \( \frac{5}{20} + \frac{7}{20} = \frac{12}{20} \).

**Work Time**

1. Explain why these equations are true.
   
   a. \( \frac{41}{16} = 2 \frac{9}{16} \)
   
   b. \( \frac{20}{16} = 1 \frac{4}{16} \)
   
   c. \( \frac{200}{16} = 12 \frac{8}{16} \)

2. Explain why these equations are true.
   
   a. \( 1 \frac{4}{16} = 1 \frac{1}{4} \)
   
   b. \( \frac{7}{4} = \frac{28}{16} \)
   
   c. \( \frac{70}{40} = \frac{28}{16} \)

3. Arrange the following six numbers in ascending order (from least to greatest). Some of the quantities may be equal.

   \[ \frac{2}{3}, \frac{2}{3} + \frac{2}{3}, \frac{2 + 2}{3 + 3}, \frac{2}{3} + \frac{2}{3} + \frac{2}{3}, \frac{3}{2}, \frac{3 + 3 + 3}{2 + 2 + 2} \]
4. Use the number lines on Handout 3: Number Lines to sketch diagrams for the following calculations.

   a. \( \frac{2}{3} + \frac{2}{3} + \frac{2}{3} \)
   b. \( \frac{2}{5} + \frac{2}{5} + \frac{2}{5} + \frac{2}{5} \)
   c. \( \frac{1}{15} + \frac{2}{15} + \frac{3}{15} + \frac{4}{15} + \frac{5}{15} \)
   d. \( 3 - \frac{4}{3} \)
   e. \( \frac{10}{15} - \frac{2}{15} \)
   f. \( \frac{40}{15} - \frac{24}{15} \)

5. a. Use fractions with denominators of halves, thirds, fourths, fifths, or fifteenth to write three addition problems that each have an answer of 2.

   Example
   Using a denominator of 15: \( \frac{10}{15} + \frac{10}{15} + \frac{10}{15} = \frac{30}{15} = 2 \)

b. For each addition problem, sketch a diagram using a number line on Handout 3.

6. a. Use fractions with denominators of halves, thirds, fourths, fifths, or fifteenth to write three subtraction problems that each have an answer of 2.

b. For each subtraction problem, sketch a diagram using a number line on Handout 3.
Lesson 8

Factors and Fractions

Preparing for the Closing

7. Explain the similarities and differences between \( \frac{8}{10} \); \( 2 + \frac{8}{10} \); 2.8; and \( \frac{28}{10} \).

8. Write each expression as the sum of two fractions.
   a. \( \frac{(a + b)}{5} \)
   b. \( \frac{(a + b)}{n} \)

9. Share your answers to problem 8 with your partner. Agree on how each addition could be represented on a number line. Prepare your explanations for the Closing.

Skills

Which numbers are factors of the number that is bold?

a. 88: 22, 24, 18, 4 
   b. 87: 23, 17, 29, 2 
   c. 86: 26, 28, 43, 3 
   d. 85: 23, 17, 29, 5 
   e. 128: 22, 32, 18, 4 
   f. 1025: 25, 41, 5, 205

Review and Consolidation

1. Use addition to write the equation that is represented by each number line.

   a. 
   b. 

2. Use subtraction to write the equation that is represented by each number line.

   a. 
   b. 
3. Lisa, Rosa, Jamal, Dwayne, and Chen each baked a pan of brownies for the school bake sale. These are shown in the diagrams.

- Darker shaded parts show the number of brownies sold at lunch.
- Lighter shaded parts show the number of brownies sold after school.
- White parts show the number of brownies left over at the end of the bake sale.

Calculate what fraction of the brownies were sold from each pan. Use addition to write equations that represent the process of putting together the two shaded parts of each diagram.

4. Five other students also baked brownies for the bake sale, but they kept some of the brownies for themselves and their friends. The crossed-out parts in the diagrams show the number of brownies that each student kept.

Calculate what fraction of the brownies were sold from each pan. Use subtraction to write equations that represent the process of removing the fractional parts that have been crossed out of each diagram.
1. Dwayne’s mom made a blueberry pie and divided it into five equal-sized pieces for the five members of the family: Mrs. Jackson, Mr. Jackson, Dwayne, and his two younger brothers.
   
   a. What fraction of the pie was left after the two younger brothers had eaten their pieces?
   
   b. What fraction of the pie was left after Dwayne’s mom had eaten her piece as well?
   
   c. Dwayne wanted two pieces, so his mom got out the knife and gave him two tenths of the pie.
      Was that fair to the other members of the family? Explain.
   
   d. Dwayne ate only one of his pieces, and his dad had not come home to eat his piece yet.
      What fraction of the pie was still uneaten?

2. Jamal found two pieces of a broken 12-inch ruler.
   
   a. How long is each of the pieces he found?
   
   b. Jamal wants to keep the longest piece of ruler.
      Could any of the missing pieces be the longest piece? Explain.
To add or subtract fractions, the fractions must have common denominators. You can make denominators the same by using equivalent fractions.

**Example**

\[ \frac{5}{6} + \frac{3}{4} = \]

Use equivalent fractions with denominators of 24, which is the product of the denominators of the two fractions.

\[ \frac{5}{6} = \frac{5 \cdot 4}{6 \cdot 4} = \frac{20}{24} \quad \text{and} \quad \frac{3}{4} = \frac{3 \cdot 6}{4 \cdot 6} = \frac{18}{24} \]

So, \( \frac{5}{6} + \frac{3}{4} = \frac{20}{24} + \frac{18}{24} = \frac{38}{24} \)

**Equivalent Fractions**

The equation is represented on this number line.

You can also use equivalent fractions with denominators that are the least common multiple of the denominators of the two fractions. Can you explain why?

**Example**

This time use equivalent fractions with denominators of 12, which is the LCM of the denominators of the two fractions.

\[ \frac{5}{6} = \frac{5 \cdot 2}{6 \cdot 2} = \frac{10}{12} \quad \text{and} \quad \frac{3}{4} = \frac{3 \cdot 3}{4 \cdot 3} = \frac{9}{12} \]

So, \( \frac{5}{6} + \frac{3}{4} = \frac{10}{12} + \frac{9}{12} = \frac{19}{12} \)

**Equivalent Fractions**
In the first example, \(\frac{5}{6} + \frac{3}{4} = \frac{38}{24}\), while in the second example \(\frac{5}{6} + \frac{3}{4} = \frac{19}{12}\).

These two answers are equivalent because \(\frac{38}{24}\) and \(\frac{19}{12}\) are equivalent fractions. \(\frac{19}{12}\) is in simplest form, with numerator and denominator having no common factors.

**Work Time**

1. Copy the following equations and fill in the missing numerators.
   
   a. \(\frac{2}{3} = \frac{}{12} = \frac{}{15} = \frac{}{60}\)
   
   b. \(\frac{3}{5} = \frac{}{15} = \frac{}{20} = \frac{}{60}\)
   
   c. \(\frac{1}{4} = \frac{}{12} = \frac{}{20} = \frac{}{60}\)

2. Use equivalent fractions from the previous problem to change the following additions into “common denominator” problems. Write the answer for each addition in simplest form.
   
   a. \(\frac{2}{3} + \frac{3}{5} = \frac{}{15} + \frac{}{15} = \frac{}{15}\)
   
   b. \(\frac{2}{3} + \frac{1}{4} = \frac{}{12}\)
   
   c. \(\frac{3}{5} + \frac{1}{4} = \frac{}{20}\)
   
   d. \(\frac{2}{3} + \frac{1}{4} + \frac{3}{5} = \frac{}{60}\)

3. Write the numerators to complete the following calculations. Write the answer for each addition in simplest form.
   
   a. \(\frac{3}{8} + \frac{5}{6} = \frac{}{48} + \frac{}{48} = \frac{}{48}\)
   
   b. \(\frac{3}{8} + \frac{1}{12} = \frac{}{96} + \frac{}{96} = \frac{}{96}\)
   
   c. \(\frac{7}{10} + \frac{8}{15} = \frac{}{150} + \frac{}{150} = \frac{}{150}\)
   
   d. \(\frac{3}{10} + \frac{25}{100} = \frac{}{1000} + \frac{}{1000} = \frac{}{1000}\)

4. Write the numerators to complete the following calculation.

\[
\frac{a}{b} + \frac{c}{d} = \frac{\text{ }}{bd} + \frac{\text{ }}{bd} = \frac{\text{ } + \text{ }}{bd}
\]
5. For the additions in problem 3:
   a. Why were those common denominators chosen?
   b. Could the additions be carried out using smaller denominators? Say why.

Skills

Which numbers are multiples of the number that is bold?

a. 12: 24, 28, 36, 44  
   d. 24: 24, 28, 36, 44

b. 13: 30, 39, 83, 156  
   e. 39: 30, 39, 83, 156

c. 14: 41, 70, 74, 126  
   f. 140: 410, 700, 740, 1260

Review and Consolidation

1. a. Use the number line with a scale marked in twenty-fourths from Handout 4: More Number Lines to represent the equation
   \[
   \frac{15}{24} + \frac{18}{24} = \frac{33}{24}.
   \]
   
   b. Explain why the equation in part a is equivalent to
   \[
   \frac{5}{8} + \frac{3}{4} = \frac{11}{8}.
   \]

2. a. Use the number line with a scale marked in eighteenths to represent the equation
   \[
   \frac{6}{18} + \frac{8}{18} = \frac{14}{18}.
   \]
   
   b. Write an equivalent equation using a smaller denominator.
   
   c. Explain how your number line represents this new equation.

3. a. Use the number line with a scale marked in sixteenths to represent the equation
   \[
   \frac{8}{16} + \frac{12}{16} = \frac{20}{16}.
   \]
   
   b. Write an equivalent equation using a smaller denominator.
   
   c. Explain how your number line represents this new equation.
4. Calculate these expressions, using equivalent fractions. Write your answers in simplest form.

\[
\begin{align*}
\text{a. } & \quad \frac{5}{7} + \frac{3}{4} \\
\text{b. } & \quad 2\frac{5}{7} + \frac{3}{4} \\
\text{c. } & \quad \frac{5}{8} + \frac{3}{5} \\
\text{d. } & \quad 2\frac{5}{8} + 1\frac{3}{5} \\
\text{e. } & \quad \frac{5}{20} + \frac{36}{100} \\
\text{f. } & \quad 1\frac{5}{20} + 10\frac{36}{100}
\end{align*}
\]

Comment
Remember that “simplest form” refers to a fraction in which the numerator and denominator have no common factors.

Homework

1. Each large square represents the number 1, and the shaded parts represent fractions. For each diagram:

• Write the fractions as equivalent fractions in simplest form.
• Add the two fractions together.

a. 

b. 

c. 

d. 

e. 

f. 

2. Lisa’s mom brought home a pizza for dinner one night. Lisa said that since her mom had worked all day, she should have one-third of the pizza. Lisa should get one-quarter, and Annie, Lisa’s younger sister, should get one-sixth, since she is the smallest.

a. What fraction of the pizza would Lisa and Annie get between them by sharing in this way?

b. What fraction of the pizza would the three family members eat?

c. What fraction of the pizza would be left over?
In class one day, Ms. Reynolds asked students to measure the length of their strides—the distance between their steps when walking.

Chen found that his stride was $\frac{6}{10}$ of a meter long.

Ms. Reynolds asked the class if anyone knew how far Chen would walk in five steps.

She drew a chalk number line on the floor from 0 to 3 meters, with a scale marked in tenths, and told him to stand at 0 and walk toward 3. Chen landed exactly on 3 with his fifth step.

Ms. Reynolds said, “Chen took 5 steps of $\frac{6}{10}$ of a meter and went 3 meters.”

Multiplying a fraction by a whole number can be shown as repeated addition.

Since division is the inverse process of multiplication, you can write four equations showing the numerical relationships between these numbers.

<table>
<thead>
<tr>
<th>Multiplication</th>
<th>Repeated Addition</th>
<th>Division (two inverse statements)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$5 \times \frac{6}{10} = 3$</td>
<td>$\frac{6}{10} + \frac{6}{10} + \frac{6}{10} + \frac{6}{10} + \frac{6}{10} = 3$</td>
<td>$3 \div 5 = \frac{6}{10}$ $3 \div \frac{6}{10} = 5$</td>
</tr>
</tbody>
</table>
1. Write the multiplication equation for each number line (scales marked in tenths).

   a.
   b.
   c.
   d.
   e.
   f.

2. Represent the calculation $9 \cdot \frac{3}{8}$ as repeated steps on a number line.
   Use a number line from 0 to 4, with a scale marked in eighths.

3. Calculate the following expressions. Write your answers in simplest form.
   You can use your number line from the previous problem to check your answers.
   a. $4 \cdot \frac{3}{8}$
   b. $6 \cdot \frac{3}{8}$
   c. $8 \cdot \frac{3}{8}$

4. Use the result for problem 2 to calculate the following expressions.
   a. $\frac{27}{8} \div 9$
   b. $\frac{27}{8} \div \frac{3}{8}$
Preparing for the Closing

5. Compare your solutions with those of your partner. Together, check that all your answers are in simplest form.

6. Use division to write two equations for each number line in problem 1.

7. A student correctly used the rule $n \cdot \frac{a}{b} = \frac{na}{b}$, but did not get the answer that the teacher wanted. Explain how this could happen.

Skills

Sketch number lines to represent the following situations.

a. Keesha’s bracelet has 28 beads. Rosa’s necklace has 75 beads more than Keesha’s bracelet.

<table>
<thead>
<tr>
<th>Keesha</th>
<th>28 beads</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rosa</td>
<td>75 more beads than Keesha</td>
</tr>
</tbody>
</table>

How many beads does Rosa’s necklace have?

b. Each morning Jamal rides his bike 2.4 miles to school, and Chen rides his bike 0.8 miles.

Compared to Chen, how much farther does Jamal ride his bike to school?

c. At a track meet, Lisa jumped 3.25 meters in the long-jump competition, and Rosa jumped 2.75 meters.

Compared to Rosa, how much farther did Lisa jump?
Review and Consolidation

1. Lisa’s class is making a film. In the film, Lisa, whose real height is 5 feet 4 inches, must look like she is 4 feet tall. The students achieve this by building a set in which everything is much bigger than normal.

Lisa’s actual height is $\frac{4}{3}$ larger than the size she needs to appear.

\[
\frac{5'4''}{4'0''} = \frac{64''}{48''} = \frac{4}{3}
\]

For Lisa to appear small, each of the props needs to be scaled to $\frac{4}{3}$ of its actual size.

Copy and complete this table, leaving three or four extra lines at the bottom.

<table>
<thead>
<tr>
<th>Normal Size</th>
<th>Large Set Design Size</th>
</tr>
</thead>
<tbody>
<tr>
<td>Doorway—90 inches high</td>
<td></td>
</tr>
<tr>
<td>Chair—36 inches high</td>
<td></td>
</tr>
<tr>
<td>Cup—9 cm wide</td>
<td></td>
</tr>
<tr>
<td>Plate—27 cm wide</td>
<td></td>
</tr>
<tr>
<td>Spear—5 feet 6 inches long</td>
<td></td>
</tr>
</tbody>
</table>

2. An identical but smaller set must be built for Jamal, whose real height is 5 feet 10 inches. The set design must make Jamal look like he is a 7-foot giant.

a. Calculate the fraction by which the props need to be scaled.

Hint: Use problem 1 as a guide.

b. Copy and complete this table, leaving three or four extra lines at the bottom.

<table>
<thead>
<tr>
<th>Normal Size</th>
<th>Small Set Design Size</th>
</tr>
</thead>
<tbody>
<tr>
<td>Doorway—90 inches high</td>
<td></td>
</tr>
<tr>
<td>Chair—36 inches high</td>
<td></td>
</tr>
<tr>
<td>Cup—9 cm wide</td>
<td></td>
</tr>
<tr>
<td>Plate—27 cm wide</td>
<td></td>
</tr>
<tr>
<td>Spear—5 feet 6 inches long</td>
<td></td>
</tr>
</tbody>
</table>
3. Measure the normal size of three or four other objects, and then calculate the sizes they would have to be in both the large set design and the small set design. Add your results to the tables.

Homework

1. a. Sketch a number line of \( \frac{2}{3} + \frac{2}{3} + \frac{2}{3} + \frac{2}{3} + \frac{2}{3} \).

   b. Use multiplication to write an equation for your diagram.

2. Explain why \( \frac{12}{3} = \frac{4}{1} \).

3. How would you change the number line you sketched for problem 1 to make it represent \( 6 \cdot \frac{8}{12} = \frac{48}{12} \)?

4. Use division to write two equations that are equivalent to \( 5 \cdot \frac{2}{3} = \frac{10}{3} \).
Lesson 11

Multiplying Fractions

Goal

To multiply fractions, and to represent products as areas of rectangles.

Concept Book

See pages 149–150 in your Concept Book.

This area diagram shows two number lines. The horizontal number line is marked in quarters. The vertical number line is marked in fifths.

The unit square is the dark square in the diagram.

It is shown as \(1 = \frac{20}{20}\), twenty small rectangles, which each have an area of \(\frac{1}{20}\) of a square unit.

The large rectangle shows the product \(\frac{7}{5} \cdot \frac{7}{4} = \frac{49}{20}\). It has an area of \(\frac{49}{20} = 2 \frac{9}{20}\) square units, made up of the 49 small rectangles, which each also have an area of \(\frac{1}{5} \cdot \frac{1}{4} = \frac{1}{20}\) of a square unit.

The product of any two rational numbers can be represented by an area diagram.

- The numerator of the answer is the total number of small rectangles.
- The denominator of the answer is the number of small rectangles in the unit square.

To multiply two fractions, multiply the numerators and multiply the denominators.

In symbols: \(\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}\).
1. The rule for multiplying fractions is \( \frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd} \).

Copy and complete the sentence: To multiply two fractions,....

2. Each of these diagrams represents the multiplication of two fractions. The unit square is shown in dark shading. In some cases, it is partly hidden by a rectangle with lighter shading.

![Diagram A](image1)

![Diagram B](image2)

![Diagram C](image3)

![Diagram D](image4)

![Diagram E](image5)

![Diagram F](image6)

a. Which area diagram represents \( \frac{3}{4} \cdot \frac{5}{6} = \frac{15}{24} = \frac{5}{8} \)?

b. Write the equation represented by each of the other diagrams.

c. Explain how the numerators and denominators of the answers are represented on the diagrams.

d. Which of the diagrams represent fractions that are less than 1, and which represent fractions that are greater than 1? Explain how this is shown on the diagrams.
Preparing for the Closing

3. a. How is the numerator of each product shown in the diagrams?
   b. How is the denominator of each product shown in the diagrams?

4. For the calculation of $3 \frac{3}{4} \cdot 3 \frac{1}{5}$, Chen argued that $3 \cdot 3 = 9$ and $\frac{3}{4} \cdot \frac{1}{5} = \frac{3}{20}$, so the answer is $9 \frac{3}{20}$.

   Explain the mistake that he made, and show the correct way to perform the calculation.

5. The product of two fractions is 1. What does this tell you about their numerators and denominators?

Skills

Solve.

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>a. $250 + 250 + 250$ =</td>
<td>b. $x + x + x =$</td>
<td>c. $(2 \cdot 50) + (6 \cdot 50)$ =</td>
</tr>
<tr>
<td>d. $2n + 6n =$</td>
<td>e. $(4 \cdot 45) - 60 + 45 =$</td>
<td>f. $4a - 60 + a =$</td>
</tr>
</tbody>
</table>

Review and Consolidation

1. Write the following numbers in the form $\frac{a}{b}$, where $a$ and $b$ are whole numbers.
   a. $1\frac{7}{8}$
   b. $3\frac{1}{5}$
   c. $2.4$

2. Calculate the following expressions, using the answers to the previous problem and equivalent fractions. Write your answers in simplest form.
   a. $1\frac{7}{8} \cdot 3\frac{1}{5}$
   b. $3\frac{1}{5} \cdot 2.4$
   c. $1\frac{7}{8} \cdot 2.4$

3. Calculate.
   a. $1\frac{7}{8} \cdot \frac{8}{15}$
   b. $3\frac{1}{5} \cdot \frac{5}{16}$
   c. $\frac{5}{12} \cdot 2.4$

4. Use multiplication to write three more equations, each with a product of 1.
1. What is the rule for multiplying fraction \( \frac{a}{b} \) by fraction \( \frac{c}{d} \)?

2. Some of these fractions are in simplest form and some are not. Write all of them in simplest form.
   - a. \( \frac{9}{24} \)
   - b. \( \frac{27}{24} \)
   - c. \( \frac{27}{35} \)
   - d. \( \frac{42}{35} \)
   - e. \( \frac{35}{27} \)
   - f. \( \frac{36}{27} \)

3. Calculate these expressions, using equivalent fractions. Write your answers in simplest form.
   - a. \( \frac{1}{8} \cdot \frac{9}{3} \)
   - b. \( \frac{3}{4} \cdot \frac{9}{6} \)
   - c. \( \frac{9}{5} \cdot \frac{3}{7} \)
   - d. \( \frac{14}{5} \cdot \frac{3}{7} \)
   - e. \( \frac{5}{9} \cdot \frac{7}{3} \)
   - f. \( \frac{2}{9} \cdot \frac{18}{3} \)
The fraction $\frac{6}{8}$ can be represented in the following eight ways:

- An equivalent fraction in simplest form, $\frac{3}{4}$
- A position on the number line

![Number line with the fraction $\frac{6}{8}$ marked between 0 and 1.]

- The shaded part of a unit square

![Unit square with 6 out of 8 parts shaded.]

- A repeated addition, $\frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{2}{8} + \frac{2}{8} + \frac{2}{8}$
- A sum of two fractions with common denominators, $\frac{1}{8} + \frac{5}{8}$
- A sum of fractions with two different denominators, $\frac{1}{2} + \frac{2}{8}$
- A product of a whole number and a fraction, $2 \cdot \frac{3}{8}$
- A product of two fractions, $\frac{2}{4} \cdot \frac{3}{2}$
Work Time

Work in groups of four students.
Each person will need four blank cards for problem 2.

1. Represent the fraction $\frac{10}{12}$ in all eight ways listed on page 48.

2. Choose a fraction of your own, and write a different representation of it on each of your four cards. Leave one side of each card blank.

3. Put all 16 of your group’s cards face down on the floor and play a “Memory Pairs” matching game. Give extra points to any person who picks up both pairs of cards that represent the same number.

Preparing for the Closing

4. Some of the representations in problem 1 have more than one correct answer. Discuss this with your group.

5. Take a set of four cards from another group, and write the other four representations of their chosen fraction on the backs of the cards.

6. This rectangle is made up of 6 squares; each square contains 4 parts.

$\frac{6}{4}$ of the rectangle is shaded. Is this statement, true, false, or can you not tell?
Lesson 12

Skills

Write the value of each expression when \( x = 27 \).

\[
\begin{align*}
a. \quad & \frac{5x}{3} \\
b. \quad & \frac{35 - x}{3} \\
c. \quad & 25x \\
d. \quad & 1404 \div x \\
e. \quad & 3x + 20 \\
f. \quad & 3x + 6x - 20
\end{align*}
\]

Review and Consolidation

1. Represent the fraction \( \frac{10}{16} \) in each of these ways.

a. An equivalent fraction in simplest form
b. A position on the number line
c. The shaded part of a unit square
d. A repeated addition
e. A sum of two fractions with common denominators
f. A sum of two fractions with different denominators
g. A product of a whole number and a fraction
h. A product of two fractions

2. Represent the number \( \frac{1}{1} \) in each of these ways.

a. An equivalent fraction
b. A repeated addition
c. A sum of two fractions with common denominators
d. A sum of two fractions with different denominators
e. A product of a whole number and a fraction
f. A product of two fractions
g. Any number (represent using a letter)
3. What advice would you give to a student who performed these calculations?

\[
\frac{5}{8} + \frac{7}{24} = \frac{(5 \cdot 24) + (7 \cdot 8)}{8 \cdot 24} = \frac{120 + 56}{192} = \frac{176}{192} = \frac{176}{192}
\]

\[
\frac{5}{16} + \frac{3}{16} = \frac{(5 \cdot 16) + (3 \cdot 16)}{16 \cdot 16} = \frac{80 + 48}{256} = \frac{128}{256}
\]

Homework

1. Choose a fraction different from the one you chose in Work Time. Represent it in each of these ways.
   a. An equivalent fraction
   b. A position on the number line
   c. The shaded part of a unit square
   d. A repeated addition
   e. A sum of two fractions with common denominators
   f. A sum of two fractions with different denominators
   g. A product of a whole number and a fraction
   h. A product of two fractions
For each addition equation $a + b = c$, there are two inverse equations involving subtraction. They are $c - b = a$ and $c - a = b$.

**Example**

\[
\frac{10}{12} + \frac{9}{12} = \frac{17}{12}
\]

The inverse equations are \( \frac{17}{12} - \frac{10}{12} = \frac{9}{12} \) and \( \frac{17}{12} - \frac{9}{12} = \frac{10}{12} \).

To represent these equations on the number line, the scale on the number line must be in twelfths.

To determine if one fraction is greater than another, you need to subtract the fractions.

**Example**

Which is greater, \( \frac{3}{4} \) or \( \frac{5}{6} \)?

If you subtract \( \frac{3}{4} - \frac{5}{6} = \frac{9}{12} - \frac{10}{12} = -\frac{1}{12} \), the answer is negative.

This means that \( \frac{3}{4} \) is less than \( \frac{5}{6} \). You can write \( \frac{3}{4} \) is less than \( \frac{5}{6} \) as: \( \frac{3}{4} < \frac{5}{6} \).
1. Explain why each equation or inequality is true.
   a. \( \frac{9}{15} - \frac{10}{15} = \frac{-1}{15} \)
   b. \( \frac{9}{15} = \frac{3}{5} \)
   c. \( \frac{10}{15} = \frac{2}{3} \)
   d. \( \frac{3}{5} < \frac{2}{3} \)

2. a. Calculate \( \frac{3}{5} - \frac{5}{8} \).
   b. Calculate \( \frac{2}{3} - \frac{5}{8} \).
   c. Use less than signs (<) to arrange \( \frac{2}{3}, \frac{3}{5}, \) and \( \frac{5}{8} \) in ascending order (from least to greatest).
   d. Arrange the fractions \( \frac{4}{5}, \frac{5}{6}, \frac{6}{7}, \) and \( \frac{7}{8} \) in ascending order.
   e. Do the subtraction calculations to check your answer to part d.

Preparing for the Closing

3. a. Explain the rule \( \frac{a}{b} - \frac{c}{d} = \frac{ad - bc}{bd} \) and why it was not used in the calculation of \( \frac{3}{4} - \frac{5}{6} \) in the beginning of the lesson.
   b. Use the rule to calculate \( \frac{7}{8} - \frac{3}{4} \).
   c. Explain how to calculate \( \frac{7}{8} - \frac{3}{4} \) without using the rule.
Lesson 13

Skills

Solve.

a. Jamal earned $25 a day at his summer job.
How much did he earn in 15 days?

b. Mrs. Jackson wanted to buy 50 erasers for her classroom.
How much did she pay if each eraser cost $0.37?

c. Rosa filled 6 glasses with 1 bottle of apple juice.
How many glasses could she fill with 5 bottles?

Review and Consolidation

1. Lisa calculated \( \frac{11}{6} - \frac{7}{6} \) as \( \frac{66}{36} - \frac{42}{36} = \frac{24}{36} \).

   a. Is the answer correct?
   b. What advice would you give to Lisa?

2. Lisa also calculated \( \frac{11}{20} - \frac{7}{30} \) as \( \frac{11 \cdot 30}{20 \cdot 30} - \frac{7 \cdot 20}{30 \cdot 20} = \frac{330 - 140}{600} = \frac{190}{600} \).

   a. Is the answer correct?
   b. What advice would you give to Lisa this time?

3. Lisa also calculated \( \frac{12}{20} - \frac{15}{30} \) as \( \frac{12 \cdot 30}{20 \cdot 30} - \frac{15 \cdot 20}{30 \cdot 20} = \frac{360 - 300}{600} = \frac{60}{600} \).

   a. Is the answer correct?
   b. What advice would you give to Lisa this time?

4. Annie, Lisa’s sister, who knows much less about fractions, said that \( \frac{7}{8} + \frac{7}{8} = \frac{14}{16} \)
and \( \frac{7}{8} - \frac{7}{8} = 0 \). What advice would you give to Annie?
Homework

1. a. Interpret this diagram as an equation involving the subtraction of fractions with denominators of 20.
   
   b. Rewrite this equation using equivalent fractions in simplest form.

2. Calculate.
   
   a. \( \frac{3}{4} - \frac{5}{7} = \)
   
   b. \( \frac{3}{4} - \frac{6}{8} = \)
   
   c. \( \frac{3}{4} - \frac{7}{9} = \)
   
   d. \( \frac{4}{5} - \frac{7}{9} = \)
   
   e. \( \frac{4}{5} - \frac{8}{10} = \)
   
   f. \( \frac{4}{5} - \frac{9}{11} = \)

3. Use your results from the previous problem to write \( \frac{3}{4}, \frac{5}{7}, \frac{6}{8}, \frac{7}{9}, \frac{4}{5}, \frac{8}{10}, \) and \( \frac{9}{11} \) in descending order (from greatest to least).
Suppose $p, q, r,$ and $s$ are rational numbers and that $p + q + r = s$.

This can be shown on a number line.

<table>
<thead>
<tr>
<th>The distance between:</th>
<th>Can be labeled either:</th>
<th>Since:</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A$ and the origin</td>
<td>$p + q + r$ or $s$</td>
<td>$p + q + r = s$</td>
</tr>
<tr>
<td>$B$ and the origin</td>
<td>$p$ or $s - r - q$</td>
<td>$p = s - r - q$</td>
</tr>
<tr>
<td>$C$ and the origin</td>
<td>$p + q$ or $s - r$</td>
<td>$p + q = s - r$</td>
</tr>
<tr>
<td>$A$ and $C$</td>
<td>$r$ or $s - q - p$</td>
<td>$r = s - q - p$</td>
</tr>
<tr>
<td>$B$ and $C$</td>
<td>$q$ or $s - p - r$</td>
<td>$q = s - p - r$</td>
</tr>
</tbody>
</table>

If you find values of $p, q, r,$ and $s$ that satisfy any one of the equations, then the same values should satisfy the other equations as well.
Work Time

1. a. Perform the calculations to show that \(2\frac{1}{4} - 2\frac{3}{4} = 3 + 5\).

b. Perform the calculations to show that \(\frac{2}{3} + \frac{3}{4} + \frac{5}{6} = 2\frac{1}{4}\).

c. Perform the calculations to show that \(2\frac{1}{4} - 2\frac{3}{4} - \frac{3}{4} = \frac{5}{6}\).

2. Perform the calculations to show that the fractions \(\frac{1}{4}, \frac{3}{8}, \frac{5}{16}\), and \(\frac{15}{16}\) satisfy all five of the equations listed in the beginning of the lesson.

Preparing for the Closing

3. a. Sketch a number line of the equation \(\frac{1}{4} + \frac{3}{8} + \frac{5}{16} = \frac{15}{16}\).

   Use a scale of sixteenths on your number line.

b. Which equivalent equations are also represented by the same diagram? Explain.

c. Your diagram could have been sketched with the numbers \(\frac{1}{4}, \frac{3}{8}\), and \(\frac{5}{16}\) shown in a different order on the number line.

   What equations would have been represented by these different diagrams?

Skills

Keesha bought six apples for \(x\) dollars each. She also bought a pineapple for $5.

a. Write an equation for the total cost of the fruit in terms of \(x\).

b. If an apple cost $2, what was the total cost of the fruit?

c. If \(x = 3\), what was the total cost of the fruit?
Lesson 14

Review and Consolidation

1. Copy the square below and fill in the missing numbers.

The rule is that the following twelve sums must be the same:

- The sum of the five numbers in any row (↔)
- The sum of the five numbers in any column (↑)
- The sum of the five numbers across each of the two diagonals (×)

<table>
<thead>
<tr>
<th>19/16</th>
<th>7/16</th>
<th>5/4</th>
<th>13/16</th>
<th>1/16</th>
</tr>
</thead>
<tbody>
<tr>
<td>5/8</td>
<td>3/16</td>
<td>1</td>
<td>9/16</td>
<td></td>
</tr>
<tr>
<td>3/8</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>15/16</td>
<td>1/2</td>
<td>21/16</td>
<td></td>
</tr>
<tr>
<td>23/16</td>
<td></td>
<td>17/16</td>
<td>5/16</td>
<td></td>
</tr>
</tbody>
</table>

Homework

1. a. Copy this table. Fill in the five empty spaces by adding the numbers in each row and each column. Check that you get the same answer for the bottom right corner by adding the row and the column.

   \[
   \begin{array}{cc}
   \frac{5}{6} & \frac{1}{2} \\
   \frac{1}{3} & \frac{2}{9} \\
   \end{array}
   \]

b. Copy the table again. This time, fill in the empty spaces by calculating differences instead of sums. Check that you get the same answer for the bottom corner by subtracting the row and the column.

   \[
   \begin{array}{cc}
   \frac{5}{6} & \frac{1}{2} \\
   \frac{1}{3} & \frac{2}{9} \\
   \end{array}
   \]
Work Time

One Saturday, Jamal, Dwayne, and Chen went for a hike in the woods. They planned to climb to the top of a mountain and back again before dark.

When they got to the waterfall, they saw that the trail split into two. Dwayne wanted to walk along the river to the pond, but Chen thought it would be faster to get to the top of the mountain if they went past the big rock.

1. a. Which is the shorter path from the waterfall to the top of the mountain? Support your answer using the distances shown in the diagram.

b. Jamal, Dwayne, and Chen can hike the uphill portion of each trail at only $\frac{4}{5}$ of the speed at which they can hike the lower portions of the trails.

Which path from the waterfall to the top of the mountain will take less time? Support your answer using the information about speeds and distances that is given.
2. Lisa wanted to know how much shorter the distance was from her home to the school than the distance from her home to the stadium to the school.

Help Lisa determine the answer. Use the distances shown in the diagram.

Preparing for the Closing

3. For both parts of problem 1, explain the operation you chose and why that operation made sense to solve the problem.

4. Discuss your solutions to the Work Time problems with your partner, and prepare explanations and calculations to present during the Closing.

Skills

Solve.

a. $2 \times 89 =$  

b. $20 \times 89 =$  

c. $200 \times 89 =$  

d. $0.2 \times 89 =$  

e. $178 \div 2 =$  

f. $0.178 \div 2 =$  

g. $1.78 \div 2 =$  

h. $0.178 \div 0.2 =$  

Review and Consolidation

1. Mark five points anywhere on a sheet of paper.
   • Label the points A through E.
   • Sketch the 10 lines that join every point to each of the others.
   • Swap your paper with that of your partner, and use it to answer parts a and b.

   a. Measure the length of each line to the nearest sixteenth of an inch.

   b. Identify the four lines that join the five points with the shortest possible total distance.

   c. With your partner, discuss your solutions, and suggest ways that a problem of this type could be involved in connecting classrooms to a computer network.

Homework

1. \(x, y, \) and \(z\) are proper fractions between 0 and 1. Find values for \(x, y, \) and \(z\) in each problem that will make the equation true.

   a. \(x + y = 1\)       b. \(x + y + z = 1\)       c. \(x - y - z = 1\)
   d. \(xy = 1\)            e. \(x \div y = 1\)            f. \(xy + z = 1\)
   g. \(xy + xz = 1\)       h. \(x(y + z) = 1\)       i. \(x \div yz = 1\)
   j. \(x \div y \div z = 1\)

2. a. In which parts of problem 1 can \(x\) and \(y\) be the same fraction?

   b. In which part of problem 1 must \(x\) and \(y\) be the same fraction?
Lisa, Rosa, Jamal, and Dwayne went on an early morning jog and then stopped by Dwayne’s house to do their math homework. The topic they were studying was dividing by fractions.

Lisa said, “I don’t understand why six divided by one-half is twelve. How can you get a larger number when you divide by a smaller one?”

“That’s a good question,” replied Jamal. “Here’s how I think about it. If I have six objects and I divide them by three, I get two groups of three objects.”

\[
\begin{align*}
\text{Start with 6} & \quad \text{Divide by 3} \\
& \quad (\text{Think: How many 3s in 6?}) \\
& \quad 6 \div 3 = 2 \\
& \quad (\text{There are two 3s in 6.}) \\
\end{align*}
\]

“But, if I have six objects and I divide them by \(\frac{1}{2}\), I get twelve groups. There are 12 halves.”

\[
\begin{align*}
\text{Start with 6} & \quad \text{Divide by } \frac{1}{2} \\
& \quad (\text{Think: How many } \frac{1}{2}\text{s in 6?}) \\
& \quad 6 \div \frac{1}{2} = 12 \\
& \quad (\text{There are twelve } \frac{1}{2}\text{s in 6.}) \\
\end{align*}
\]

“Oh, now that makes sense,” said Lisa.
When you compare the equation \( 3 \div \frac{6}{10} = 5 \) with the equation \( 3 \cdot \frac{10}{6} = \frac{30}{6} = 5 \), you see that both equations turn 3 into 5.

The numbers \( \frac{6}{10} \) and \( \frac{10}{6} \), with their numerators and denominators interchanged, are called *reciprocals* of each other.

Dividing by a fraction is the same as multiplying by the reciprocal of the fraction.

**Example**

\[
\frac{3}{4} \div \frac{6}{10} = \frac{3}{4} \cdot \frac{10}{6} = \frac{30}{24} = \frac{5}{4} = 1\frac{1}{4}.
\]

Note that the answer is greater than 1, since \( \frac{3}{4} \) is greater than \( \frac{6}{10} \).

As you know, multiplication can be written as repeated addition. Since division is the inverse of multiplication and subtraction is the inverse of addition, division can be written as repeated subtraction.

**Example**

\[
3 \div \frac{6}{10} = 5 \text{ can be written as } 3 - \frac{6}{10} - \frac{6}{10} - \frac{6}{10} - \frac{6}{10} - \frac{6}{10} = 0.
\]

**Work Time**

1. a. Calculate \( 6 \div \frac{3}{4} \).

   b. Write the equivalent equation using multiplication.

   c. Explain why the answer to part a is greater than 6.
2. a. For $\frac{6}{10} \div \frac{3}{4}$, should the answer be greater than or less than $\frac{6}{10}$? Should the answer be greater than or less than 1? Say why.

b. Perform the calculation.

c. Write the equivalent equation using multiplication.

3. Calculate.

   a. $\frac{8}{5} \div \frac{2}{5} = \phantom{0}$
   
   b. $\frac{7}{6} \div \frac{7}{3} = \phantom{0}$

   c. $\frac{7}{6} \div \frac{7}{12} = \phantom{0}$

   d. $\frac{9}{4} \div \frac{12}{5} = \phantom{0}$

   e. $\frac{12}{5} \div \frac{9}{4} = \phantom{0}$

   f. $\frac{12}{5} \div \frac{4}{9} = \phantom{0}$

4. Which of these expressions are equal to $\frac{6}{10} \div \frac{3}{4}$?

   a. $\left(\frac{6}{10} \div 3\right) \div 4$

   b. $\left(\frac{6}{10} \div 3\right) \cdot 4$

   c. $\frac{6}{10} \cdot \frac{4}{3}$

   d. $\frac{10}{6} \cdot \frac{3}{4}$

   e. $(6 \cdot 4) \div (3 \cdot 10)$

**Preparing for the Closing**

5. Write a word problem to match each equation.

   a. $\frac{3}{4} \div 6 = \frac{1}{8}$

   b. $\frac{3}{4} \div \frac{1}{10} = \frac{7}{2}$

   c. $\frac{3}{4} \div \frac{6}{10} = \frac{1}{4}$

6. What could you do to check the answers to problem 3?

7. Lisa wrote $\frac{3}{4} \div \frac{6}{10} = \frac{4}{3} \cdot \frac{6}{10} = \frac{24}{30} = \frac{4}{5}$.

   a. What mistake did she make?

   b. What advice would you give to Lisa?

8. Explain the rule for calculating $\frac{a}{b} \div \frac{c}{d}$. 

64 | Factors and Fractions
Skills

Mrs. Jackson bought 5 bottles of tomato juice. She gave the cashier $30 and received $x$ dollars in change.

a. Write an equation for the cost of a bottle of juice.

b. If she received $10 in change, what was the price of the juice?

c. If a bottle of juice cost $1.50, how much change did she receive?

Review and Consolidation

1. Calculate these expressions. Write your answers in simplest form.

   a. \( \frac{8}{5} \div \frac{4}{15} \)
   
   b. \( \frac{6}{5} \div \frac{3}{20} \)
   
   c. \( \frac{2}{3} \div \frac{3}{4} \)
   
   d. \( \frac{4}{15} \div \frac{8}{5} \)
   
   e. \( \frac{5}{6} \div \frac{20}{3} \)
   
   f. \( \frac{3}{5} \div 2 \frac{1}{2} \)

2. Check the answers to the previous problems by multiplying the answers by the original divisors.

3. A cup holds \( \frac{3}{8} \) liters of liquid when it is full.

   How many cupfuls can you get from a bottle containing \( 2 \frac{1}{4} \) liters?

4. The gas mileage of a certain car is 30 miles to the gallon.

   What is that in kilometers to the liter? (5 miles ≈ 8 kilometers, and \( 3 \frac{3}{4} \) liters ≈ 1 gallon)

Homework

1. a. What is the rule for dividing one fraction by another?

   b. Complete this rule using symbols: \( \frac{a}{b} \div \frac{c}{d} = \)

2. Use fractions to write two examples for each of these equations.

   a. Fraction \( \div \) fraction = 1

   b. Fraction \( \div \) fraction = 2

   c. Fraction \( \div \) fraction = \( \frac{1}{2} \)

   d. Fraction \( \div \) fraction = \( \frac{3}{4} \)
Calculations involving three or more fractions can be simplified by applying some of the number properties.

Example

To calculate \( \frac{3}{4} + \frac{2}{3} + \frac{5}{8} \), the common denominator does not have to be \( 4 \cdot 3 \cdot 8 = 96 \).

The lowest common denominator for adding these three fractions is 24.

\[
\frac{3}{4} = \frac{3 \cdot 6}{4 \cdot 6} = \frac{18}{24}, \quad \frac{2}{3} = \frac{2 \cdot 8}{3 \cdot 8} = \frac{16}{24}, \quad \frac{5}{8} = \frac{5 \cdot 3}{8 \cdot 3} = \frac{15}{24}
\]

This means that

\[
\frac{3}{4} + \frac{2}{3} + \frac{5}{8} = \frac{18}{24} + \frac{16}{24} + \frac{15}{24} = \frac{49}{24} = 2 \frac{1}{24}.
\]

Example

To calculate \( \frac{3}{4} \cdot \frac{2}{3} \cdot \frac{5}{8} \), the numerators and denominators can be kept small by using the commutative property of multiplication to change the order of the numerators.

This means that

\[
\frac{3}{4} \cdot \frac{2}{3} \cdot \frac{5}{8} = \frac{3 \cdot 2 \cdot 5}{4 \cdot 3 \cdot 8} = \frac{3 \cdot 2 \cdot 5}{3 \cdot 4 \cdot 8} = \frac{3 \cdot 2 \cdot 5}{1 \cdot 2 \cdot 8} = \frac{5}{16}.
\]

Example

The calculation \( \left( \frac{2}{3} \cdot \frac{3}{4} \right) + \left( \frac{2}{3} \cdot \frac{5}{8} \right) \) can be simplified by using the distributive property to write it as \( \frac{2}{3} \left( \frac{3}{4} + \frac{5}{8} \right) \). This equals \( \frac{2}{3} \cdot \frac{11}{8} = \frac{22}{24} = \frac{11}{12} \).
1. Work with a partner to copy and complete this table. There are two expressions involving fractions in each row. Decide whether the two expressions are equivalent without doing any calculations, and say which number property is involved.

<table>
<thead>
<tr>
<th>Expression 1</th>
<th>Expression 2</th>
<th>Equivalent (Yes/No)</th>
<th>Explanation</th>
</tr>
</thead>
</table>
| \[
\left(\frac{3}{4} - \frac{1}{3}\right) - \frac{1}{6}
\] | \[
\frac{3}{4} - \left(\frac{1}{3} - \frac{1}{6}\right)
\] | No                  |             |
| \[
\frac{3}{4} + \frac{2}{5} - \frac{3}{10}
\] | \[
\frac{3}{4} + \left(\frac{2}{5} - \frac{3}{10}\right)
\] | Yes                 |             |
| \[
\frac{2}{5} \div \frac{10}{10}
\] | \[
\frac{2}{5} \cdot \frac{20}{5}
\] | Yes                 |             |
| \[
\frac{2}{5} \div \frac{3}{10}
\] | \[
\frac{3}{10} \div \frac{2}{5}
\] | Yes                 |             |
| \[
\frac{2}{3} \div \frac{4}{9}
\] | \[
\frac{3}{2} \div \frac{2}{9}
\] | Yes                 |             |
| \[
\frac{7}{5} \div \frac{3}{10}
\] | \[
\frac{7}{5} \cdot \frac{10}{3}
\] | Yes                 |             |
| \[
\frac{7}{5} \div \frac{5}{7}
\] | \[
\frac{5}{7} \div \frac{7}{5}
\] | Yes                 |             |
| \[
\frac{7}{5} \cdot \frac{0}{3}
\] | \[
\frac{7}{6} \div \frac{0}{2}
\] | Yes                 |             |
| \[
\frac{5}{2} \div \frac{3}{10}
\] | \[
\frac{5}{2} \div \left(\frac{3}{5} \cdot \frac{3}{10}\right)
\] | Yes                 |             |
| \[
\left(\frac{5}{3} \div \frac{2}{5}\right) \div \frac{3}{10}
\] | \[
\frac{5}{3} \div \left(\frac{2}{5} \div \frac{3}{10}\right)
\] | Yes                 |             |
| \[
\frac{3}{5} (6 + 4)
\] | \[
\left(\frac{6}{3} \cdot \frac{3}{5}\right) + \left(\frac{4}{3} \cdot \frac{3}{5}\right)
\] | Yes                 |             |
| \[
\frac{3}{7} \left(\frac{5}{4} - \frac{3}{8}\right)
\] | \[
\frac{3}{7} \cdot \frac{5}{4} - \left(\frac{3}{7} \cdot \frac{3}{8}\right)
\] | Yes                 |             |
Preparation for the Closing

2. Working with your partner, calculate the expressions in problem 1.

3. Prepare your explanations and calculations to present during the Closing.

Skills

Solve.

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>7(12 + 4) =</td>
<td>b</td>
<td>8(14 + 5) =</td>
</tr>
<tr>
<td>c</td>
<td>9(12 + 5) =</td>
<td>d</td>
<td>25(3 + 4) =</td>
</tr>
<tr>
<td>e</td>
<td>7(2a + 4) =</td>
<td>f</td>
<td>a(14 + 5) =</td>
</tr>
<tr>
<td>g</td>
<td>4(4 + 3a) =</td>
<td>h</td>
<td>5(4a + 4) =</td>
</tr>
</tbody>
</table>

Review and Consolidation

Use fractions to provide examples that show the following number properties.

1. An equation involving repeated addition can be written as a whole number multiplied by another number.

2. \((a - b) - c = a - (b + c)\)

3. \((a ÷ b) ÷ c = a ÷ bc\)

4. \(a(b + c) = ab + ac\)

5. \((a - b) ÷ c = (a ÷ c) - (b ÷ c)\)

6. The product of a number and the reciprocal of the number is 1.

7. A fraction with a denominator less than 10 can be written as an equivalent fraction with a denominator greater than 10.

8. The average of two numbers is greater than one of the given numbers and less than the other.
Homework

Look at each statement, and decide whether it is true or false. If a statement is false, change it to make a true statement.

1. \( \frac{2}{5} + \frac{2}{5} = \frac{4}{10} \)

2. Any fraction with the same nonzero number for both the numerator and the denominator has a value of 0.

3. A mixed number greater than 1 is a combination of a natural number and a fraction, and can be rewritten in the form \( \frac{a}{b} \), where \( a \) and \( b \) are natural numbers.

4. \( 8 - \frac{5}{2} = \frac{3}{3} \)

5. \( \frac{5}{2} \cdot \frac{1}{4} = 10 \frac{1}{8} \)

6. \( \frac{5}{6} + \frac{3}{4} = \frac{10}{12} + \frac{9}{12} = \frac{19}{12} = 1 \frac{7}{19} \)

7. \( \frac{3}{7} \div \frac{2}{5} = \frac{3 \cdot 2}{7 \cdot 5} \)
1. Of the first ten natural numbers, how many are prime numbers and how many are composite numbers? **Hint:** Natural numbers are the counting numbers (1, 2, 3, 4, ...).

2. The prime factorization of 78 is $2 \cdot 3 \cdot 13$.
   a. List all of the factors of 78.
   b. List two numbers that are common factors of 30 and 78.
   c. List two numbers that are common multiples of 30 and 78.
   d. Use two different methods to calculate the product of the least common multiple and the greatest common factor of 30 and 78.

3. Calculate the following expressions. Write your answers in simplest form.
   a. $\frac{2}{5} + \frac{2}{5} + \frac{2}{5} + \frac{2}{5} + \frac{2}{5}$
   b. $6 \cdot \frac{2}{5}$
   c. $\frac{2}{5} + \frac{2}{5} + \frac{2}{5} + \frac{2}{5} + \frac{2}{5} + \frac{2}{5}$
   d. $6 \cdot \frac{2}{5}$
   e. $2 \left( \frac{3}{4} - \frac{2}{5} \right)$
   f. $\left( 2 \cdot \frac{3}{4} \right) - \left( 2 \cdot \frac{2}{5} \right)$
   g. $\frac{5}{2} \cdot \frac{4}{3} \cdot \frac{3}{4} \cdot \frac{2}{5}$
   h. $\frac{7}{8} + \frac{7}{8} + \frac{7}{8} + \frac{7}{8}$
   i. $\frac{7 + 7 + 7 + 7}{8 + 8 + 8 + 8}$
   j. $\frac{7 \cdot 4}{8 \cdot 4}$

4. Look over your answers to problem 3. What are the connections between them?
5. Measure the length of the piece of wire shown.

![Image of a piece of wire]

a. Write your answer in sixteenths of an inch.
b. Write your answer (in inches) as an equivalent fraction in simplest form.

6. Complete the following rules for operations with fractions.

a. \( \frac{a}{b} + \frac{c}{d} = \)  

b. \( \frac{a}{b} - \frac{c}{d} = \)  

c. \( \frac{a}{b} \cdot \frac{c}{d} = \)  

d. \( \frac{a}{b} ÷ \frac{c}{d} = \)  

7. Write all the equations that are represented by this number line.

![Image of a number line]

8. a. Perform the calculations to complete the equation: \( \frac{5}{4} - \frac{3}{8} = \)  

b. Sketch a number line of this equation.

c. Use subtraction to write a problem about distances, measured in miles, to match this equation.

9. a. Which number is greater, \( \frac{4}{5} \) or \( \frac{4}{6} \)? Explain why.

b. Which number is greatest, \( \frac{3}{5}, \frac{6}{10}, \) or \( \frac{4}{5} \)? Explain why.

c. Which number is greatest, \( \frac{3}{4}, \frac{4}{5}, \) or \( \frac{5}{6} \)? Explain why.

Preparing for the Closing

10. Compare your answers to the Work Time problems with those of your partner. Be prepared to provide your solutions and explanations during the Closing.
Lesson 18

Skills

Solve. Try to find a way to make the problems easier, so you can solve them in your head.

a. 46 • 14 = 

b. 47 • 9.8 =

c. 405 • 2.5 =

d. 1200 • 33 =

e. 67 • 33 =

f. 85 • 1.03 =

Review and Consolidation

1. Calculate.

a. \( \frac{7}{8} + \frac{7}{8} = \)

b. \( \frac{7}{3} + \frac{3}{8} = \)

c. \( \frac{7}{3} - \frac{5}{6} = \)

d. \( 2\frac{3}{5} + 1\frac{3}{4} = \)

2. a. Sketch an area diagram to represent the calculation of \( \frac{4}{3} \cdot \frac{5}{2} \).

   Explain how the answer is represented on the diagram.

b. Write the equations involving division that are represented on the same diagram.

3. a. Calculate \( \frac{4}{9} \cdot 2\frac{1}{4} \), and express the answer in simplest form.

   b. What is the reciprocal of \( \frac{4}{7} \)?

   c. Calculate \( 1 \div 1\frac{3}{7} \).

4. Calculate \( \frac{9}{20} \left( \frac{4}{3} \cdot \frac{5}{2} \right) \), and express the answer in simplest form.

5. Which of these fractions is equal to \( \frac{4}{9} \div \frac{7}{3} \)?

   A  \( \frac{28}{27} \)  

   B  \( \frac{63}{12} \)  

   C  \( \frac{27}{28} \)  

   D  \( \frac{12}{63} \)  

6. a. Calculate \( \frac{5}{2} \left( \frac{3}{2} + \frac{3}{4} \right) \).

   b. Calculate \( \frac{5}{3} \cdot \frac{3}{2} + \left( \frac{5}{3} \cdot \frac{3}{4} \right) \).

   c. Which number property explains why your answers to parts a and b should be the same?
1. Rosa likes eating trail mix.

She bought two bags and ate \( \frac{1}{8} \) of a bag on Monday, \( \frac{3}{16} \) of a bag on Tuesday, \( \frac{1}{4} \) of a bag on Wednesday, \( \frac{1}{2} \) of a bag on Thursday, \( \frac{1}{8} \) of a bag on Friday, and the rest over the weekend.

<table>
<thead>
<tr>
<th>Day</th>
<th>Monday</th>
<th>Tuesday</th>
<th>Wednesday</th>
<th>Thursday</th>
<th>Friday</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fraction</td>
<td>( \frac{1}{8} ) of a bag</td>
<td>( \frac{3}{16} ) of a bag</td>
<td>( \frac{1}{4} ) of a bag</td>
<td>( \frac{1}{2} ) of a bag</td>
<td>( \frac{1}{8} ) of a bag</td>
</tr>
</tbody>
</table>

How much trail mix did Rosa eat over the weekend?

2. Jamal also likes trail mix. He bought two bags as well.

On Monday, he ate \( \frac{1}{6} \) of his two bags.

On Tuesday, he ate \( \frac{1}{5} \) of what was left over from Monday.

On Wednesday, he ate \( \frac{1}{4} \) of what was left over from Tuesday.

On Thursday, he ate \( \frac{1}{3} \) of what was left over from Wednesday.

On Friday, he ate \( \frac{1}{2} \) of what was left over from Thursday.

Jamal ate the rest of the trail mix over the weekend.

How much trail mix did Jamal eat over the weekend?
**Lesson 19**

**Learning from the Progress Check**

**Goal**

To consolidate the concepts included in the Progress Check.

**Concept Book**

See pages 139–153 in your Concept Book.

**Work Time**

1. **Equations using addition and multiplication**
   - a. Write an equation using addition for this number line.
   - b. Write an equation using multiplication for this number line.

2. **Comparing unit-square diagrams**
   - Explain the similarities and differences between these two unit-square diagrams.

3. **Equations for area diagram**
   - a. Write two equations using multiplication.
   - b. Write two equations using division.
4. Write these decimals as equivalent fractions in simplest form.
   a. 0.45
   b. 0.375
   c. 0.1 + 0.05 + 0.05

5. a. Sketch a number line of \(\frac{3}{4} + \frac{3}{4} + \frac{3}{4} + \frac{3}{4} = 3\)
   b. Write the other equations that are also represented by the same diagram.

6. Calculate the following expressions. Write your answers as fractions in simplest form.
   a. \(\frac{3}{4} + \frac{3}{4}\)
   b. \(\frac{3}{5} + \frac{3}{5} + \frac{3}{5}\)
   c. \(\frac{7}{15} + \frac{3}{5}\)
   d. \(\frac{13}{15} - \frac{2}{5}\)
   e. \(2\frac{3}{4} - 1\frac{5}{6}\)
   f. \(\frac{3}{4} + \frac{4}{5} + \frac{5}{6}\)

7. a. Which number is greater, \(\frac{5}{3}\) or \(\frac{5}{4}\) ? Explain why.
   b. Which number is greater, \(\frac{4}{5}\) or \(\frac{4}{6}\) ? Explain why.
   c. Which number is greatest, \(\frac{3}{8}\), \(\frac{4}{9}\), or \(\frac{5}{10}\) ? Explain why.

Preparing for the Closing

8. Write six different types of problems (addition, subtraction, multiplication, mixed operations, etc.) involving fractions. The answer to each problem must be 1.
Lesson 19

Skills

Solve. Try to find a way to make the problems easier, so you can solve them in your head.

- a. \(24 \div 10 = \)
- b. \(240 \div 12 = \)
- c. \(360 \div 12 = \)
- d. \(36 \div 1.2 = \)
- e. \(18 \div 10 = \)
- f. \(180 \div 20 = \)
- g. \(360 \div 20 = \)
- h. \(0.67 \div 10 = \)

Review and Consolidation

1. Write the equations that are represented by the area diagram.

2. Calculate the following expressions. Write your answers in simplest form.

- a. \(\frac{4}{5} \cdot 10\)
- b. \(\frac{4}{5} \cdot \frac{15}{8}\)
- c. \(\frac{4}{5} \cdot 2\frac{1}{4}\)
- d. \(\frac{4}{5} \div \frac{12}{5}\)
- e. \(\frac{8}{5} \div \frac{2}{5}\)
- f. \(\frac{3}{5} \cdot \frac{5}{12} \cdot \frac{4}{9}\)

3. Write an example, using fractions, for these number properties, and then do the calculations to check.

- a. The product of a number and the reciprocal of the number is 1.
- b. \((a + b) - c = a + (b - c)\)
- c. \(a(b + c) = ab + ac\)
- d. \((a \div b) \div c = a \div bc\)
4. a. One night a week, Dwayne cooks dinner for his family. He decides to try a meatloaf recipe he has found in one of his mother’s cookbooks. The recipe makes enough meatloaf to serve 4 people, but there are five in the Jackson family. Also, Dwayne wants to make a little extra to take to school for lunch. He decides to make enough meatloaf to serve 6 people.

The cookbook recipe calls for $\frac{1}{4}$ pounds of ground beef and 3 onions.

How many pounds of meat and how many onions does Dwayne need in order to make enough meatloaf to serve 6 people?

b. Explain the operation that you chose and why that operation made sense to solve Dwayne’s problem.

Homework

1. Write and solve three problems that are similar to ones that you found difficult during the last two lessons.

2. What have you learned during the last two lessons?
Lesson 20
Shortest Time

Goal
To apply the concept of adding fractions to solve a problem.

Concept Book
See pages 23, 147–148 in your
Concept Book.

Work Time

1. This diagram shows traveling times in hours for flights between six cities.

a. List the different ways of getting from Atlanta to Chicago, and calculate the total time in the air for each possible route.

b. Dwayne’s aunt is a sales representative based in Atlanta. She must visit the five other cities shown in the diagram for one day each and then return home at the end of the five days.

Which path involves the least time in the air?

c. How much more traveling time would be involved if she came home every night?

Preparing for the Closing

2. Discuss your solutions with your partner and prepare explanations for the Closing.
Skills

Solve.

a. 20% of 100
b. 30% of 90
c. 5% of 350
d. 50% of 450
e. 20% of 0.1
f. 30% of 0.09
g. 5% of 0.35
h. 50% of 4.5

Review and Consolidation

1. Keesha has four 2-hour DVDs and she wants to use them to record ten TV programs. The length of each program (in hours) is given in the table.

Can she do it without using different DVDs for parts of the same program?

If she can, explain how.

<table>
<thead>
<tr>
<th>Program</th>
<th>Length (hours)</th>
<th>Program</th>
<th>Length (hours)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Presidential speech</td>
<td>$\frac{1}{6}$</td>
<td>Reality show</td>
<td>$\frac{2}{3}$</td>
</tr>
<tr>
<td>Variety program</td>
<td>$\frac{5}{6}$</td>
<td>Comedy</td>
<td>$\frac{1}{2}$</td>
</tr>
<tr>
<td>Cartoon</td>
<td>$\frac{1}{6}$</td>
<td>Local news</td>
<td>$\frac{1}{2}$</td>
</tr>
<tr>
<td>Fantasy adventure</td>
<td>$1\frac{11}{12}$</td>
<td>Thriller</td>
<td>$\frac{1}{2}$</td>
</tr>
<tr>
<td>Music program</td>
<td>$\frac{2}{3}$</td>
<td>Adventure</td>
<td>$1\frac{5}{6}$</td>
</tr>
</tbody>
</table>

DVDs

- 2 hrs
- 2 hrs
- 2 hrs
- 2 hrs
Homework

Copy the diagrams and fill in the missing sums and products.

Follow this example:

\[
\begin{array}{c}
\text{Product} \\
\frac{1}{2} & \frac{4}{5} \\
\text{Sum} \\
\frac{1}{2} & \frac{13}{10}
\end{array}
\]

1. \[
\frac{2}{5} \times \frac{1}{3} = \frac{2}{5} \times \frac{1}{3}
\]
2. \[
\frac{3}{4} \times \frac{3}{4} = \frac{3}{4} \times \frac{3}{4}
\]
3. \[
\frac{6}{9} \times \frac{1}{2} = \frac{6}{9} \times \frac{1}{2}
\]
4. \[
\frac{3}{5} \times \frac{1}{3} = \frac{3}{5} \times \frac{1}{3}
\]
5. \[
\frac{1}{2} \times \frac{7}{10} = \frac{1}{2} \times \frac{7}{10}
\]
6. \[
\frac{3}{4} \times \frac{5}{8} = \frac{3}{4} \times \frac{5}{8}
\]
7. \[
\frac{3}{4} \times 2 = \frac{3}{4} \times 2
\]
8. \[
\frac{1}{7} \times \frac{1}{2} = \frac{1}{7} \times \frac{1}{2}
\]
9. \[
\frac{5}{7} \times 8 = \frac{5}{7} \times 8
\]
10. \[
\frac{1}{6} \times \frac{4}{3} = \frac{1}{6} \times \frac{4}{3}
\]
11. \[
\frac{1}{7} \times \frac{1}{3} = \frac{1}{7} \times \frac{1}{3}
\]
12. \[
\frac{4}{5} \times \frac{1}{5} = \frac{4}{5} \times \frac{1}{5}
\]
THE UNIT IN REVIEW

CONCEPT BOOK

GOAL
To review multiples, factors, and fractions.

Work Time

Choose the correct response for each of the nine multiple-choice problems. Copy the table, and use it to summarize your responses. Explain the mistakes that could lead to the incorrect choices.

<table>
<thead>
<tr>
<th>Problem</th>
<th>Correct Answer and Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
<tr>
<td>etc.</td>
<td></td>
</tr>
</tbody>
</table>

Multiple-Choice Problems

1. \( \frac{3}{8} + \frac{3}{8} = \)
   - A \( \frac{3}{4} \)
   - B \( \frac{3}{16} \)
   - C \( \frac{6}{16} \)
   - D 0.375

2. \( 5 \cdot \frac{2}{3} = \)
   - A \( \frac{10}{3} \)
   - B \( \frac{10}{15} \)
   - C \( \frac{5}{2} \)
   - D \( \frac{2}{15} \)

3. The equation \( \frac{4}{5} \cdot \frac{2}{3} = \frac{8}{15} \) is equivalent to:
   - A \( \frac{8}{15} \div \frac{3}{2} = \frac{4}{5} \)
   - B \( \frac{4}{5} \div \frac{8}{15} = \frac{2}{3} \)
   - C \( \frac{2}{3} \div \frac{8}{15} = \frac{4}{5} \)
   - D \( \frac{4}{5} \div \frac{3}{2} = \frac{8}{15} \)
4. \( \frac{5}{8} - \frac{3}{5} = \)

A. \( \frac{2}{3} \)  
B. \( \frac{1}{40} \)  
C. \( \frac{2}{40} \)  
D. \( \frac{49}{40} \)

5. \( \frac{3}{5} + \frac{3}{5} + \frac{3}{5} + \frac{3}{5} + \frac{3}{5} = \)

A. \( \frac{15}{25} \)  
B. \( \frac{53}{5} \)  
C. \( 3 \)  
D. \( 5 \)

6. In simplest form, \( \frac{5}{6} \cdot \frac{3}{4} = \)

A. \( \frac{15}{24} \)  
B. \( \frac{19}{12} \)  
C. \( \frac{5}{8} \)  
D. \( \frac{13}{5} \)

7. \( \frac{5}{8} ÷ \frac{7}{12} = \)

A. \( \frac{35}{96} \)  
B. \( \frac{14}{15} \)  
C. \( \frac{15}{14} \)  
D. \( \frac{56}{60} \)

8. \( 2\frac{2}{3} \cdot 3\frac{3}{4} = \)

A. \( (2 \cdot 3) + \left( \frac{2}{3} \cdot \frac{3}{4} \right) \)  
B. \( (2 \cdot 3) + \left( \frac{2}{3} \cdot \frac{3}{4} \right) + \left( 2 \cdot \frac{3}{4} \right) + \left( 3 \cdot \frac{2}{3} \right) \)  
C. \( (2 \cdot 3) + \left( \frac{2}{3} \cdot \frac{3}{4} \right) + \left( 2 \cdot \frac{2}{3} \right) + \left( 3 \cdot \frac{3}{4} \right) \)  
D. \( \frac{4}{3} \cdot \frac{9}{4} \)

9. \( 1\frac{1}{5} ÷ \frac{6}{5} = \)

A. 0  
B. \( \frac{1}{6} \)  
C. \( \frac{1}{2} \)  
D. 1
Preparing for the Closing

10. Compare your answers to the multiple-choice problems with those of your partner. Reach an agreement about all of your choices and reasons.

Skills

Museum Attendance

a. What was the increase in attendance at the museum from January to February?

b. There was a decrease in attendance from March to April—what was it?

c. Which month had the highest attendance?

d. Which month had the second highest attendance?

e. Which month had the lowest attendance?

Review and Consolidation

Assessing Your Work

In this unit, you have learned some important concepts and skills.

Multiples and factors:

• Identifying prime and composite numbers
• Prime factorization
• Determining greatest common factors
• Determining least common multiples
Lesson 21

How to represent the four operations with fractions:

- Number lines
- Area diagrams

Operations with fractions:

- Adding and subtracting fractions that have equal denominators
- Adding and subtracting fractions that have different denominators
- Multiplying and dividing fractions

Assess your work by thinking about each point listed above.

1. First review your work. Then choose three pieces, one about multiples and factors, one about representing operations with fractions, and one about operations with fractions.

2. Write a brief explanation of why you chose each piece and how each demonstrates your understanding of the concept it represents.

Homework

Use this time to prepare for the End-of-Unit Assessment.

- Consult the Concept Book to review the main ideas.
- Check your understanding of earlier work on fractions in Unit 2: The Number System.
- Write what you have learned about adding and subtracting fractions. Include number line illustrations.
- Write what you have learned about multiplying and dividing fractions. Include area illustrations.
- Check the distance and time problems that were solved by adding and subtracting fractions.
- Find mistakes you made in your work, and write out explanations and corrections.
1. In each problem, calculate which percent is greater.
   a. 50% of 600 or 50% of 800?
   b. 50% of 600 or 40% of 800?
   c. 5% of 600 or 4% of 800?

2. This input-output table gives the area and base of several different rectangles. Notice that all of the rectangles have an area of 12 square centimeters, but each has a different base.

<table>
<thead>
<tr>
<th>Area (cm²)</th>
<th>12</th>
<th>12</th>
<th>12</th>
<th>12</th>
<th>12</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td>Base (cm)</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>n</td>
</tr>
<tr>
<td>Height (cm)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

   a. Copy the table into your notebook and fill in the height for each rectangle.
   b. What is the rule for finding the height of a rectangle if the area is 12 cm² and the base is known?

3. You have some money in each of two pockets.
   Write an expression for each of these situations.
   a. Add ten cents to the money in each pocket.
      How much money do you have now?
   b. Now that you have added the ten cents, double the money in each pocket.
      How much money do you have now?

4. Solve.
   a. \( \frac{2}{15} + \frac{2}{15} + \frac{2}{15} = \) 
   b. \( 5 \cdot \frac{2}{15} = \) 
   c. \( \frac{4}{15} + \frac{4}{15} + \frac{2}{15} = \) 
   d. \( \frac{3}{5} = \frac{15}{15} \)
Factors and Fractions

5. If \( a = b \), then the following are true:
\[
\begin{align*}
  a + c &= b + c \\
  ac &= bc
\end{align*}
\]

Describe these addition and multiplication properties in your own words.

6. a. Sketch a number line to represent the repeated addition \( 0.7 + 0.7 + 0.7 \).

b. Write the sum of this addition.

c. Write the multiplication equation that this number line represents.

7. a. Write \( 2\frac{1}{7} \) as an improper fraction.

b. Write \( \frac{9}{5} \) as a mixed number.

c. Write 0.84 as a fraction.

d. Write \( \frac{7}{8} \) as a decimal.

8. Explain the similarities and the differences between three quarters of a pizza and twelve sixteenths of the same size pizza.

9. Write three fractions, with different denominators, that add up to 1.

10. The students of Monroe High School wanted to have an End-of-the-Year Dance before summer break. To pay for the dance, different clubs donated money to cover the costs of the dance. Look at the pie chart.

   a. How much money was raised for the dance?

   b. Which club donated the most money?

   c. Which three clubs donated exactly half of the money needed?

   d. The Student Council donated $90. Which two groups together donated as much as the Student Council?
CALIFORNIA MATHEMATICS CONTENT STANDARDS

Page numbers in red are found in the Concept Book.

**Number Sense**

Gr. 3 NS: 3.1  Compare fractions represented by drawings or concrete materials to show equivalency and to add and subtract simple fractions in context (e.g., 1/2 of a pizza is the same amount as 2/4 of another pizza that is the same size; show that 3/8 is larger than 1/4). 29–34; 86–87

Gr. 3 NS: 3.2  Add and subtract simple fractions (e.g., determine that 1/8 + 3/8 is the same as 1/2). 29–34; 86–87

Gr. 4 NS: 1.7  Write the fraction represented by a drawing of parts of a figure; represent a given fraction by using drawings; and relate a fraction to a simple decimal on a number line. 29–34, 48–51; 143–146

Gr. 4 NS: 4.0  Students know how to factor small whole numbers.  1–5; 120–122

Gr. 4 NS: 4.1  Understand that many whole numbers break down in different ways (e.g., 12 = 4 • 3 = 2 • 6 = 2 • 2 • 3).  1–5; 122

Gr. 4 NS: 4.2  Know that numbers such as 2, 3, 5, 7, and 11 do not have any factors except 1 and themselves and that such numbers are called prime numbers.  6–9; 123–124

Gr. 5 NS: 1.4  Determine the prime factors of all numbers through 50 and write the numbers as the product of their prime factors by using exponents to show multiples of a factor (e.g., 24 = 2 • 2 • 2 • 3 = 2^3 • 3).  10–14, 23–25; 124–126

Gr. 5 NS: 2.0  Students perform calculations and solve problems involving addition, subtraction, and simple multiplication and division of fractions and decimals.  52–58, 62–84; 147–148

Gr. 5 NS: 2.3  Solve simple problems, including ones arising in concrete situations, involving the addition and subtraction of fractions and mixed numbers (like and unlike denominators of 20 or less), and express answers in the simplest form.  35–38, 48–51; 147–148

Gr. 5 NS: 2.4  Understand the concept of multiplication and division of fractions.  39–51, 62–65; 149–153

Gr. 5 NS: 2.5  Compute and perform simple multiplication and division of fractions and apply these procedures to solving problems.  70–77, 81–84; 144–153

Gr. 6 NS: 1.0  Students compare and order positive and negative fractions, decimals, and mixed numbers. Students solve problems involving fractions, ratios, proportions, and percentages.  52–55; 147–148

Gr. 6 NS: 2.1  Solve problems involving addition, subtraction, multiplication, and division of positive fractions and explain why a particular operation was used for a given situation.  59–61; 147–148

Gr. 6 NS: 2.2  Explain the meaning of multiplication and division of positive fractions and perform the calculations (e.g., 58 ÷ 1516 = 58 • 1615 = 23).  10–14; 149–153

Gr. 6 NS: 2.4  Determine the least common multiple and the greatest common divisor of whole numbers; use them to solve problems with fractions (e.g., to find a common denominator to add two fractions or to find the reduced form for a fraction). 15–28, 35–38, 48–58; 135–138
Mathematical Reasoning

Gr. 4 MR: 1.1  Gr. 4 NS: 4.2
Analyze problems by identifying relationships, distinguishing relevant from
irrelevant information, sequencing and prioritizing information, and observing
patterns.  6–9; 123–124

Gr. 5 MR: 1.0  Gr. 5 NS: 2.0
Students make decisions about how to approach problems.  78–80; 147–148

Gr. 6 MR: 1.2  Gr. 6 NS: 2.4
Formulate and justify mathematical conjectures based on a general description of the
mathematical question or problem posed.  26–28; 135–138

Gr. 6 MR: 2.5  Gr. 6 NS: 2.1
Express the solution clearly and logically by using the appropriate mathematical
notation and terms and clear language; support solutions with evidence in both verbal
and symbolic work.  26–28, 59–61; 135–138, 147–148
A
area diagram 44; 149–150

C
common denominator 35, 48, 66; 147
common factor 19, 23, 36, 38; 128–129, 137
common multiple 15; 133–135
composite number 6, 10, 23; 123–124

denominator 29, 35–36, 38, 44, 48, 63;
139, 142, 147–148
common 35, 48, 66; 147
lowest common 66
division of fractions 39, 62–63; 152–153

E
equivalent equation 39; 355
equivalent fraction 35, 48, 52; 64,
145–146, 148
exponent 10–11; 12, 128

F
factor 1–2, 10, 19, 23; 120–138
common 19, 23, 36, 38; 128–129, 137
greatest common (GCF) 19, 23; 129–130
prime 10–11; 123–124
factorization 1, 10–11; 122–124
prime 10–11, 14, 19, 23; 124–128, 136–138

INDEX
Page numbers in red are found in the Concept Book.

factor pair 11; 120, 122
factor tree 11, 14, 23; 125, 127
fraction
adding 29–30, 35–36, 48, 66; 86–87,
147–148
dividing 62–63; 56–57
equivalent 35, 48, 52; 64, 145–146, 148
multiplying 39, 44, 48, 63; 149
subtracting 29–30, 52; 86–87, 147–148
fundamental theorem of arithmetic 10;
126–128

greatest common factor (GCF) 19, 23;
129–130
inverse 39, 52, 56, 63; 15–16, 98–99
least common multiple (LCM) 19, 23, 35;
135–138
multiple 1–2, 15, 35; 120–123
least common (LCM) 19, 23, 35;
135–138
multiplication 1–2, 39, 44, 63, 66; 89–90
table 2; 121
INDEX

N
natural number 1–2, 6–7; 117–118
number line 2, 29–30, 35, 39–40, 44, 48, 52, 56; 71–72, 86–87
number properties
  commutative property of multiplication 66; 15, 20, 89–90
  distributive property 66; 15–18
numerator 36, 38, 44, 63, 66; 139, 142

P
prime factor 10–11; 123–124
prime factorization 10–11, 14, 19, 23; 124–128, 136–138
prime number 6; 123–126
product 1, 10–11, 15, 19, 35, 44, 48; 89

R
reciprocal 63; 16–17, 150–151
repeated addition 39, 48, 63; 89–111
repeated subtraction 63; 94

S
simplest form 38; 146

U
unit square 30, 44, 48; 149

W
whole number 29, 39, 48; 45, 117

Page numbers in red are found in the Concept Book.