Ramp-Up to Algebra

Positive and Negative Numbers

Unit 4

AMERICA'S CHOICE®

Acknowledgments

Field Test:

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EXTENDING THE NUMBER LINE

LESSON 1

CONCEPT BOOK	GOAL
See pages 185–187 in your *Concept Book.*	To recognize the need for negative numbers, and to place negative numbers on the number line.

Negative numbers are numbers that are less than zero.

By contrast, *positive numbers* are numbers that are greater than zero.

Example

Negative 4.5, written with a negative sign (–) in front of the 4.5: –4.5

Positive 4.5, written with a positive sign (+) in front: +4.5
Most of the time, you simply write this: 4.5

The number zero (0) is neither positive nor negative.

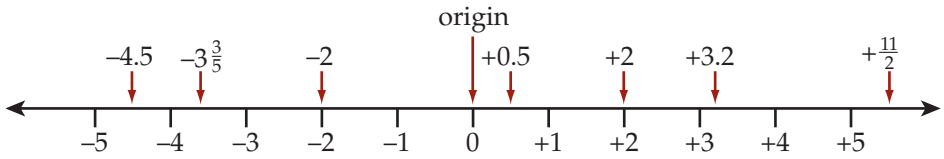
The point for zero on a number line is call the *origin*.

All positive numbers on a number line are located to the right of the origin.

All negative numbers on a number line are located to the left of the origin.

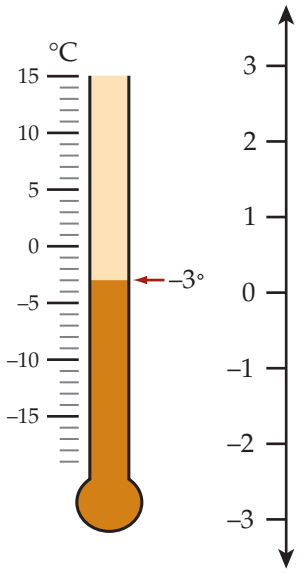
Example

On this number line, four positive numbers are marked with arrows.
These are to the right of the origin, 0.
Three negative numbers are also marked; these are to the left of the origin.



Sometimes, a number line is sketched vertically, with positive numbers above the origin and negative numbers below the origin.

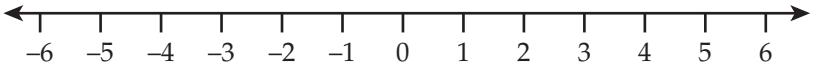
The y-axis on a graph (far right) is a vertical number line. This thermometer diagram is also a vertical number line.



Example

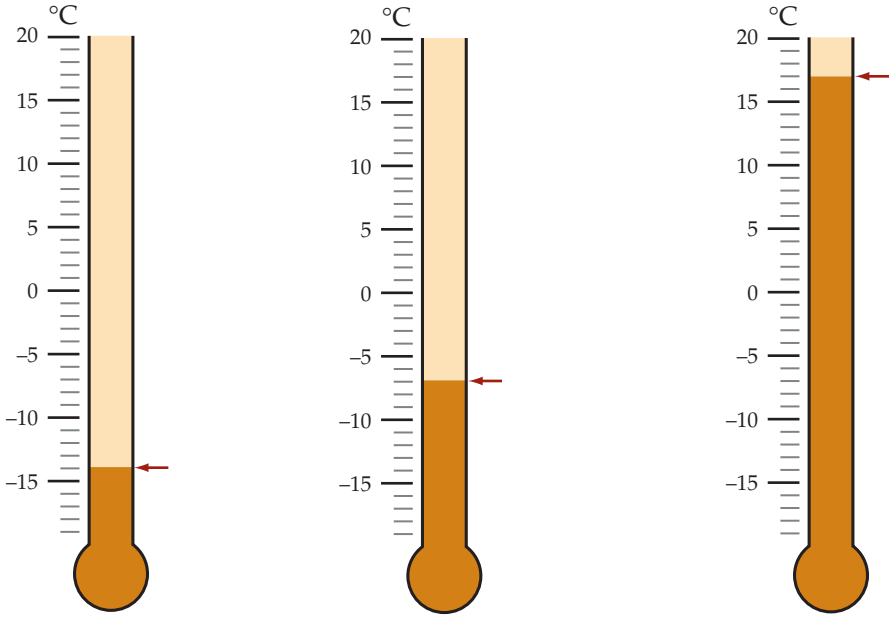
The temperature on the thermometer is written as –3°. It can be read as, "negative three degrees," or "three degrees below zero."

Positive numbers are usually written on the number line without the + sign, as shown on the horizontal and vertical number lines below and at the far right:

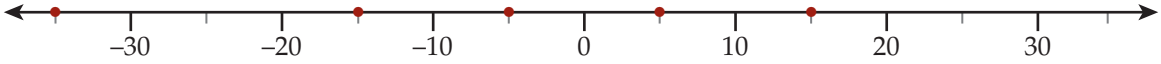


Work Time

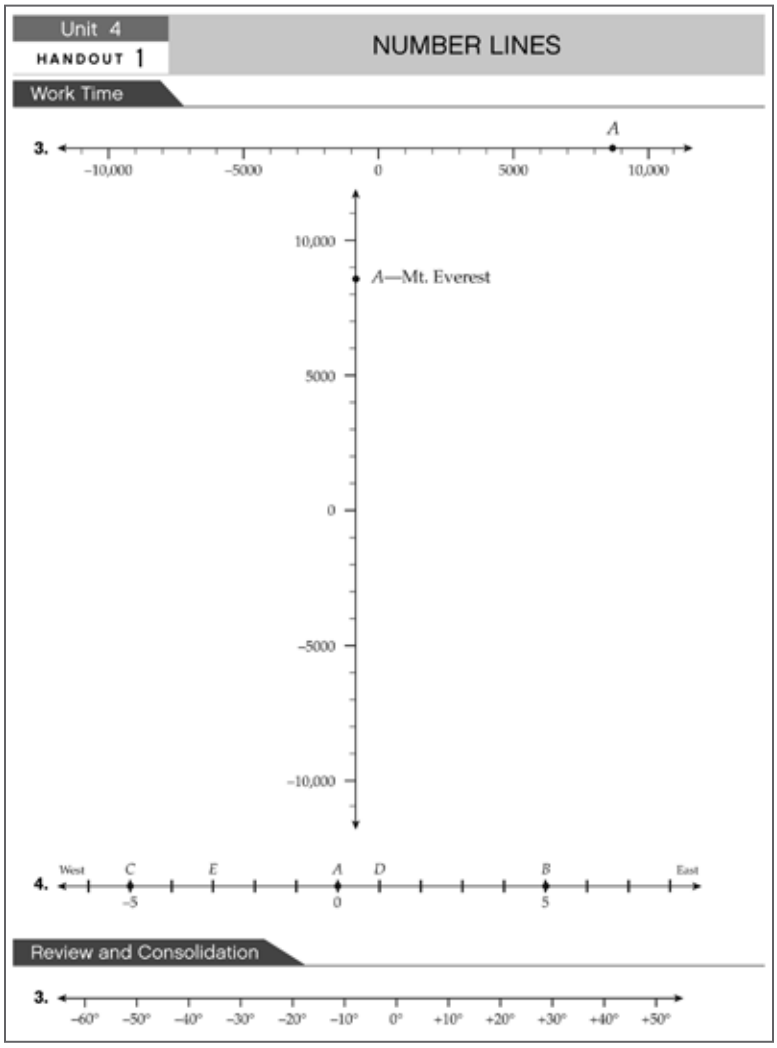
1. List the temperatures shown by the arrows in the diagrams below.



2. This number line has a scale marked in tens. List the numbers at the five given points.



3. Your teacher will give you a copy of Handout 1: *Number Lines* for use with problems in this lesson.



Dwayne and Lisa were studying elevation in their Earth Studies class. They needed to see relationships between the altitudes of different places. They decided that the best way to do this was to mark the altitude of each place as a point on both a horizontal number line and a vertical number line.

Below are the points which Dwayne and Lisa assigned. Using Handout 1, mark these points on horizontal and vertical number lines, each with a scale marked from –12,000 to 12,000 meters.

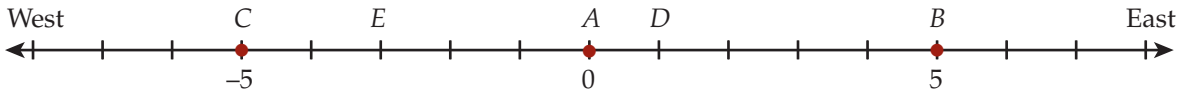
If you look at the handout, you will see that Point *A*, Mt. Everest, has already been marked for you.

Point *A*	Mt. Everest, in Tibet/Nepal, is 8850 m above sea level.
Point *B*	Mt. McKinley, the tallest mountain in the United States, has a height of 6195 m.
Point *C*	The highest point in California, Mt. Whitney, has a height of 4419 m.
Point *D*	Sea level.
Point *E*	The lowest point in California, Death Valley, is 86 m below sea level.
Point *F*	The deepest point in the Gulf of Mexico is 3787 m below sea level.
Point *G*	The Mariana Trench, in the Pacific Ocean, has a depth of 11,033 m.

4. This number line shows the position of five train stations.

Keesha takes the train from station *A*.
Station *B* is 5 miles to the east of station *A*.
Station *C* is 5 miles to the west of station *A*.

The locations of stations *A*, *B*, and *C* are labeled on the number line.



a. Handout 1 includes the number line. On it, label the points for stations *D* and *E*, using positive numbers for east and negative numbers for west.

b. A new station, *F*, is to be built 8 miles to the east of station *A*. Mark and label the point for station *F* on the number line.

c. What is the distance between station *A* and station *B*?

d. What is the distance between station *A* and station *C*?

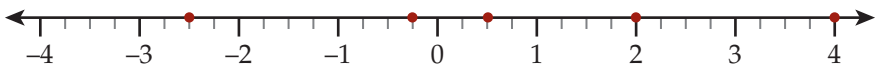
e. What is the distance between station *C* and station *B*?

f. Station *G* is the same distance from station *A* as station *E*, but in the opposite direction. Mark and label the point for station *G* on the number line.

Preparing for the Closing

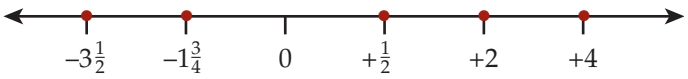
5. Three students were given the task of sketching a number line to show the numbers $4, 2, \frac{1}{2}, -1\frac{3}{4}$, and $-3\frac{1}{2}$. Here are their answers.

Student A



Student B

Student C



a. What advice would you give each student to overcome the mistakes he/she made?

b. Compare and discuss your answer to part a with your partner.

c. Sketch the number lines showing the numbers correctly placed.

6. With your partner's help, make a list of situations in which negative numbers are used. Use situations other than those described in the Work Time problems.

Skills

Find the missing numerator or denominator, and then express each fraction as a decimal.

a. $\frac{2}{5} = \frac{\square}{10} = \square$

b. $\frac{3}{5} = \frac{6}{\square} = \square$

c. $\frac{4}{\square} = \frac{8}{10} = \square$

d. $1\frac{1}{5} = \frac{12}{\square} = \square$

e. $\frac{20}{50} = \frac{\square}{100} = \square$

f. $\frac{30}{50} = \frac{60}{\square} = \square$

g. $\frac{40}{50} = \frac{\square}{100} = \square$

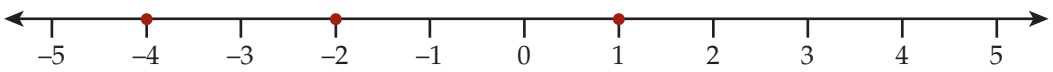
h. $\frac{1}{50} = \frac{2}{\square} = \square$

Review and Consolidation

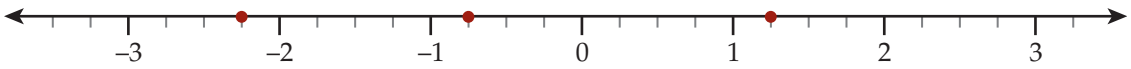
1. When you write 3, do you mean +3 (positive 3) or –3 (negative 3)?

2. What is the value of each point on the following number lines?

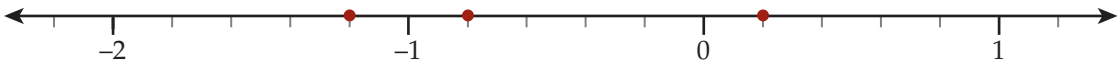
a.



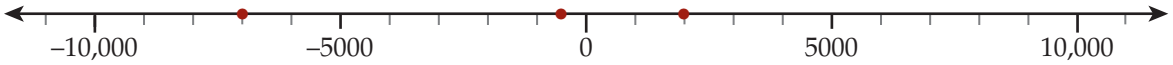
b.



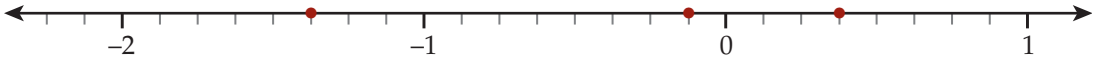
c.



d.



e.

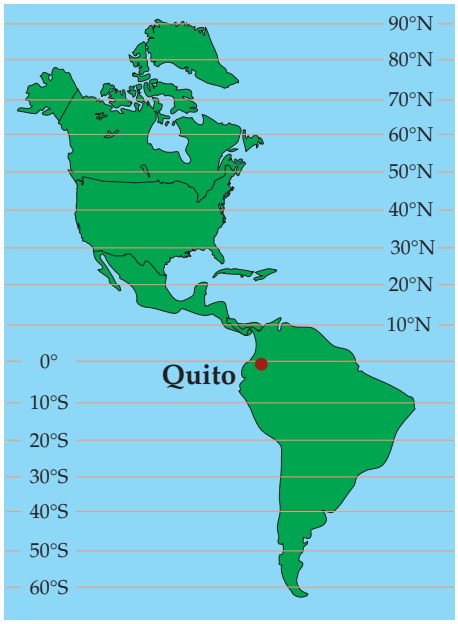


3. Lines of latitude run parallel to the equator. The latitude at the equator is 0°.

Every city in the world has a latitude, which is the number of degrees, north or south, of the equator.

This table lists the latitudes of some cities in North and South America:

City	Latitude
Cape Horn	62° South
Lima	12° South
Miami	26° North
Panama City	9° North
Pittsburgh	41° North
Quito	0°
Toronto	44° North



Hint: Latitude lines are horizontal.

a. Using the number line at the bottom of Handout 1, mark and label the point for each of the cities.

Use positive numbers for north and negative numbers for south.

b. Use your number line to help you determine the difference in degrees of latitude between Panama City and each of the other six cities.

Copy this table, fill in your answers, and explain your choices.

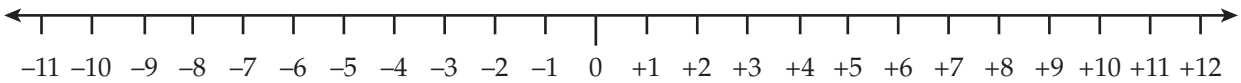
Distance from Panama City	City	Explanation
Closest		
Second closest		
Third closest		
Fourth closest		
Farthest		

4. Standard time at different places around the world varies from 12 hours ahead of Greenwich Mean Time (0 hours in England) to 12 hours behind.

Five cities with different standard times are shown in the following table.

City	Sydney	Los Angeles	London	New York	New Delhi
Time Difference from Greenwich	10 hours ahead	8 hours behind	0 hours	5 hours behind	5.5 hours ahead

a. Sketch a number line like this, mark, and label the point for each of the times shown in the table. Use positive numbers for "ahead" and negative numbers for "behind."



b. Use your number line to help find the time difference between New York and each of the other four cities. Copy and complete this table, explaining your answers.

City	Hours Ahead or Behind New York	Explanation
London		
Los Angeles		
Sydney		
New Delhi		

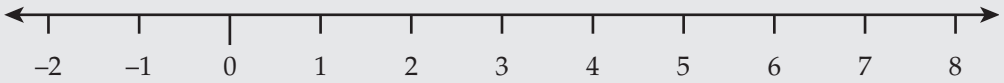
Homework

1. Lines of longitude run parallel to the Prime Meridian, which is an imaginary vertical line passing through Greenwich, England. The longitude at the Prime Meridian is 0°.

Every city in the world has a longitude, which is the number of degrees, east or west, of the Prime Meridian. This table lists the longitudes of some French cities.

City	Longitude
Le Mans	0.2° West
Nancy	6.2° East
Paris	2.3° East
Rennes	1.7° West
Strasbourg	7.8° East

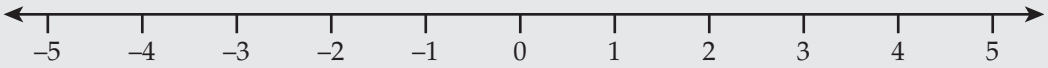
a. Sketch a number line like this. Mark and label the point for each of the cities listed in the table. Use positive numbers for east and negative numbers for west.



b. Use your number line to help you determine the difference in degrees of longitude between Paris and each of the other four cities. Copy and complete this table, explaining your answers.

Distance from Paris	City	Explanation
Closest		
Second closest		
Third closest		
Farthest		

2. Sketch a time line like this, with your date of birth at zero, stretching from 5 years before your birth (–5) to your fifth birthday (+5).



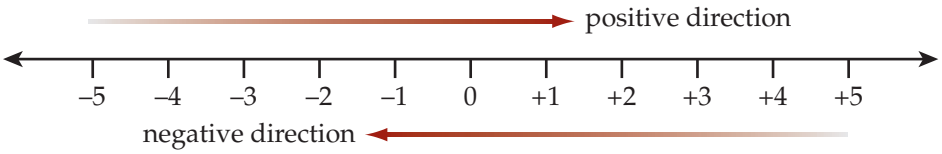
Choose at least five important events that occurred during that 10-year interval, marking and labeling each event as points on the time line. (Some ideas are: when you learned to walk, when your brother or sister was born, the years your team won the World Series, or presidential election years.)

PUTTING NUMBERS IN ORDER

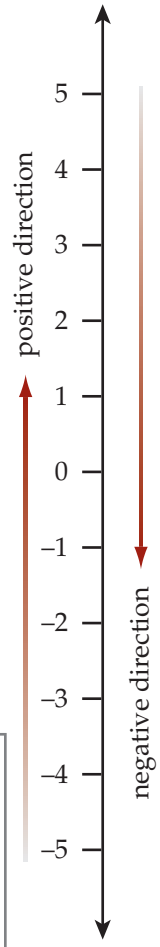
LESSON 2

CONCEPT BOOK	GOAL
See pages 187–189 in your *Concept Book*.	To compare positive and negative numbers and zero, using is less than (<), is less than or equal to (≤), is greater than (>), and is greater than or equal to (≥).

On horizontal number lines, like those shown in Lesson 1 with trains and time zones, the direction to the right is called the *positive direction*, and the direction to the left is called the *negative direction*.



On vertical number lines, like those shown in Lesson 1 with thermometers and latitudes, the direction up is called the *positive direction*, and the direction down is called the *negative direction*.



For any two points on a horizontal number line, the value of the point on the right is always greater than the value of the point on the left.

For any two points on a vertical number line, the value of the higher point is always greater than the value of the lower point.

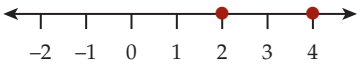
The symbol for *is greater than* is >.

Example

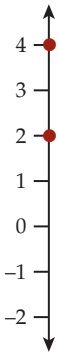
The point for 4 is above or to the right of the point for 2, so 4 > 2.

4 > 2

4 is greater than 2.



4 is to the right of 2.



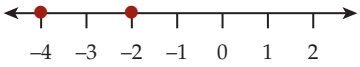
4 is above 2.

Example

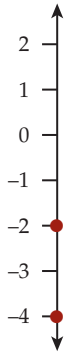
The point for –2 is above or to the right of the point for –4, so –2 > –4.

–2 > –4

–2 is greater than –4.



–2 is to the right of –4.



–2 is above –4.

For any two points on a horizontal number line, the value of the point on the left is always less than the value of the point on the right.

For any two points on a vertical number line, the value of the lower point is always less than the value of the higher point.

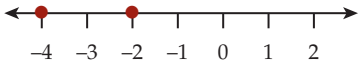
The symbol for *is less than* is <.

Example

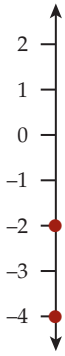
The point for –4 is below or to the left of the point for –2, so –4 < –2.

–4 < –2

–4 is less than –2.



–4 is to the left of –2.



–4 is below –2.

Example

If –4 < –2 and –2 < 5:

- You can write the numbers in ascending order as –4 < –2 < 5.
- You can write the numbers in descending order as 5 > –2 > –4.

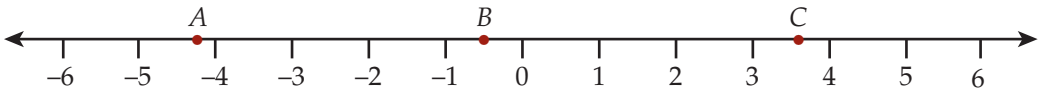
The phrases *is less than or equal to* (≤) and *is greater than or equal to* (≥) are also used.

Example

- The statement $x \leq 0$ is true if x represents any negative number or 0, since 0 = 0, and any negative number is less than zero.
- The statement $x \geq 0$ is true if x represents any positive number or 0, since 0 = 0, and any positive number is greater than zero.

Work Time

1. Points A, B, and C are labeled on this number line.



a. Which of the following numbers are values for A, B, and C?

–0.50 3.60 3.50 –5.80 –3.50 3.09 –4.20

b. Sketch the number line. Mark and label all seven numbers from part a.

c. List the numbers that are greater than –1.

d. List the numbers that are less than –1.

e. List the numbers that are greater than or equal to 3.09

f. List the numbers that are less than or equal to 3.09

g. Copy the following and place all seven numbers in the proper order. The least and the greatest numbers have already been placed.

–5.80 < ☐ < ☐ < ☐ < ☐ < ☐ < 3.60

h. Copy the following and place all seven numbers in the proper order. The least and the greatest numbers have already been placed.

3.60 > ☐ > ☐ > ☐ > ☐ > ☐ > –5.80

2. Decide whether the following statements are true or false.

a. $-1 > -\frac{3}{4}$ b. $-5 > -4$ c. $-5 < 7$ d. $5 < -4$

e. $-30 > 0$ f. $-0.2 < -0.03$ g. $+3.5 \leq 3.5$ h. $-3.5 \geq -3\frac{1}{2}$

Preparing for the Closing

3. A student reasoned that +3 is greater than +2, so –3 will be greater than –2. Was the student correct? Say why.

4. A student reasoned that 100 is much bigger than 5, so –100 > –5. Was the student correct? Say why.

5. A student reasoned that $\frac{15}{20} > \frac{3}{4}$, because the numbers 15 and 20 are greater than the numbers 3 and 4. Was the student correct? Say why.

6. A student reasoned that if $a > b$, then $b < a$. Was the student correct? Say why.

Skills

Express each fraction as a decimal.

a. $\frac{6}{8}$ b. $\frac{7}{8}$ c. $\frac{0}{8}$ d. $\frac{14}{8}$

e. $\frac{17}{8}$ f. $\frac{17}{4}$ g. $\frac{17}{2}$ h. $\frac{17}{34}$

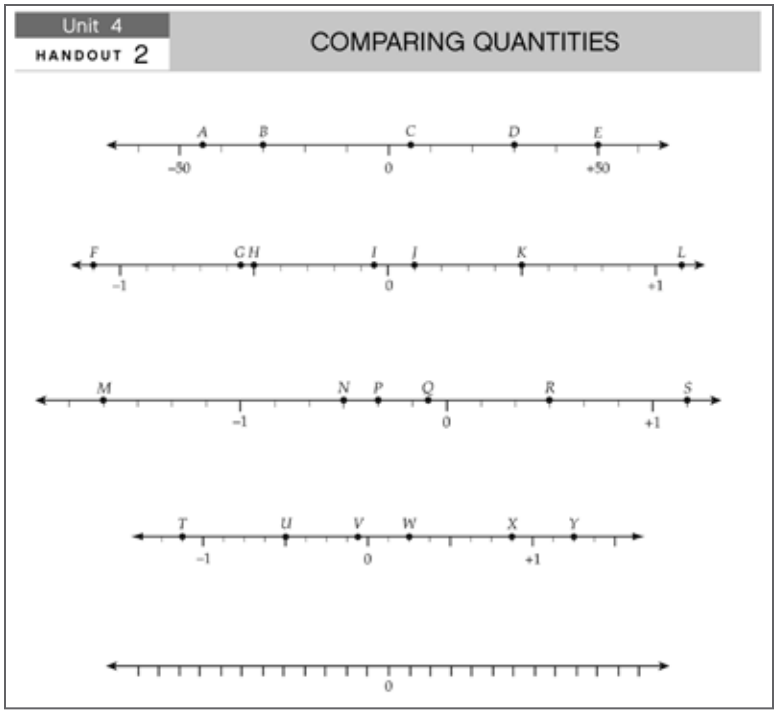
Review and Consolidation

1. Your teacher will give you a copy of Handout 2: *Comparing Quantities*, which shows points marked on some number lines.

a. On each number line, write the value for each labeled point.

b. Below each number line, use *less than* symbols (<) between the numbers written in ascending order, from least to greatest.

2. The last number line is blank. Use it to label some points and a scale. Trade number lines with your partner. Below your partner's number line, use *greater than* symbols (>) between the numbers written in descending order, from greatest to least.



Homework

1. a. Sketch a vertical number line (like a thermometer), mark a suitable scale that stretches from –15° to +15°, and label the following six temperatures.

11° 3° –5.5° –10° –8° –14°

b. List the temperatures that are greater than –4°.

c. List the temperatures that are less than –4°.

d. List the temperatures that are greater than or equal to –5.5°.

e. List the temperatures that are less than or equal to –5.5°.

f. Copy the following and place all six temperatures in the proper order.

☐ < ☐ < ☐ < ☐ < ☐ < ☐

g. Copy the following and place all six temperatures in the proper order.

☐ > ☐ > ☐ > ☐ > ☐ > ☐

LESSON 3

ADDING WITH NEGATIVE NUMBERS

GOAL	CONCEPT BOOK
To use the number line to add positive and negative numbers.	See pages 189–191 in your *Concept Book.*

Dwayne, Lisa, Rosa, and Jamal met at their favorite spot, the Valdez Restaurant, to do their math homework. That night, the homework was about adding and subtracting positive and negative numbers.

As they were working on their addition problems, Jamal reminded the group, "When you add, you get a sum. You can represent a sum on the number line as the distance from 0. You start at the first addend on the number line. Since you are adding a positive number, you move that number of units in the positive direction. Your answer is the distance you are from zero."

Comment

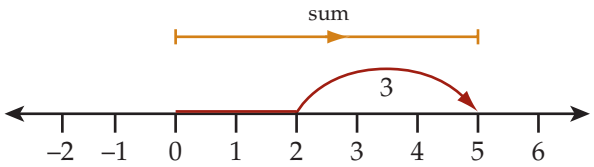
- Addend + addend = sum (distance from zero)
- When adding a **positive** number, you move to the **right**.

Jamal then showed his friends two examples of adding a positive number to any number:

2 + 3 =

Start at 2.

Move 3 in the positive direction.



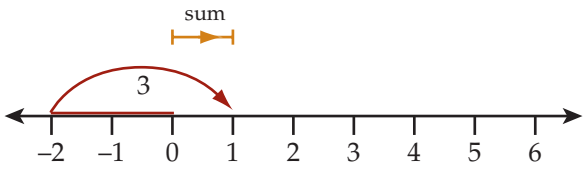
The distance from 0 is 5, or 5 in the positive direction.

2 + 3 = 5

(–2) + 3 =

Start at –2.

Move 3 in the positive direction.



The distance from 0 is 1, or 1 in the positive direction.

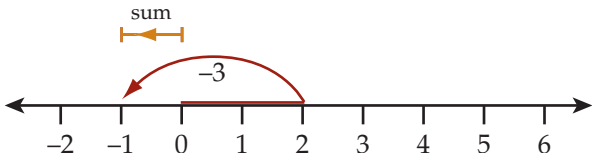
(–2) + 3 = 1

"Adding a negative number is just as easy." Jamal explained. "The only difference is you need to move in a negative direction from the first number." Again, he showed an example:

2 + (–3) =

Start at 2.

Move 3 in the negative direction because it is a –3 you are adding.



The distance from 0 is –1, or 1 in the negative direction.

2 + (–3) = –1

Comment

When adding a **negative** number, you move to the **left**.

He then challenged his friends to try some similar problems.

Work Time

1. Sketch number lines to illustrate these calculations.

a. $2 + (-6) =$

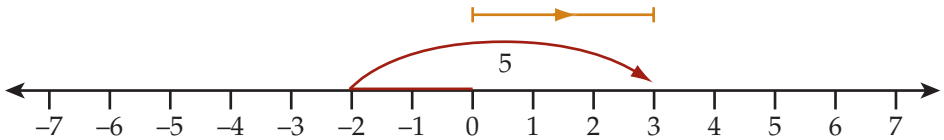
b. $(-2) + (-6) =$

c. Positive $1\frac{1}{2}$ plus negative 4 equals

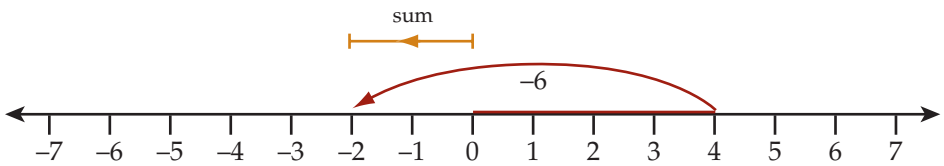
d. $\left(-2\frac{1}{2}\right) + \left(-3\frac{1}{2}\right) =$

2. Write the equation for each number line.

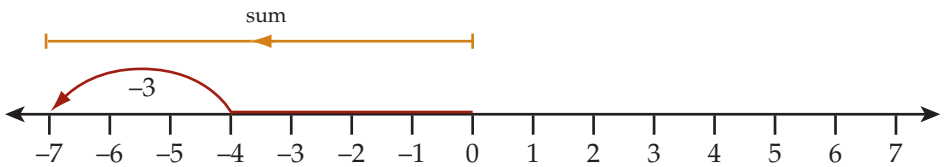
a.



b.



c.



Preparing for the Closing

3. Decide whether the following statements are *always true*, *sometimes true*, or *never true*.

a. If $a + b$ is a positive number, then both a and b are positive numbers.

b. If $a + b$ is a negative number, then both a and b are negative numbers.

4. In your own words, describe the technique for adding positive and negative numbers.

5. Play a game with your partner. Your teacher will give you game pieces with negative numbers and positive numbers. Lay the pieces face down.

a. Start at zero, and at each turn, flip over a game piece, and add the number it shows. The loser is the first person whose score goes outside the range –7 to +7.

b. At the end of the game, write the equation for the step that lost the game.

c. Is the losing score in this game more likely to be positive or negative? Say why.

Skills

Express each of the following decimals as fractions in their simplest form.

a. 0.4 b. 0.48 c. 0.5 d. 0.58

e. 2.58 f. 0.6 g. 0.68 h. 0.75

Review and Consolidation

1. Some students were playing the game in Work Time problem 6, and the first two numbers that came up were –1 and +3.

a. Sketch a number line that stretches from –7 to +7, using arrows to illustrate the first two numbers that were added.

b. Write the equation that is represented by the situation.

c. The next number that came up was –4. Write the new equation.

d. Here are the next numbers that came up.
Continue adding until the game ends.

–2 –1 +3 –4 –2

e. Write the equation for the step that lost the game.

f. Use your completed number line to calculate
(–1) + (+3) + (–4) + (–2) + (–1) + (+3) + (–4) + (–2) =

g. Calculate (–1) + (–4) + (–2) + (–1) + (–4) + (–2) + 3 + 3 =

h. Explain why the answers to parts f and g should be the same.

2. Calculate.

a. (–5) + 8 = b. (–8) + 5 = c. 4 + (–5) + 6 =

d. (–4) + 5 + (–6) = e. (–2.3) + (–0.9) = f. (–2.3) + 0.9 =

Homework

1. Copy this table.
Starting at 0, add the number listed, and fill in the answer for each step.
Stop when the answer is greater than +7 or less than –7.

Add		3	4	–2	–7	–5	2	3	–4	3	–5	8
Answer	0	3	7									

Example

0 + 3 = 3; 3 + 4 = 7; 7 + (–2) = ■

2. Calculate.

a. (–7) + 8 =

b. (–15) + 5 =

c. 10 + 10 + 18 =

d. (–4) + (–5) + (–6) =

e. (–4.3) + (–0.8) =

f. (–4.3) + 0.8 =

SUBTRACTING WITH NEGATIVE NUMBERS

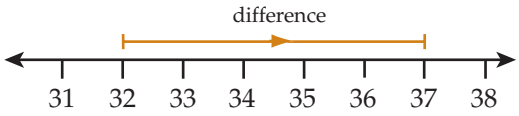
LESSON 4

CONCEPT BOOK	GOAL
See pages 191–193 in your *Concept Book.*	To use the number line to subtract positive and negative numbers.

The study group met again at the Valdez Restaurant. The homework that day focused on subtracting positive and negative numbers.

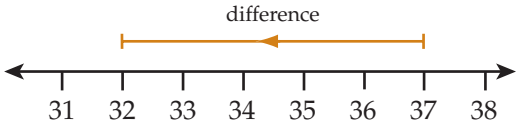
Dwayne said, "I think I understand this. When you subtract, you get a difference. We know this *difference* is the distance between the two numbers on the number line.

"Remember the example, 37 – 32 = 5. The distance between 37 and 32 is the difference, 5. Jamal reminded us yesterday how important direction is when dealing with positive and negative numbers. We never thought about it when subtracting two positive numbers, but the direction is from the number being subtracted to the first number." Dwayne drew an arrow pointing from 32 to 37.



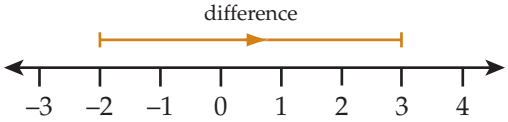
"As Ms. Reynolds would say, when you have $a - b$, the difference is the distance between a and b in the direction of b to a."

"So, if we subtracted 32 – 37, the distance would still be 5, but the direction would be from 37 to 32, or a negative direction. Thus, 32 – 37 = –5."



"Here is one that looks tricky, but it is really easy when you use number lines."

3 – (–2) =



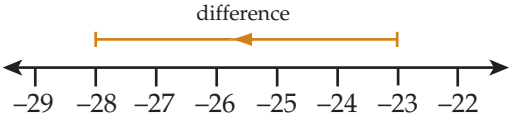
The distance between –2 and 3 is 5.

The direction is from –2 to 3, or a positive direction.

The difference is 5. 3 – (–2) = 5

"Or this one looks tough."

(–28) – (–23) =



The distance between –28 and –23 is 5.

The direction is from –23 to –28, or a negative direction.

The difference is –5. (–28) – (–23) = –5

Work Time

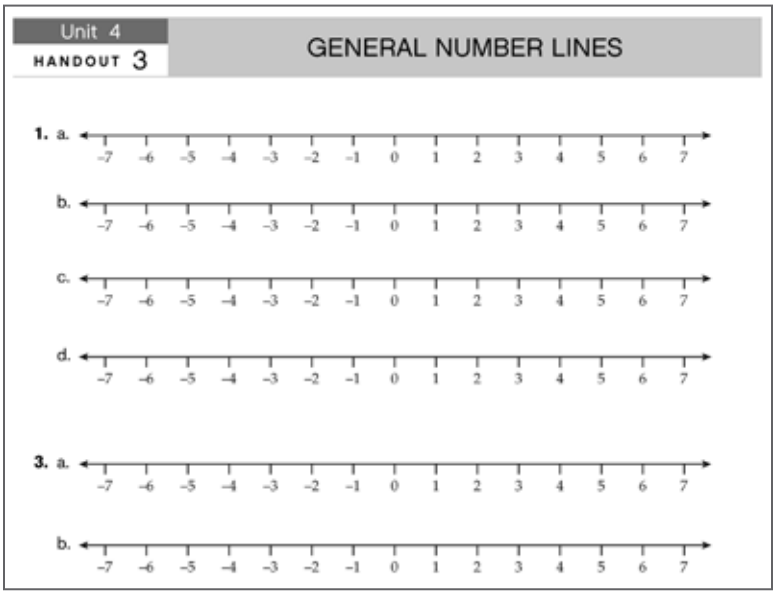
1. Your teacher will give you a copy of Handout 3: *General Number Lines*. Using the blank number lines on the handout, sketch the difference, including direction, for each of the following calculations.

a. Positive 1.5 minus positive 6.5 equals

b. (–2.5) – (–2.5) =

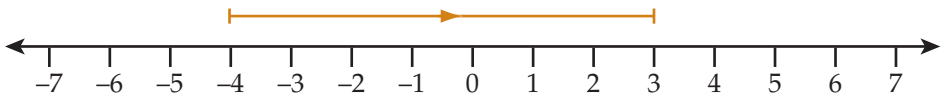
c. 2 – (–6) =

d. 2 – (–6) – 4 =

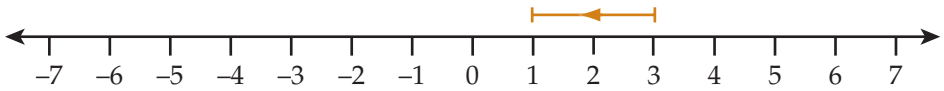


2. Write the equation for each number line.

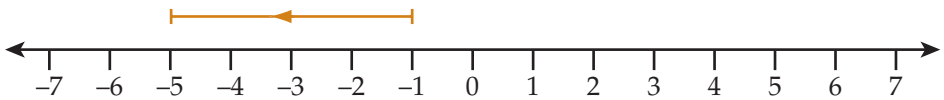
a.



b.



c.



3. Calculate.

a. (–7) – (–8) =

b. (–100) – (–100) =

c. 10 – 10 – 18 =

d. (–4) – (–5) – (–6) =

e. (–4.2) – 0.2 =

f. 4.2 – 0.2 =

Preparing for the Closing

4. In your own words, describe the technique for subtracting positive and negative numbers.

5. Play a game with your partner. Your teacher will give you game pieces with negative numbers and positive numbers. Lay the pieces face down.

a. Start at zero, and at each turn, flip over a game piece, and subtract the number it shows. The loser is the first person whose score goes outside the range –7 to +7.

b. At the end of the game, write the equation for the step that lost the game.

c. Is the losing score in this game more likely to be positive or negative? Say why.

Skills

Solve and give each solution in its simplest form.

a. $\frac{3}{8} + \frac{2}{8} + \frac{1}{8} =$

b. $\frac{3}{8} - \frac{2}{8} =$

c. $\frac{15}{8} - \frac{1}{8} =$

d. $10\frac{1}{8} + \frac{7}{8} =$

e. $\frac{3}{16} + \frac{2}{16} + \frac{1}{16} =$

f. $\frac{3}{16} - \frac{2}{16} =$

g. $\frac{15}{16} - \frac{1}{16} =$

h. $10\frac{1}{16} + \frac{7}{16} =$

i. $\frac{3}{16a} + \frac{2}{16a} + \frac{1}{16a} =$

j. $\frac{3}{16a} - \frac{2}{16a} =$

k. $\frac{15}{16a} - \frac{1}{16a} =$

l. $10\frac{1}{16a} + \frac{7}{16a} =$

Review and Consolidation

1. Some students were playing the game in Work Time problem 5. They started at 0.

a. The first number turned over was +1, and the second number was –3. Remember the rules are to subtract each of the numbers. What number did they calculate?

b. The next number to be turned over was +4. Write the new equation, and solve.

2. Write the equation for the following number line.

3. Using the number lines on Handout 3: *General Number Lines*, sketch the difference, including direction, for each of the following calculations.

a. (–3) – 4 = b. (–5) – (–3) =

4. Calculate.

a. (–5) – 8 = b. (–8) – 5 = c. (–4) – (–5) = d. (–6) – (–5) – (–4) =

e. (–5) – (–8) = f. 3 – (–4 + 5 – 6) = g. (–2.2) + (–1.9) = h. (–2.2) – (–1.9) =

Homework

1. Copy this table.
Starting at 0, subtract the number listed, and fill in the answer for each step.
Stop when the answer is greater than +7 or less than –7.

Subtract		–3	–4	2	7	5	–2	–3	4	–3	5	–8
Answer	0	3	7									

Example

0 – (–3) = 3; 3 – (–4) = 7; 7 – 2 = ■

2. Calculate.

a. (–7) – 8 = b. (–15) – 5 = c. 10 – 10 – 19 =

d. (–1) – (–2) – (–3) = e. (–4.2) – (–0.8) = f. (–4.2) – 0.8 =

ADDING AND SUBTRACTING

LESSON 5

CONCEPT BOOK	GOAL
See pages 189–193 in your *Concept Book.*	To understand and use the equivalence relationship between addition and subtraction.

It is important that you can write an expression as both an addition and a subtraction. Every expression can be written either as addition or subtraction using positive and negative numbers. Being able to move between the two operations will help you when you are working with algebraic expressions.

Subtracting a negative number is the same as adding a positive number.

Example

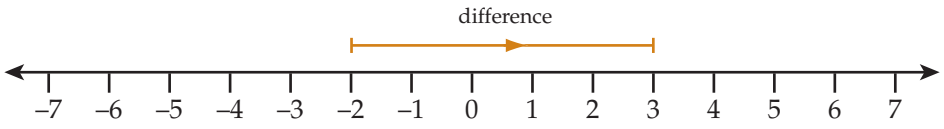
3 – (–2) is the same as 3 + 2.

In words:

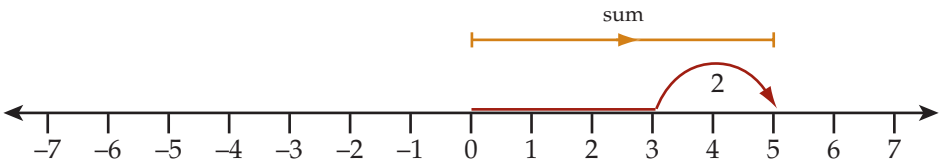
3 – (–2)

positive three minus negative two

If you look at the 3 – (–2) on the number line, you know the difference is 5:



If you look at 3 + 2 on the number line, the difference is also 5.



Using the same reasoning, you can prove that:

Subtracting any number is the same as adding its opposite (or adding the negative of that number.)

Example

$3 - 2 = 3 + (-2)$

Work Time

1. Convert the following to equations involving addition and solve.

a. $3 - (-4) =$ b. $(-9) - (-7) =$ c. $(-3) - (-8) =$ d. $7 - (-4) =$

2. Convert the following to equations involving subtraction and solve.

a. $3 + (-4) =$ b. $(-9) + (-7) =$ c. $(-3) + (-8) =$ d. $7 + (-4) =$

3. Calculate.

a. $3.1 - (-4.05) =$

b. $(-9.03) - (-6.951) =$

c. $(-3.1) - (-8.09) =$

d. $6.9 + (-4.1) =$

e. $(-5.11) - 12.13 - (-8.2) =$

f. $(-5.11) + 12.13 + (-8.2) =$

g. $6.2 - 5.1 - (-3.2) - 8.1 - (-3.15) - (-8) =$

h. $5.1 + (-3.1) - 9.1 - (-7.05) + 5 + (-12) =$

4. Write the following expression, using symbols.

Negative 15, plus positive 23, minus positive 18, minus negative 29

5. Use the words *plus*, *minus*, *positive*, and *negative* to write the following expression.

$31 - (-52) + (-28) + 16$

Preparing for the Closing

6. A student said, "To subtract negative numbers, you just turn all the negative numbers into positive numbers and then add."

Is this student correct? Say why.

Skills

Solve and give each solution in its simplest form.

a. $\frac{2}{3} + \frac{3}{9} =$

b. $\frac{2}{3} + \frac{4}{9} =$

c. $\frac{2}{3} + \frac{5}{9} =$

d. $\frac{2}{3} + \frac{6}{9} =$

e. $\frac{3}{7} - \frac{2}{56} =$

f. $\frac{3}{7a} - \frac{2}{56a} =$

g. $\frac{3}{7} - \frac{24}{56} =$

h. $\frac{3}{7x} - \frac{24}{56x} =$

Review and Consolidation

1. Convert the following to expressions involving addition, and solve.

a. $230 - (-7) =$

b. $(-230) - (-17) =$

c. $(-230) - (-27) =$

d. $(-230) - (-7) =$

2. Convert the following to expressions involving subtraction, and solve.

a. $230 + (-7) =$

b. $(-230) + (-17) =$

c. $(-230) + (-27) =$

d. $(-230) + (-7) =$

3. Calculate.

a. $15.2 - (-16.15) =$

b. $(-8.04) - (-5.567) =$

c. $(-10.2) - (-20.18) =$

d. $7.8 + (-2.3) =$

4. Lisa and Rosa went shopping. Lisa did not want to take her purse, so she gave Rosa $50.00 to hold.

- At the movies, Rosa gave Lisa a $20 bill to pay for the movie and a drink.
- Lisa gave Rosa the change, which was $4.50.
- The next day, she got her allowance and gave Rosa an additional $10.00 to hold as they went shopping.
- At the mall, Lisa bought a bracelet for $10.50 and a purse for $12.50.

When Rosa gave Lisa back the rest of her money to put in her new purse, how much did she give her?

5. a. Make up problems that involve addition and subtraction of negative numbers. Give these problems to your partner to solve.

b. When you have both finished solving all your problems, compare answers, and discuss the solution methods that each of you used.

c. Write a short report on your preferred methods and how to avoid possible errors.

Homework

1. Write the following expression, using symbols.

25, plus negative 67, minus positive 12, minus negative 20

2. Use the words *plus*, *minus*, *positive*, and *negative* to write the following expression.

$-85 + (-10) - (-25) + 16$

3. Calculate the answers to problems 1 and 2 by the most efficient method, without using a calculator.

4. Calculate.

a. $(-5) - 8 + (-1) =$

b. $(-8) - 5 + (-2) =$

c. $4 - 5 + 6 =$

d. $(-4) + (-5) - (-6) =$

e. $(-5) - (-8) + (-2) =$

f. $(-5) - (-8) + (-2) - [(-4) - (-5) - 6] =$

LESSON 6

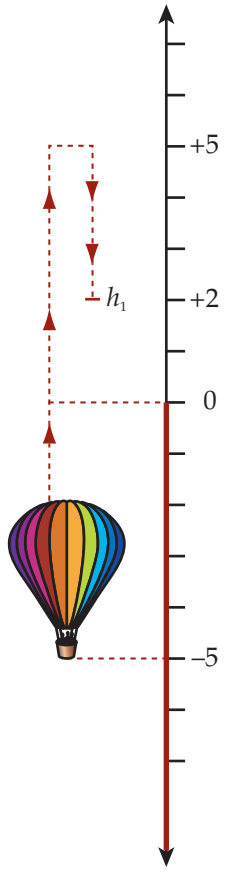
BALLOON MODEL

GOAL	CONCEPT BOOK
To model addition and subtraction of positive and negative numbers.	See pages 185–193 in your *Concept Book.*

Work Time

The height of an imaginary balloon above (positive) or below (negative) its normal operating height is influenced by:

- Weights: Each 1-unit change in weight moves the balloon up (losing weight) or down (gaining weight) by 1 meter.
- Hot air: Each 1-unit change in the amount of hot air in the balloon moves the balloon up (gaining hot air) or down (losing hot air) by 1 meter.
- Whether weight or hot air is *added* to the balloon, or *taken away* (subtracted) from the balloon.



Comment

Balloon goes up (↑) when:
- Hot air is added
- Weight is taken away

Balloon goes down (↓) when:
- Hot air is taken away
- Weight is added

Your teacher will give you a copy of Handout 4: *Height of Balloon*, containing a vertical number line stretching from +6 m to –16 m, where 0 represents the normal operating height of the balloon.

Track the motion of the balloon for problems 1–5 using the number line. The motion for problem 1 is shown on the number line at right.

1. The balloon starts at a height of 5 m below its normal operating height, and then 10 units of hot air and 3 units of weight are added.

a. Explain why the new height of the balloon, h_1, can be given by the description:

h_1 equals negative 5, plus positive 10, plus negative 3.

b. Calculate h_1.

c. Check your answer against the diagram.

2. Starting from the previous height of h_1, the balloon has 8 units of hot air and 2 units of weight taken away.

a. Explain why the new height of the balloon, h_2, can be given by the description:
$h_2 = h_1 - (+8) - (-2)$

b. Calculate h_2.

3. Starting from the previous height of h_2, the balloon has 2 units of hot air taken away and 6 units of weight added.

a. Explain why the new height of the balloon, h_3, can be given by the description:

$h_3 = h_2 - (+2) + (-6)$

b. Calculate h_3.

4. Starting from the previous height of h_3, the balloon has 7 units of hot air added and 5 units of weight taken away.

a. Write a description for the new height of the balloon, h_4.

b. Calculate h_4.

5. Starting from the previous height of h_4, the changes described in problem 3 (2 units of hot air taken away and 6 units of weight added) are repeated twice.

a. Write a description for the new height of the balloon, h_5.

b. Calculate h_5.

Preparing for the Closing

6. Subtracting a negative number gives the same result as adding its opposite, a positive number.

a. Give an example of this, and illustrate it on a vertical number line.

b. Interpret your example in terms of the balloon model.

7. Adding a negative number gives the same result as subtracting its opposite, a positive number.

a. Give an example of this, and illustrate it on a vertical number line.

b. Interpret your example in terms of the balloon model.

8. Does the balloon model help you understand addition and subtraction of positive and negative numbers? Explain.

Skills

Estimate and then solve.

a. 15.41 + 6.6 =
b. 15.41 + 16.6 =
c. 15.41 + 26.6 =
d. 20 – 4.59 =
e. 20.2 – 4.59 =
f. 200.2 – 4.59 =

Review and Consolidation

Write equations to match each of the following stories based on the balloon model.

1. The balloon starts at a height of 8 meters below its normal operating height.
   5 units of hot air are added into the balloon.

   To what height does the balloon move?

2. The balloon starts at a height of 5 meters below its normal operating height.
   15 units of weight are then thrown (taken away) from the balloon.

   To what height does the balloon move?

3. The balloon starts at a height of 8 meters above its normal operating height.
   12 units of weight are then added to the balloon.

   To what height does the balloon move?

4. The balloon starts at a height of 6 meters below its normal operating height.
   18 units of hot air are then released (taken away) from the balloon.

   To what height does the balloon move?

5. The balloon starts at a height of 6 meters above its normal operating height.
   4 units of hot air and 7 units of weight are then released (taken away) from the balloon.

   To what height does the balloon move?

6. The balloon starts at a height of 9 meters above its normal operating height.
   8 units of hot air are then released (taken away) from the balloon, and 12 units of weight are added.

   To what height does the balloon move?

7. The balloon starts at a height of 15 meters below its normal operating height.
   The release (taking away) of 3 units of hot air is repeated 5 times, and then 9 units of weight are added.

   What is the height of the balloon at the end?

Homework

1. a. Calculate (–3) + (4 • 2).

b. Sketch the calculation on a number line.

2. a. Calculate 5 – (4 • 2).

b. Sketch the calculation on a number line.

3. Explain what is common to both problems 1 and 2.

LESSON 7

REVIEWING ADDITION AND SUBTRACTION

GOAL

To review what you have learned about positive and negative numbers.

CONCEPT BOOK

See pages 185–193 in your *Concept Book*.

Putting It Together

To prepare for this lesson, review your *Concept Book* and the work in your notebook.

In particular, look carefully at Homework problem 1 of both Lesson 3 and Lesson 4. The answers you should have obtained are as follows.

For adding:

Add		3	4	–2	–7	–5	2	3	–4	3	–5	8
Answer	0	3	7	5	–2	–7	–5	–2	–6	–3	–8	GAME OVER

For subtracting:

Subtract		–3	–4	2	7	5	–2	–3	4	–3	5	–8
Answer	0	3	7	5	–2	–7	–5	–2	–6	–3	–8	GAME OVER

Work Time

1. In your own words, describe what you learned from the identical answers in the two tables.

2. Subtracting a positive number gives the same result as adding its inverse (a negative number): $a - b = a + (-b)$. Say why.

3. Subtracting a negative number gives the same result as adding its inverse (a positive number): $a - (-b) = a + b$. Say why.

4. a. $-\frac{1}{10} \leq -\frac{1}{11}$

   Say why.

   b. $-0.1 \geq -\frac{1}{10}$

   Say why.

5. Calculate the following expressions by adding the inverse of the number in parentheses.

a. $5 - (-3) =$ b. $(-5.5) - (-4) =$ c. $4 - (-6.5) =$ d. $(-4) - (-6) =$

6. Calculate.

a. $225 - 15 =$ b. $225 - 10 =$ c. $225 - 5 =$

d. $225 - 0 =$ e. $225 - (-5) =$ f. $225 - (-10) =$

g. Do you see a pattern? Explain.

h. Why are the answers for problems e and f greater than the answer for problem d?

7. Calculate.

a. $(-25) + (-10) =$ b. $(-25) + (-5) =$ c. $(-25) + 0 =$

d. $(-25) + 5 =$ e. $(-25) + 10 =$ f. $(-25) + 15 =$

g. Do you see a pattern? Explain.

Preparing for the Closing

8. A student claimed that $b - a$ is the negative of $a - b$.

a. Check that this is true for $a = -4.5$ and $b = 2.5$.

b. Is the statement true for all values of a and b? Say why.

9. Which two numbers have a sum of 8 and a difference of 22?

Skills

Solve and give each solution in its simplest form.

a. $\frac{1}{2} + 0.5 + \frac{5}{8} =$ b. $\frac{1}{4} + 0.6 + \frac{5}{8} =$ c. $2.5 + 2.6 + \frac{5}{8} =$

d. $4.625 - \frac{5}{16} =$ e. $\frac{37}{8} - 0.25 =$ f. $\frac{40}{8} - 0.25 =$

Review and Consolidation

1. Decide whether the following statements are true or false. Say why.

a. Negative 5 is less than negative 2

b. $4 < 4$

c. $-3 \geq -3$

d. Negative 2 is less than positive 2

e. Negative one hundred is greater than negative one

f. Negative one hundred is less than or equal to negative one

g. $0.50 > 0.5$

h. $0.3333 \geq \frac{1}{3}$

i. $(-5.5) - (-8.5) = 5.5 - 8.5$

j. $(-3) + (-5) = (-3) - 5$

k. $(-8) - (-5) = (-5) - (-8)$

l. $(-7) + 6 = 6 + (-7)$

2. Calculate.

a. $245 - 15 =$

b. $225 + 15 =$

c. $(-300) - 5 =$

d. $(-300) + (-5) =$

e. $300 - 5 =$

f. $300 + (-5) =$

g. $450 - (-50) =$

h. $450 + 50 =$

i. $25 + 4.5 - 2.5 - (-2) =$

j. $5 + (-5) + 6 + (-12) + 6 =$

Homework

1. Without using a calculator, use your preferred method to do these calculations.

a. $(-5) - (-3.4) =$

b. $0.9 - (-0.2) =$

c. $4.1 - 6.2 =$

d. $(-3.6) - 6.4 =$

2. Each of the calculations in problem 1 can be described by the equation $a - b = c$. Write equivalent addition equations in the form $a + (-b) = c$.

3. Rearrange terms to simplify the calculation:

$+3 - (-4) + (+5) - (+1) + (-4) - (-7) + (+8) - (-3) =$

4. True or false?

a. $3 - (-5) = (-3) - (-5)$

b. $3 + (-5) = 3 - (-5)$

c. $5 + (-3) = 5 - 3$

d. $(-3) + (-5) = (-5) + (-3)$

e. $(-3) - (-5) = (-3) + 5$

f. $(-3) - (-5) = (-5) - (-3)$

LESSON 8

MULTIPLYING AND DIVIDING

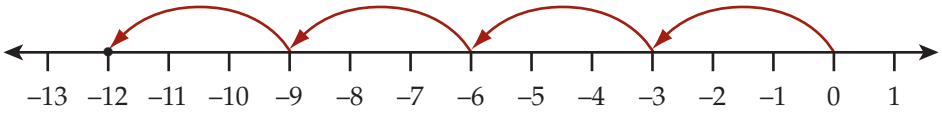
GOAL	CONCEPT BOOK
To multiply and divide positive and negative numbers.	See pages 195–198 in your *Concept Book*.

Ms. Reynolds' class was still studying positive and negative numbers. Dwayne, Chen, Keesha, and Rosa met to do their homework.

Keesha said, "We all know how to multiply and divide with positive numbers, and we know that the answers to these problems are always positive: 4 • 3 = 12. Or you could say, +4(+3) = (+12). Also, 3 = 12 ÷ 4 is the same as +3 = (+12) ÷ (+4)."

Chen recalled learning about multiplying and dividing negative numbers in school in Beijing. There, they used the number line to help them.

Chen said, "The expression 4(–3) can be thought of as (–3) + (–3) + (–3) + (–3)," and he sketched this number line:



"So, 4 • (–3) = –12 (a negative number). For this multiplication, you know the two equivalent division equations are: (–12) ÷ 4 = –3 and (–12) ÷ –3 = 4.

"In other words, for any multiplication fact, $ab = c$, there are two division facts that are based on the same relationship, $c \div b = a$ and $c \div a = b$.

"To multiply two negative numbers, remember that $-a$ is the inverse of a. Therefore, the product of –4(–3) can be considered the inverse of the product of 4(–3).

Comment

For $ab = c$	Ex: 4 • (–3) = –12
$c \div b = a$	–12 ÷ (–3) = 4
$c \div a = b$	–12 ÷ 4 = –3

"Another way to look at it is:

$$\begin{aligned} -4(-3) &= (-1 \bullet 4)(-3) \\ &= -1(4 \bullet -3) \\ &= -[4(-3)] \end{aligned}$$

"Therefore, $-4(-3) = 12$ (a positive number).

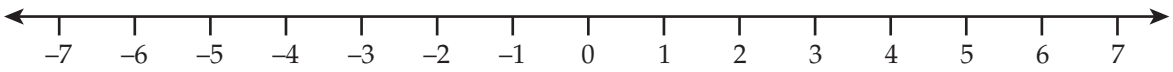
"The related division problems are $12 \div -4 = -3$ and $12 \div -3 = -4$."

Chen concluded by making a table of rules:

Number	Operation	Number	Result
Positive Number	•	Positive Number	= Positive Number
	÷		
Negative Number	•	Negative Number	
	÷		
Negative Number	•	Positive Number	= Negative Number
	÷		
Positive Number	•	Negative Number	
	÷		

Work Time

1. a. Copy this number line and sketch arrows to illustrate $3(-2) = -6$.



b. Locate the number that is the negative of $3(-2)$.

2. Copy and complete this table by finding the missing products and the differences between each two equations that are next to each other.

$3 \bullet 11 = 33$	
	11
$2 \bullet 11 = 22$	
	☐
$1 \bullet 11 =$ ☐	
	☐
$0 \bullet 11 =$ ☐	
	☐
$-1 \bullet 11 =$ ☐	
	☐
$-2 \bullet 11 =$ ☐	

3. Calculate.

a. $4 \bullet 8 =$

b. $0.1(-0.1) =$

c. $(-10) \bullet 0.1 =$

d. $(-4) \div (-8) =$

e. $(-0.1) \div (-0.1) =$

f. $40 \div (-8) =$

4. Explain why each of the following is true.

a. $(-10)(-10)(-10) = -1000$

b. $(-10)(-10)(-10)(-10) = 10{,}000$

c. $(-10)(-10)(-10)(-10)(-10) = -100{,}000$

d. It is easy to determine the sign in any further calculation of this type. Say why.

Preparing for the Closing

Work with a partner on these problems.

5. Write your own explanation of why a negative number multiplied by a negative number gives a positive number.

6. A student explained $(-7) + 2 = -5$ and $(-7) \bullet 2 = -14$ by saying, "A negative number and a positive number gives a negative number."

Was the explanation correct? If not, what should the student have said?

Skills

Add or subtract. Give each answer as a mixed number.

a. $4\frac{2}{3} + 1\frac{5}{9} =$

b. $5\frac{2}{3} + 1\frac{7}{9} =$

c. $6\frac{2}{3} + 1\frac{8}{9} =$

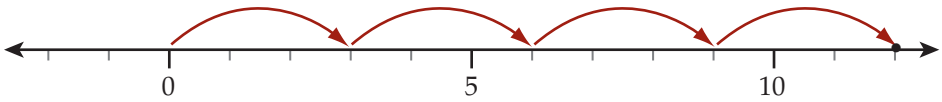
d. $4\frac{3}{4} - 1\frac{5}{8} =$

e. $5 - 1\frac{7}{8} =$

f. $6\frac{1}{2} - 1\frac{1}{8} =$

Review and Consolidation

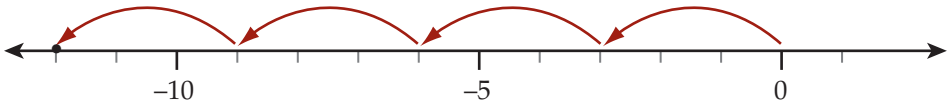
1.



a. Write a repeated addition equation to match the number line.

b. Write your repeated addition equation as a multiplication equation.

2.



a. Write a repeated addition equation to match the number line.

b. Write your repeated addition equation as a multiplication equation.

3. Calculate.

a. $(-6) \bullet 9 =$

b. $8(-7) =$

c. $(-9)(-7) =$

d. $\left(-\frac{3}{4}\right)\left(-\frac{4}{5}\right) =$

e. $\left(-\frac{3}{4}\right)\left(-\frac{5}{4}\right) =$

f. $\left(-\frac{3}{4}\right)\left(\frac{4}{5}\right) =$

g. $0.25(-0.8) =$

h. $0.24 \div (-0.8) =$

i. $(-0.8) \div 0.24 =$

Homework

1. Copy this table, and fill in the cells by performing the operation shown at the top of each column. Use the values for a and b that are given in each row. The first row has been done for you.

Use the completed table to answer parts b through e.

a.

a	b	$a + b$	$a - b$	ab	$a \div b$
6	–3	3	9	–18	–2
–6	3				
3	–6				
–3	6				
6	3				
–6	–3				
3	6				
–3	–6				

b. For which pairs of values of a and b does ab equal a negative number?

c. What do the pairs of values that you listed in part b have in common?

d. For which pairs of values of a and b does $a \div b$ equal a positive number?

e. What do the pairs of values that you listed in part d have in common?

ORDER OF OPERATIONS

LESSON 9

CONCEPT BOOK	GOAL
See page 198 in your *Concept Book.*	To calculate expressions involving more than one operation using the order of operations.

The convention used for order of operations is:

- Parentheses or brackets first. ( ) [ ]
- Then, exponents.
- Then, multiplication and division (•, ÷), working from left to right.
- Then, addition and subtraction (+, –) last, working from left to right.

The operations inside of parentheses are calculated first. If the expression has more than one set of parentheses or brackets, calculate the operations inside the innermost set first.

Note that the order of operations given above is the convention used to evaluate numerical expressions with scientific calculators. This is an arbitrary convention that only applies to numerical expressions. It is needed, for example, when evaluating the numerical value of an algebraic expression for substitutions.

Example

When evaluating $a(b-1)^2 + c$ with $a = 3$, $b = 5$, and $c = 2$:

$3 \bullet (5-1)^2 + 2$	Parentheses
$3 \bullet 4^2 + 2$	Exponents
$3 \bullet 16 + 2$	Multiplication/Division
$48 + 2$	Addition/Subtraction
50	

Work Time

1. Work with a partner. You will need scissors.
   - Your teacher will give you a copy of Handout 5: *True or False?*
   - Cut out all of the cards.
   - Sort the cards into two piles by taking turns selecting a card and deciding whether the equation is true or false. Try working out the values of the expressions to help you decide.
   - For each card, explain to your partner how you know the equation is true or false. Your partner should either agree with your explanation or challenge it if the explanation is not clear and complete.
   - Look for patterns in your answers.

Unit 4 HANDOUT 5 — TRUE OR FALSE?

A1 $8 + 4 = 4 + 8$	A2 $8 - 4 = 4 - 8$
A3 $8 \bullet 4 = 4 \bullet 8$	A4 $8 \div 4 = 4 \div 8$
B1 $8 + 4 + 2 = 8 + (4 + 2)$	B2 $8 \bullet 4 \bullet 2 = 8 \bullet (4 \bullet 2)$
B3 $8 - 4 - 2 = 8 - (4 - 2)$	B4 $8 \div 4 \div 2 = 8 \div (4 \div 2)$
C1 $8 \bullet (4 + 2) = 8 \bullet 4 + 2$	C2 $8 \bullet (4 + 2) = 8 \bullet 4 + 8 \bullet 2$
C3 $8 \bullet (4 - 2) = 8 \bullet 4 - 2$	C4 $8 \bullet (4 - 2) = 8 \bullet 4 - 8 \bullet 2$
D1 $(8 + 4) \div 2 = 8 \div 2 + 4 \div 2$	D2 $(8 - 4) \div 2 = 8 \div 2 - 4 \div 2$
D3 $(8 + 4) \div 2 = 8 + 4 \div 2$	D4 $(8 - 4) \div 2 = 8 - 4 \div 2$

Preparing for the Closing

2. a. What number properties told you that A1 and A3 were true?

   b. Write the property using letters.

3. Would the statement on B2 still be true if you used different numbers, like 12, 6, and 3?

4. What have you found out about cards C1–C4?

5. What have you found out about cards D1–D4?

6. a. What would a calculator give as an answer for the expression $1 + 2 \bullet 3 + 4$?

   b. Add parentheses to the expression $1 + 2 \bullet 3 + 4$ to make the answer 13.

   c. Add parentheses to the expression $1 + 2 \bullet 3 + 4$ to make the answer 15.

   d. Add parentheses to the expression $1 + 2 \bullet 3 + 4$ to make the answer 21.

Skills

Solve.

a. $\frac{1}{5} \bullet 36 =$

b. $\frac{2}{5} \bullet 36 =$

c. $\frac{4}{5} \bullet 36 =$

d. $37 \bullet \frac{1}{5} =$

e. $37 \bullet \frac{2}{5} =$

f. $37 \bullet \frac{4}{5} =$

Review and Consolidation

1. Add parentheses to the expression to reflect the order of operations and then calculate its value.

a. $32 \div 10 - 2$

b. $(25 + 8) \div 3 - 5 \bullet 3$

c. $20 + 5 \bullet 3$

d. $77 \div (11 - 4) \bullet 13$

e. $15 - (24 \div 6) + 3 \bullet 2$

f. $43 - 39 \div 3 + 7$

g. $8 + 9 \bullet 8$

h. $196 \div 14 \bullet 3$

i. $196 \div (14 \bullet 3)$

j. $\frac{3^3}{11 - 5}$

k. $2.5(3.2 + 1.6) - (3.8 \div 1.9)$

2. a. Make up some similar problems and share them with your partner.

b. Check that you both get the same answers.

Homework

1. $40 + 7 \bullet 3$

a. Add parentheses to make this expression equal 61.

b. Add parentheses to make this expression equal 141.

c. Which answer uses the order of operations convention?

2. $3 \bullet 4 + 14 - 5 - (-2) \bullet 7$

a. Add parentheses to make this expression equal –23.

b. Add parentheses to make this expression equal 5.

c. Add parentheses to make this expression equal 35.

d. Which answer uses the order of operations convention?

3. $24 \div 8 - 2 \bullet 2$

a. Add parentheses to make this expression equal –1.

b. Add parentheses to make this expression equal 8.

c. Add parentheses to make this expression equal 2.

d. Which answer uses the order of operations convention?

MIXED OPERATIONS

LESSON 10

CONCEPT BOOK	GOAL
See pages 12–13, 198 in your *Concept Book*.	To calculate expressions involving more than one operation, using the distributive property.

Parentheses (and brackets) are used in two different ways in this unit.

- In the expression $5 + (-7)$, parentheses are used to indicate that the number to be added to 5 is negative 7.
- In the expression $a(b + c)$, parentheses are used to indicate that b and c are to be added together before multiplying the result by a.

Example

Calculate $a(b + c)$ for the values $a = -6$, $b = 5$, and $c = -7$.

Substitute the values.

$$a(b + c)$$

$$(-6)[5 + (-7)]$$

Reduce the expression to a multiplication of two numbers.

$$(-6)[5 + (-7)] = (-6)(-2)$$

Since a negative number multiplied by a negative number gives a positive number, the final answer is +12.

Work Time

1. a. Calculate $ab + ac$ for the values $a = -6$, $b = 5$, and $c = -7$.

b. The answer for part a should be the same as the answer for the example in the introduction to the lesson.

What is the name of the property that describes this equality?

c. $a(b + c) = (b + c)a$

What is the name of the property that describes this equality?

2. a. Use the distributive property to rewrite $-2(x - 7)$ without using parentheses.

b. Calculate the value of $-2(x - 7)$ for $x = -4$.

c. Check that you get the same answer by substituting $x = -4$ in your answer to part a.

3. a. Use the distributive property to rewrite $-x(x - 7)$ without using parentheses.

Hint: Remember that $x \cdot x = x^2$. You say "x squared."

b. Calculate the value of $-x(x - 7)$ for $x = -4$.

c. Check that you get the same answer by substituting $x = -4$ in your answer to part a.

4. a. Which of the following expressions are equal to $(5 - x) \div y$? Say why.

Ⓐ $y \div (5 - x)$ Ⓑ $\frac{5}{y} - \frac{x}{y}$ Ⓒ $\frac{1}{y}(5 - x)$ Ⓓ $\frac{5 - x}{y}$

b. Check your answers to part a by calculating the value of each expression for $x = -3$ and $y = -2$.

5. a. Calculate $\frac{a}{c + d}$ for the values $a = 15$, $c = 4$, and $d = -7$.

Hint: $\frac{a}{c + d}$ is just another way of writing $a \div (c + d)$

b. Calculate $\frac{b}{c + d}$ for the values $b = -6$, $c = 4$, and $d = -7$.

c. Express $\frac{a}{c + d} + \frac{b}{c + d}$ as a single fraction.

d. Calculate your answer to part c for the values $a = 15$, $b = -6$, $c = 4$, and $d = -7$.

e. Do the calculations to check that your answer to part d is equal to the sum of your answers to parts a and b.

Preparing for the Closing

Check your work with your partner, and then answer the following problems together.

6. Which number property justifies the equation $\frac{1}{a}(b + c) = (b + c)\frac{1}{a}$?

7. What are the similarities and differences between these three number properties?

$$c(a + b) = ca + cb$$

$$(a + b)c = ac + bc$$

$$(a + b) \div c = (a \div c) + (b \div c)$$

8. a. Write an expression for "negative 8 divided by the quantity positive 3 plus negative 5."

b. What are some common mistakes that could be made in calculating the expression in part a?

c. What are some other common mistakes that can be made when calculating with long expressions that include negative numbers?

9. Read aloud an expression involving three numbers—including negative numbers—and two operations. See if your partner can write it down the way you meant it—and then see if your partner gets the same answer as you.

Skills

Solve and write your answers in simplest form.

a. $\frac{3}{5} \bullet \frac{1}{2} =$ b. $\frac{4}{5} \bullet \frac{1}{2} =$ c. $\frac{6}{5} \bullet \frac{1}{2} =$ d. $\frac{9}{5} \bullet \frac{1}{8} =$

e. $\frac{3}{7} \bullet \frac{3}{8} =$ f. $\frac{7}{3} \bullet \frac{8}{3} =$ g. $\frac{7}{3} \bullet \frac{3}{8} =$ h. $\frac{7}{3} \bullet \frac{3}{6} =$

Review and Consolidation

1. Calculate.

a. $-3 + [8 - (-12)] =$

b. $(-3) + 8 - (-12) =$

c. $-6 - (-8 + -4) =$

d. $(-6) - (-8) + (-4) =$

e. $(-12) \div [(-9)(-4)] =$

f. $[(-12) \div (-9)](-4) =$

g. $-10\ (-5 \bullet -2) =$

h. $[(-10)(-5)] - [(-10) \bullet 2] =$

2. Calculate.

a. $(-0.3) + [0.8 - (-1.2)] =$

b. $-0.3 + 0.8 - (-1.2) =$

c. $(-0.6) - [(-0.08) + (-0.4)] =$

d. $-0.6 - (-0.08) + (-0.4) =$

e. $(-1.2) \div [(-0.9)(-0.4)] =$

f. $[(-1.2) \div (-0.9)](-0.4) =$

g. $(-10)[(-0.9) - 0.11] =$

h. $[-10(-0.9)] - [(-10) \bullet 0.11] =$

3. a. Make up some similar problems and share them with your partner.

b. Check that you both get the same answers.

Homework

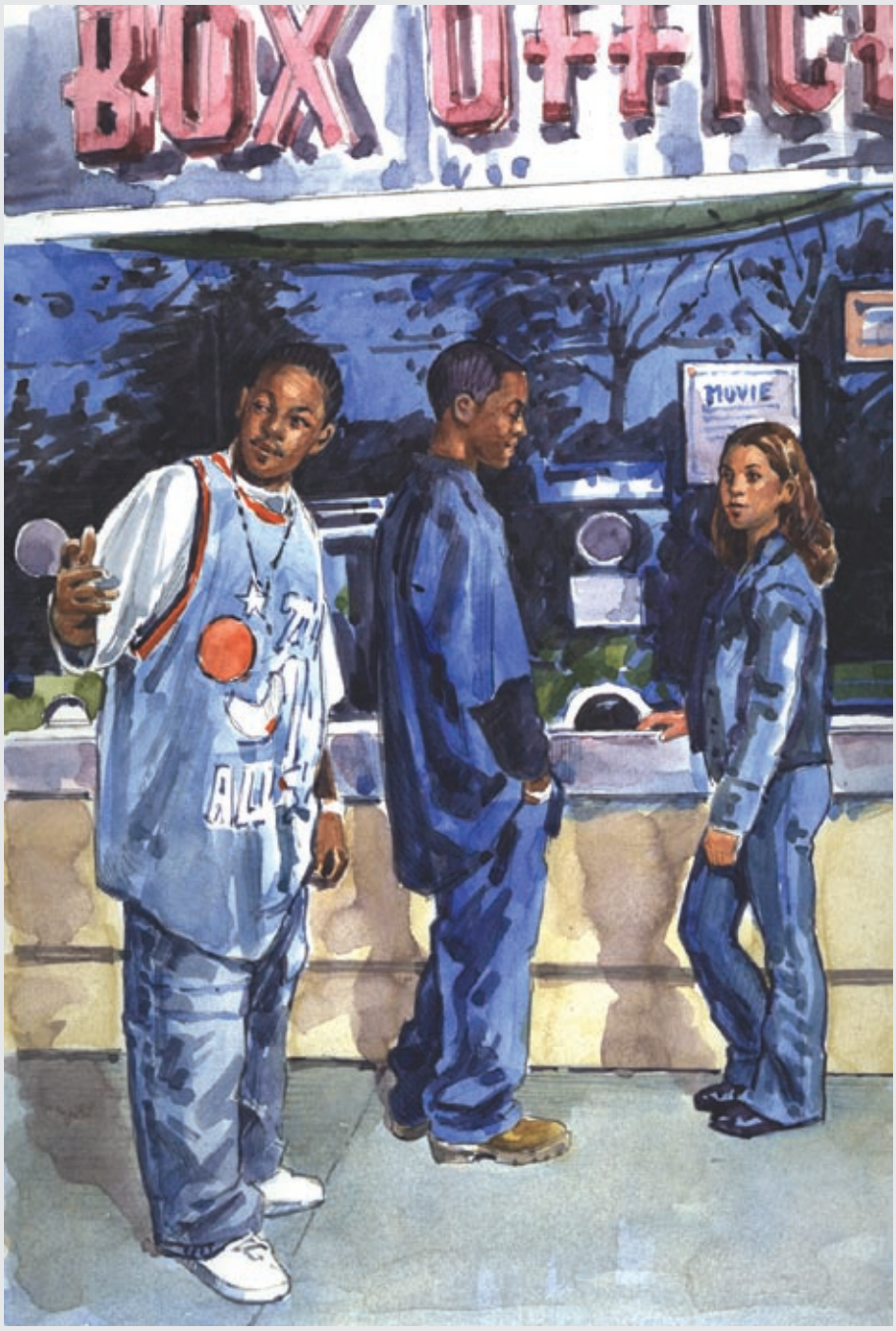
1. Dwayne's parents gave him $120 for his birthday so he could take Lisa, Rosa, Jamal, and his other friends to the movies.

   The theater tickets cost $7 each.

   Dwayne also purchased seven buckets of popcorn at $5 each.

   So many friends showed up that Dwayne had to spend an additional $69 of his own money to help pay for everything.

   How many of Dwayne's friends went to the movies?



LESSON 11

NUMBER PROPERTIES

GOAL	CONCEPT BOOK
To apply the number properties to positive and negative numbers.	See pages 15–19, 194–195, 197–198 in your *Concept Book.*

You have been using the number properties.

Here, you will look at how they apply to negative numbers and to the operations of subtraction and division.

The Commutative Property of Addition
The Commutative Property of Multiplication

In words	In symbols	Example
The order in which two numbers are added or multiplied does not affect the sum or product.	$a + b = b + a$ $ab = ba$	$(-2) + (-4) = (-4) + (-2)$ $-2 - 4 = -4 - 2$ $(-2) \bullet (-4) = (-4) \bullet (-2)$

The Associative Property of Addition
The Associative Property of Multiplication

In words	In symbols	Example
The sum or product of any three numbers is the same, no matter how they are grouped using parentheses.	$(a + b) + c = a + (b + c)$ $(ab)c = a(bc)$	$(-2 + -4) + -3 = -2 + (-4 + -3)$ $(-2 - 4) + -3 = -2 + (-4 - 3)$ $(-2 \bullet -4) \bullet -3 = -2 \bullet (-4 \bullet -3)$

The Inverse Property of Addition
The Inverse Property of Multiplication

In words	In symbols	Example
Subtracting any number from the same number equals 0.	$a - a = 0$	$(-2) - (-2) = 0$ $-2 + 2 = 0$
Dividing any number by the same number equals 1.	$a \div a = 1$	$(-2) \div (-2) = 1$
Multiplying any number by its inverse equals 1.	$a \bullet \frac{1}{a} = 1$	$(-2) \bullet \frac{1}{(-2)} = 1$

The Distributive Property		
In words	**In symbols**	**Example**
Multiplying a number by the sum of two numbers in parentheses is the same as multiplying the number by each number inside the parentheses and adding the resulting products.	$a(b + c) = ab + ac$	$-2(-4 + -3) = (-2 \bullet -4) + (-2 \bullet -3)$ $-2(-4 - 3) = 2(4) + 2(3)$

Note:

If you have a problem that involves adding or subtracting and also multiplying or dividing, and it does not have parentheses to indicate which operation to perform first, insert parentheses so you first multiply or divide, and then add or subtract.

$4 - 3 \bullet 2$ is the same as $4 - (3 \bullet 2)$.

$2 \bullet 3 - 4$ is the same as $(2 \bullet 3) - 4$, but it is not the same as $2 \bullet (3 - 4)$.

Work Time

1. Copy this table, and fill in the cells by substituting values in each expression shown at the top of each column. Use the values for a and b that are given in each row. The first three cells have been filled in for you.

Use the completed table to answer parts b through e.

a.

a	b	$a + b$	$b + a$	$a - b$	$b - a$	ab	ba	$a \div b$	$b \div a$
8	2	10	10	6					
–8	2								
8	–2								
–8	–2								

b. Does $a + b = b + a$ for all the values of a and b in the table?

c. Does $a - b = b - a$ for all the values of a and b in the table?

d. Does $ab = ba$ for all the values of a and b in the table?

e. Although $a \div b$ does not equal $b \div a$, what do they have in common?

2. Copy this table, and fill in each cell by substituting values in the expressions shown at the top of each column.

Use the completed table to answer parts b and c.

a.

a	$a - a$	$a \div a$
–1.75		
–11		
–2		
–0.5		

b. Does $a - a = 0$ for all positive and negative values of a?

c. Does $a \div a = 1$ for all positive and negative values of a?

3. Copy this table, and fill in each cell by substituting values in the expressions shown at the top of each column.

Use the completed table to answer part b.

a.

a	b	c	$a(b + c)$	$ab + ac$
–4	–7	–1		
–3	–2	–5		
–1.75	–2.5	–3.5		
–1.5	–2	–4		

b. Is the distributive property true for problems involving negative numbers? Say why.

Preparing for the Closing

4. Consider the equation $a - b = -(b - a)$. Which can be negative? Say why.

Ⓐ Only a Ⓑ Only b Ⓒ Both a and b Ⓓ Neither a nor b

5. Which number property is illustrated in problem 2?

6. Which number property is illustrated in problem 3?

Skills

Solve and write your answers in simplest form.

a. $\frac{3}{5} \div \frac{1}{2} =$

b. $\frac{4}{5} \div \frac{1}{2} =$

c. $\frac{6}{5} \div \frac{1}{2} =$

d. $\frac{9}{5} \div \frac{1}{8} =$

e. $\frac{3}{7} \div \frac{3}{8} =$

f. $\frac{7}{3} \div \frac{8}{3} =$

g. $\frac{7}{3} \div \frac{3}{8} =$

h. $\frac{7}{3} \div \frac{3}{6} =$

Review and Consolidation

1. Calculate the following pairs of expressions.

a. $(-5) - 8$ and $8 - (-5)$

b. $6 - (-7)$ and $(-7) - 6$

c. $4 - 12$ and $12 - 4$

d. $(-8) - (-20)$ and $(-20) - (-8)$

e. Can you write a general equation using letters that would make each pair of expressions equal?

2. The distributive property for subtraction is $a(b - c) = ab - ac$.

a. Do the calculations to show that it is true for $a = -7$, $b = -4$, and $c = 6$.

b. Do the calculations to show that it is true for $a = -9$, $b = 4$, and $c = -6$.

c. Do the calculations to show that it is true for $a = 8$, $b = -4$, and $c = 6$.

d. Do the calculations to show that it is true for $a = 5$, $b = -4$, and $c = -6$.

3. Calculate.

a. $1(-2) =$

b. $(-2)(-2) =$

c. $(-2)(-2)(-2) =$

d. $(-2)(-2)(-2)(-2) =$

e. $(-2)(-2)(-2)(-2)(-2) =$

f. $(-2)(-2)(-2)(-2)(-2)(-2) =$

g. How does the number of factors determine the sign of the product in a multiplication calculation?

h. Between which two integers is $1 \div (-3) \div (-3) \div (-3) \div (-3) \div (-3)$?

Homework

1. Using the values $a = -3$, $b = 4$, and $c = -5$, do the calculations to find out if these equations are true.

a. $(a + b) + c = a + (b + c)$

b. $(ac)b = a(cb)$

c. $a(b + c) = ab + ac$

d. $(a \div b) \div c = a \div (b \div c)$

e. $(a \div b) \div c = a \div (bc)$

PROGRESS CHECK

LESSON 12

CONCEPT BOOK	GOAL
See pages 185–198 in your *Concept Book.*	To review working with positive and negative numbers.

Work Time

1. Compare these four numbers: –0.01; –0.1; –1; 0

a. Which is the greatest?

b. Which is the least?

2. a. Replace the words with symbols in the following:

Positive three minus negative five is greater than zero.

b. Replace the symbols with words in the following equation: $(-4) + 5 = 1$.

3. Compare these five numbers: –5; –3; –1; 0.2; 2

a. How many of the numbers are < -2?

b. How many of the numbers are ≥ -1?

c. Which two numbers are closest to each other on the number line?

4. a. Convert $(-1) + (-5) = -6$ to an equation involving subtraction.

b. Use the equation in part a to explain why subtracting a number gives the same result as adding its opposite.

5. Calculate.

a. $(-7) + 3 =$

b. $(-5) - 18 =$

c. $(-8)(-9) =$

d. $18 \div (-6) =$

e. $5 + (-12) - 9 - (-4) =$

f. $[6(-3)] - [(-4) \div 8] =$

g. $[6(-3) - (-4)] \div 8 =$

h. $4 + [(-1)(-1)(-1)] - [3(-1)] =$

6. Calculate the following by substituting the values $a = -3$, $b = 5$, and $c = -2$.

a. $a(b - c)$

b. $ab - ac$

7. a. Explain why $-1.0 < -0.1$

b. Explain why $(-1.0)(-1.0) > -0.1$

8. Explain why positive three minus negative five is greater than zero.

9. Replace the symbols with words in the statement $(-4) - (-5) = 1$.

10. Calculate the distance on the number line between:

a. -3 and -1

b. -1 and 0.2

c. 0.2 and 2.0

11. a. Sketch a number line that represents the calculation $(-4) + (-3)$.

b. What would be different and what would be the same in a number line that represented the calculation $(-4) - 3$?

12. In your own words, explain the rules for multiplying and dividing positive and negative numbers.

Skills

a. Rosa ate $\frac{1}{8}$ of a pizza and gave $\frac{1}{2}$ of the remainder to her brother, Carlos.

What fraction of the pizza did she give away?

b. Dwayne bought $\frac{7}{8}$ kg of shrimp. He cooked $\frac{1}{5}$ of them for dinner.

What was the weight of the shrimp he cooked?

Homework

1. Compare these four numbers: –5; –2; 1; $-\frac{1}{2}$

a. Which is the greatest?

b. Which is the least?

c. Which is the farthest from zero?

d. How many are $< \frac{1}{2}$?

e. How many are ≥ –0.5?

2. a. Replace the words with numbers and symbols in this statement.

Negative three minus negative five is greater than zero.

b. Replace the numbers and symbols with words in the equation 4 + (–5) = –1.

3. Calculate.

a. (–4) + 3 =

b. (–25) – 18 =

c. (–8)(–7) =

d. 18 ÷ (–72) =

e. 5 + (–12) – 7 – (–4) =

f. $[6(-4)] + [(-4) \div 8] =$

g. $[6(-4) + (-4)] \div 8 =$

h. $[(-1)(-1)(-1)] + [3(-1)] =$

LESSON 13

LEARNING FROM THE PROGRESS CHECK

GOAL

To review and learn from the Progress Check.

CONCEPT BOOK

See pages 185–198 in your *Concept Book*.

Work Time

1. Assessing Your Work:

a. Select those pieces of work from this unit that you believe are the best examples of your explanations of the rules for adding, subtracting, multiplying, and dividing with positive and negative numbers.

For example, one of these rules is, "Subtraction is the same as *add the opposite*." Another rule is, "The product of two negative numbers is a positive number."

The pieces of work that you select could be explanations that use arrows on number lines or that show your understanding of number properties.

b. Identify the pieces of work in the manner described by your teacher.

c. Write a brief explanation of why you chose these pieces and how they demonstrate your knowledge and ability.

Work with a partner on the remaining problems.

2. a. Make up four problems that each involve exactly one positive number and one negative number.

- One problem should be an addition problem.
- One should be a subtraction problem.
- One should be a multiplication problem.
- One should be a division problem.

Hint: See Lesson 12 Work Time problems 5a through 5d for examples.

b. Read each of your problems aloud in words, and see if your partner can write the correct answers by listening to what you say.

Putting It Together

3. a. Make up an example of four numbers that meets all of these conditions.

- The least of the four numbers is the farthest from zero.
- The greatest of the four numbers is the closest to zero.
- Three of the four numbers are $\leq -\frac{1}{2}$.
- Two of the four numbers are $\geq -\frac{1}{2}$.

b. Compare your example with that of your partner, and write what is the same and what is different.

4. Sketch a number line of an addition equation that starts and finishes at negative points on the number line, and then trade number lines with your partner.

Write an addition equation that matches your partner's number line.

5. a. Make up an example of two numbers, a and b, that meets all of these conditions.

- The sum, $a + b$, is a negative number.
- The difference, $a - b$, is a negative number.
- The product, ab, is a positive number.
- ab is closer to zero than b is to zero.

b. Compare your example with that of your partner, and write what is the same and what is different.

Preparing for the Closing

6. Calculate (–10)(–10)(–10)(–10).

7. –10 is multiplied by itself a given number of times. (Four times in problem 6.)

a. How do you determine the sign of the answer?

b. How do you determine the place value of the 1 in the answer?

8. With your partner, decide which of the following expressions give the same answer.

a. $a(b - c)$ b. $(ab) - c$ c. $ab - c$

d. $(ab) - (ac)$ e. $(ab - a)c$ f. $ab - ac$

g. $(b - c)a$ h. $b - (ca)$ i. $b - ca$

9. With your partner, choose some positive and negative values for a, b, and c. Use these values to check the decisions you made in the previous problem.

Skills

Solve and write your answers in simplest form.

a. $4 \div 2 =$ b. $2 \div 2 =$ c. $\frac{1}{2} \div 2 =$ d. $\frac{1}{4} \div 2 =$

e. $16 \div 4 =$ f. $8 \div 4 =$ g. $4 \div 4 =$ h. $\frac{1}{4} \div 4 =$

i. $\frac{1}{2} \div 8 =$ j. $\frac{1}{3} \div 8 =$ k. $\frac{1}{4} \div 10 =$ l. $\frac{1}{8} \div 10 =$

Review and Consolidation

1. Write the following using symbols, and then calculate.

a. Negative 5 multiplied by negative 8

b. Negative 8 divided by negative 5

c. Negative 5 divided by negative 8

d. Negative 5 added to negative 8

e. Negative 5 subtracted from negative 8

f. Negative 8 subtracted from negative 5

2. Write the following using symbols, and then calculate.

a. Positive 0.05 multiplied by negative 8

b. Negative 8 divided by positive 0.05

c. Positive 0.05 divided by negative 8

d. Positive 0.05 added to negative 8

e. Positive 0.05 subtracted from negative 8

f. Negative 8 subtracted from positive 0.05

3. Write the following using symbols, and then calculate.

a. Positive $\frac{2}{3}$ multiplied by positive $\frac{4}{5}$

b. Positive $\frac{2}{3}$ divided by positive $\frac{4}{5}$

c. Positive $\frac{4}{5}$ divided by positive $\frac{2}{3}$

d. Positive $\frac{4}{5}$ added to positive $\frac{2}{3}$

e. Positive $\frac{4}{5}$ subtracted from positive $\frac{2}{3}$

f. Positive $\frac{2}{3}$ subtracted from positive $\frac{4}{5}$

Homework

1. Which of the following expressions give the same answer?

Ⓐ $(-2)[4-(-5)] =$

Ⓑ $(-2 \bullet 4)-(-5) =$

Ⓒ $(-2) \bullet 4-(-5) =$

Ⓓ $[(-2) \bullet 4]-[-2(-5)] =$

Ⓔ $[(-2 \bullet 4)-(-2)](-5) =$

Ⓕ $(-2) \bullet 4-(-2)(-5) =$

Ⓖ $[4-(-5)](-2) =$

Ⓗ $4-[(-5)(-2)] =$

Ⓘ $4-(-5)(-2) =$

2. Do the calculations to check your answers to problem 1.

LESSON 14

IT'S COLD UP THERE

GOAL

To graph with positive and negative numbers.

CONCEPT BOOK

See pages 185–209 in your *Concept Book.*

Work Time

When Chen came to the United States from China, he flew from Beijing to San Francisco. As they took off and climbed, he noticed that both the height at which the airplane was flying and the outside air temperature were displayed on the TV monitor.

Since Chen was interested in the outside temperature, he wrote the data down.

Height	Air Temperature (°C)
Beijing: 0 ft	16
1000 ft	14
5000 ft	6

1. Assuming the temperature continued to decrease at the same rate:
   a. At what height was the outside air temperature zero degrees?
   b. Write an equation describing what happened to the temperature as the height the airplane was flying at increased.
   c. What was the temperature at a height of 30,000 feet?

2. Chen wrote: 16 + 30(–2).

Explain how this expression relates to the temperature at 30,000 feet.

3. The airplane changed height and the temperature dropped by 10 degrees. Chen wanted to know how much the height had changed.

a. Say why the answer can be found by calculating $[-10 \div (-2)]1000$ feet.

b. Say why the answer also can be found by calculating $(10 \div 2)1000$ feet.

c. Copy and complete the following calculations.

$10 \div 2 = \square$	$5 \div 2 = \square$	$0 \div 2 = \square$	$(-5) \div 2 = \square$	$(-10) \div 2 = \square$
$10 \div (-2) = \square$	$5 \div (-2) = \square$	$0 \div (-2) = \square$	$(-5) \div (-2) = \square$	$(-10) \div (-2) = \square$

Preparing for the Closing

4. If the temperature dropped two degrees every 1500 feet, what was the temperature at 30,000 feet?

Assume that the temperature at ground level was still 16°C.

Skills

a. A string of length $\frac{1}{5}$ m is cut into 5 equal parts.

What length is each piece in centimeters?

b. The perimeter of a square painting is $\frac{2}{5}$ m.

Find the length of each side in centimeters.

c. Keesha poured $\frac{3}{5}$ cup of orange juice equally into 3 glasses.

How much orange juice was there in each glass?

Review and Consolidation

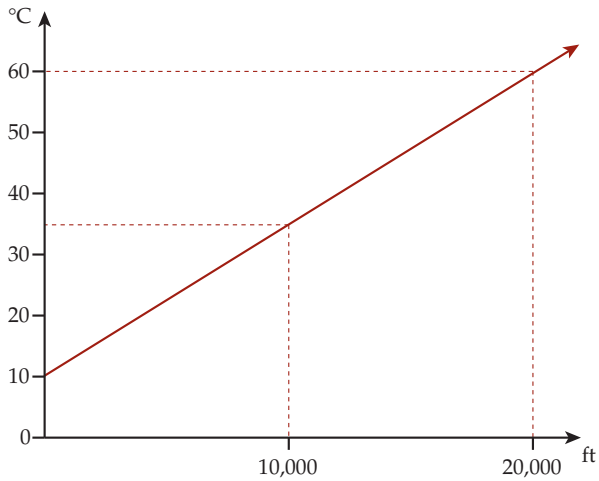
In this problem, the temperature at ground level is 12°C, and the temperature drops $2\frac{1}{2}$ degrees for every 1000 feet increase in the airplane's height.

1. Copy and complete this table.

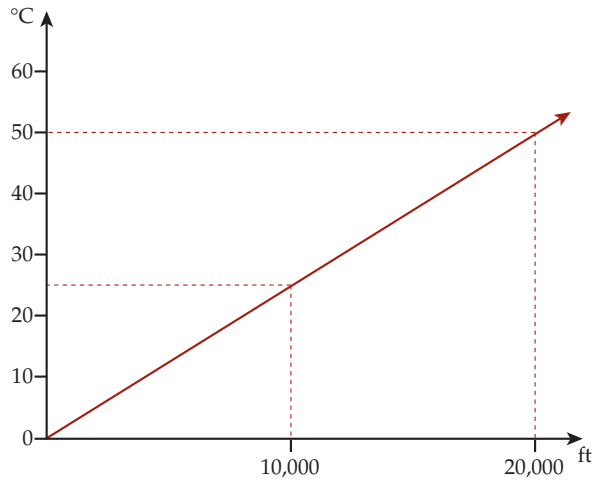
Height (ft)	0	1000	3000	5000	10,000	20,000
Air Temperature (°C)						

Four students gave these answers when they were asked to sketch a graph of the relationship between the height of the aircraft and the outside air temperature.

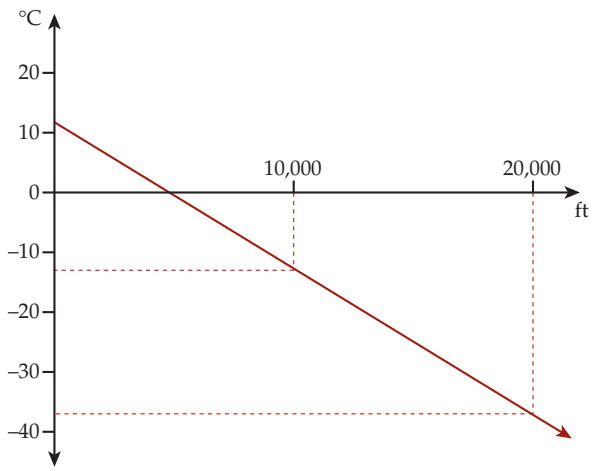
Jamal's graph



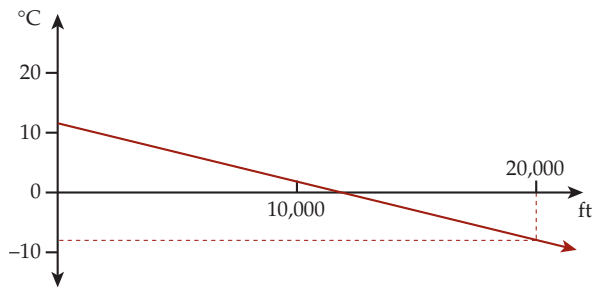
Rosa's graph



Lisa's graph



Dwayne's graph



2. a. Which graph is the correct one? Say why.

b. What is right and what is wrong with each of the other graphs? What advice would you give to each student?

On one of the graphs, the teacher wrote, "Well done! This is $t = 12 - 2.5(h \div 1000)$."

c. Whose graph was it?

d. Explain the t, the h, and the equation.

e. For this graph, what would be the height if the outside air temperature was 0°C?

On one of the graphs, the teacher wrote, "This is the graph of $t = 12 - (h \div 1000)$."

f. Whose graph was it?

g. Explain the equation.

h. For this graph, what would be the height if the outside air temperature was 0°C?

On one of the graphs, the teacher wrote, "This is the graph of $t = 10 + 2.5(h \div 1000)$."

i. Whose graph was it?

j. Explain the equation.

k. For this graph, what would be the height if the outside air temperature was 0°C?

Homework

Review the topics in this unit.

1. List three important concepts about working with negative numbers that you have learned in this unit.

2. List two topics in the *Concept Book* that you would like to learn more about.

3. Choose three problems from the lessons that you think are the most difficult. Write why you think they are difficult.

LESSON 15

WORD PROBLEMS

GOAL	CONCEPT BOOK
To solve word problems involving positive and negative numbers.	See pages 23, 185–190 in your *Concept Book.*

Work Time

1. The Race

Keesha looks forward to competing in a county race each year. This table shows the results from last summer.

Place	1st	2nd	3rd
Name	Keesha	Mario	Jin
Time	1 min 48 sec	2 min 3 sec	2 min 12 sec

At this year's meet, Mario lowered (improved) his time by 4 seconds and Jin lowered her time by 20 seconds. Keesha lowered her time by 7 seconds and finished 18 seconds ahead of Mario.

a. Create a table like the one above showing the results from this year's race.

b. What was the time difference between Keesha and Mario?

c. What was the time difference between Mario and Jin?

d. What was the time difference between Jin and Keesha?

2. A Flight Around the World

It is 11 A.M. on a Monday morning. An airplane has just taken off from New York's Kennedy International Airport, flying east to west. It will be dropping off and picking up passengers at different airports as it travels around the world.

The first leg of the flight from New York to San Francisco, California, will take 5 hours.

New York is 5 hours behind Greenwich Mean Time (GMT), and San Francisco is 8 hours behind GMT.

a. What time will it be in San Francisco when the plane lands?

Two hours after landing in San Francisco, the plane will take off, heading for Sydney, Australia. It will also fly to Hong Kong, and then to London, England, before heading back to New York.

Your task is to work out the local time of arrival at each place.

Use the following information.

- **Wait times**: There is a 3-hour stop after landing in each of the remaining cities, before taking off again.
- **Time zones**:

Sydney	10 hours ahead of GMT
Hong Kong	8 hours ahead of GMT
London	0 hours (= GMT)

- **Flight times**:

San Francisco to Sydney	$13\frac{1}{2}$ hours
Sydney to Hong Kong	9 hours
Hong Kong to London	12 hours
London to New York	$7\frac{1}{2}$ hours

- **International Date Line**:

  As the plane travels west across the Pacific Ocean, it crosses the International Date Line. Here, the local time changes from 11 hours behind GMT to 12 hours ahead of GMT—not 12 hours behind GMT. The difference is 24 hours, or one day.

  When the plane crosses the International Date Line, the crew and passengers will find that, suddenly, they have lost a whole day, as Tuesday becomes Wednesday.

b. What time will it be in Sydney when the plane lands?

c. What time will it be in Hong Kong when the plane lands?

d. What time will it be in London when the plane lands?

e. What time will it be in New York when the plane lands?

Preparing for the Closing

3. a. Discuss your solutions to the Work Time problems with your partner.

b. How long did it take for the plane to complete its around-the-world trip?

c. Discuss different approaches to solving the problems and different ways of checking the answers.

Skills

Solve.

a. $\frac{5}{7} + \frac{2}{3} =$

b. $\frac{5}{7} - \frac{2}{3} =$

c. $\frac{5}{7} \bullet \frac{2}{3} =$

d. $\frac{5}{7} \div \frac{2}{3} =$

e. $2\frac{5}{7} + 1\frac{2}{3} =$

f. $2\frac{5}{7} - 1\frac{2}{3} =$

g. $2\frac{5}{7} \bullet 1\frac{2}{3} =$

h. $2\frac{5}{7} \div 1\frac{2}{3} =$

i. $\frac{2}{3}a + \frac{2}{3}a =$

j. $\frac{2}{3}a - \frac{2}{3}a =$

k. $\frac{2}{3}a \bullet \frac{2}{3}a =$

l. $\frac{2}{3}a \div \frac{2}{3}a =$

Review and Consolidation

1. A student wrote $-10 > 1$.
What possible misunderstandings could have led to this mistake?

2. A student wrote, "Negative 8 minus 20 is equal to positive 8 plus 20."
What possible misunderstandings could have led to this mistake?

3. A student wrote $(-8)(-12) = 20$.
What possible misunderstandings could have led to this mistake?

4. A student wrote $(-8) - 8 = 16$.
What possible misunderstandings could have led to this mistake?

5. A group of students were asked to calculate $(-8) - (-6 \bullet 4)$.

- One student wrote $-2 \bullet 4 = -8$.
- A second student wrote $-8 - (-24) = -32$.
- A third student wrote $-8 - (-24) = -16$.

What possible misunderstandings could have led to these mistakes?

6. a. Calculate the value of $7 - 3x$ for $x = -6$.

b. Calculate the value of $3x - 7$ for $x = -6$.

c. Explain the relationship between part a and part b.

7. a. Calculate the value of $3x(3 - 2x)$ for $x = -4$.

b. Calculate the value of $9x - 6x^2$ for $x = -4$.

c. Explain the relationship between part a and part b.

Homework

Use this session to prepare for the End-of-Unit Assessment.

- Consult the *Concept Book* to review the main ideas.
- Find mistakes you made in your notebook, and write out explanations and corrections.
- Review the problems in Lesson 12, and correct any mistakes.
- Write what you have learned about adding and subtracting positive and negative numbers. Include number-line illustrations.
- Write what you have learned about multiplying and dividing positive and negative numbers.
- Give positive and negative number examples of the distributive property and other key number properties, such as the associative and commutative properties.

COMPREHENSIVE REVIEW

UNITS 1–4

1. Write $6x + 6y$ using parentheses.

2. Use mental strategies and place value to multiply each number by 10.

a. 4.5 b. 0.045 c. 4500

3. Calculate the following.

a. $-10 + 15 =$ b. $-4 + (-7) + (-8) =$

4. Calculate the following.

a. $\frac{1}{5} + \frac{1}{3} =$ b. $2 \bullet \frac{4}{15} =$ c. $\frac{5}{15} + \frac{5}{15} =$ d. $\left(2 \bullet \frac{2}{15}\right) \bullet 2 =$

5. Sketch a number line of $-5 + 3 + (-4)$.

6. Write each number in standard form.

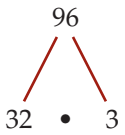
a. 5 tens, 6 ones, and 3 tenths.

b. 2 hundreds, 6 tens, and 0 ones.

c. 8 thousands, 6 tens, and 5 tenths.

7. a. Here is one way to begin a factor tree of the number 96. Show another way.

b. Complete both trees.



8. a. $15 \bullet 20 = 3 \bullet a \bullet 4$
Solve the equation for a.

b. Say what you did for each step.

9. Calculate.

a. $\frac{3}{4} - \frac{1}{8} - \frac{1}{8} - \frac{1}{8} - \frac{1}{8} - \frac{1}{8} - \frac{1}{8} =$

b. $\frac{3}{4} \div \frac{1}{8} =$

10. a. Convert the mixed number $1\frac{1}{2}$ to an improper fraction.

b. Say what you did for each step.

11. Calculate the following.

a. $\frac{3}{4} \bullet (-5) =$

b. $-4 \div (-5) =$

Page numbers in red are found in the *Concept Book*.

Number Sense

Gr. 4 NS: 1.8	Use concepts of negative numbers (e.g., on a number line, in counting, in temperature, in "owing"). 1–8, 62–65; 186–187
Gr. 4 NS: 3.0	Students solve problems involving addition, subtraction, multiplication, and division of whole numbers and understand the relationships among the operations. 36–40; 195–198
Gr. 5 NS: 1.5	Identify and represent on a number line decimals, fractions, mixed numbers, and positive and negative integers. 1–22; 185–191
Gr. 5 NS: 2.1	Add, subtract, multiply, and divide with decimals; add with negative integers; subtract positive integers from negative integers; and verify the reasonableness of the results. 14–27, 32–35, 55–61; 189–193
Gr. 6 NS: 1.1	Compare and order positive and negative fractions, decimals, and mixed numbers and place them on a number line. 9–13; 187–189
Gr. 6 NS: 2.0	Students calculate and solve problems involving addition, subtraction, multiplication, and division. 23–27, 32–35, 55–61; 189–193
Gr. 6 NS: 2.3	Solve addition, subtraction, multiplication, and division problems, including those arising in concrete situations, that use positive and negative integers and combinations of these operations. 28–31, 36–40, 45–49, 62–70; 185–193, 195–198

Algebra and Functions

Gr. 5 AF: 1.1	Use information taken from a graph or equation to answer questions about a problem situation. 62–65; 203–209
Gr. 5 AF: 1.3	Know and use the distributive property in equations and expressions with variables. 45–54; 17–18
Gr. 6 AF: 1.3	Apply algebraic order of operations and the commutative, associative, and distributive properties to evaluate expressions; and justify each step in the process. 41–54; 15–19, 198
Gr. 6 AF: 1.4	Solve problems manually by using the correct order of operations or by using a scientific calculator. 41–44; 198

Mathematical Reasoning

Gr. 6 MR: 2.3 Gr. 6 NS: 2.3	Estimate unknown quantities graphically and solve for them by using logical reasoning and arithmetic and algebraic techniques. 28–31; 185–193, 195–198
Gr. 6 MR: 2.4 Gr. 6 NS: 2.3	Use a variety of methods, such as words, numbers, symbols, charts, graphs, tables, diagrams, and models, to explain mathematical reasoning. 28–31, 62–70; 185–193, 195–198
Gr. 6 MR: 2.5 Gr. 6 NS: 2.3	Use a variety of methods, such as words, numbers, symbols, charts, graphs, tables, diagrams, and models, to explain mathematical reasoning. 28–31, 62–70; 185–193, 195–198

Page numbers in red are found in the *Concept Book*.