Ramp-Up to Algebra

Ratio and Proportionality

Unit 5

AMERICA'S CHOICE®

Acknowledgments

Field Test:

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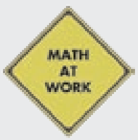

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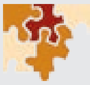

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LESSON 1

COMPARING QUANTITIES

CONCEPT BOOK	GOAL
See pages 291–292 in your *Concept Book*.	To compare quantities using subtraction and division.

Comparing Using Subtraction

You can use subtraction to compare two quantities.

Example

Ms. Reynolds has 24 students in her class, and the classroom has 6 computers. You can compare the number of students to the number of computers.

$$\underbrace{24}_{\text{number of students}} - \overbrace{6}^{\text{number of computers}} = \underbrace{18}_{\text{difference}}$$

By subtracting, you find that the difference between the number of students and the number of computers is 18. In words, "There are eighteen more students than computers."

Comparing two numbers by subtraction, $a - b$, gives a new number $(a - b)$, called the *difference*. The difference is an *absolute* comparison between two quantities.

Comparing Using Division

You can also use division to compare two quantities.

Example

Ms. Reynolds has 24 students in her class, and the classroom has 6 computers.

$$\underbrace{24}_{\text{number of students}} \div \overbrace{6}^{\text{number of computers}} = 4$$

By dividing, you find that $\frac{24}{6} = \underbrace{\frac{4}{1}}_{\text{ratio}} = 4$. In words, "There are four students for every computer."

Comparing two numbers by division, a to b, gives a new number: the *ratio* $\frac{a}{b}$, where $b \neq 0$. The ratio is a *relative* comparison between two quantities.

Example

You can use the ratio of students to computers to describe the situation in several ways:

Ms. Reynolds has 24 students in her class, and the classroom has 6 computers. The ratio is $\frac{24}{6}$. This ratio simplified is $\frac{4}{1}$.

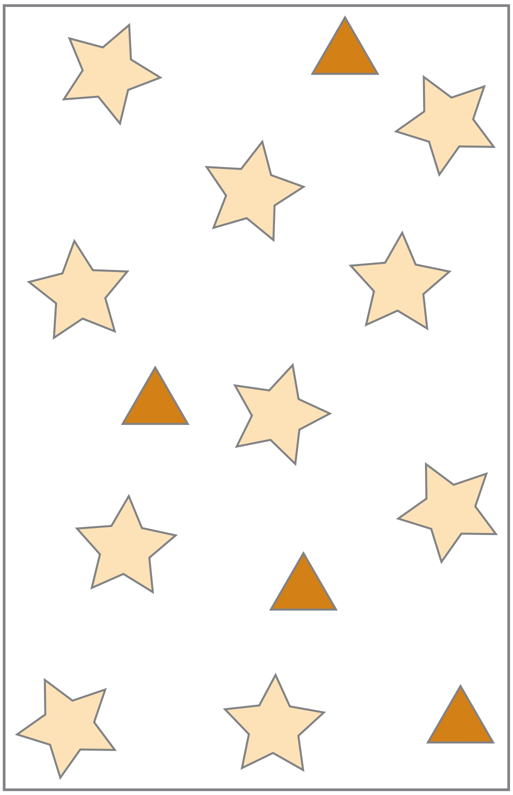
- There are 4 times as many students as computers.
- There are 4 students for every computer.
- There are 4 students per computer.
- The ratio of students to computers is 4 to 1.

Summary

A difference is the comparison of two quantities using subtraction. A ratio is the comparison of two quantities using division. Division and subtraction are different kinds of comparisons.

Work Time

1. Examine this set of stars and triangles.



a. Compare the number of stars to the number of triangles using subtraction.

b. Compare the number of stars to the number of triangles using division.

c. Sketch another set of triangles and stars. In the new set, change the numbers of triangles and stars, but keep the difference between the two numbers the same.

d. Compare the ratio of stars to triangles in your set to the ratio of stars to triangles in the given set. Are they the same? Say why.

e. Sketch another set of triangles and stars that has the same ratio of stars to triangles as in the given set.

f. Is the difference of the triangles and stars in your second set the same as for the given set? Say why.

2. Each week, Dwayne earns \$20, Jamal earns \$10, Lisa earns \$40, and Chen earns \$30.

a. Compare the amounts of money earned by Dwayne and Jamal using subtraction. Now compare the amounts of money earned by Lisa and Chen using subtraction. What do you notice?

b. Write a sentence that describes the difference in earnings for Dwayne and Jamal. Write another sentence about the difference in earnings for Lisa and Chen.

c. Write a ratio to compare the amounts of money earned by Dwayne and Jamal. Write another ratio to compare the amounts of money earned by Lisa and Chen.

d. Compare the two ratios. What do you notice?

e. Which comparison, using subtraction or division, gives better information? Say why.

3. The Valdez family was remodeling their restaurant. Mr. Valdez bought 48 chairs and 12 tables. He set up the tables and chairs so that every chair was at a table, and every table had an equal number of chairs.

a. Compare the total number of chairs to the total number of tables using subtraction.

b. Compare the total number of chairs to the total number of tables using division.

c. What does each of these comparisons tell you about the situation?

4. Mr. Valdez wanted to expand the restaurant to seat 144 people using the same ratio of chairs to tables.

a. How many tables will Mr. Valdez need for 144 chairs?

b. Did you use division or subtraction to figure out how many tables Mr. Valdez would need? Say why.

Preparing for the Closing

5. Look at your work for problems 1 and 3.

a. What information do you get by finding the difference of two quantities?

b. In which types of comparisons should you use subtraction?

c. What information do you get by finding the ratio of two quantities?

d. In which types of comparisons should you use division?

6. Look at your work for problem 4.

a. Write a rule that explains how the difference changes if the two numbers in the subtraction are each multiplied by the same number.

b. Write a rule that explains how the ratio changes if the two numbers in the division are each multiplied by the same number.

Skills

Write these inequalities in words.

a. $8 > -8$ b. $-12 < 10$ c. $-1250 > -2000$ d. $-8 < -6\frac{1}{4}$

Review and Consolidation

1. Decide whether each statement is *true* or *false*.

State which quantities should be subtracted or divided.

Justify your answer.

> **Example**
>
> To find out how much the temperature fell between 2 PM and 10 PM, you should subtract.
>
> *True*. You would subtract the temperature at 10 PM from the temperature at 2 PM. Subtraction will give you the number of degrees the temperature fell.

a. To find out how many more boys than girls there are in a class, you should divide.

b. To find out how many students will ride on each bus for the ninth-grade field trip, you should divide.

c. To find out how many more cats there are than dogs, you should subtract.

d. To find out how many parking spaces there are for each store at a mall, you should subtract.

2. In the math room at Monroe High School, there are six chairs around each table.

a. Write the ratio of chairs to tables.

b. Write four different statements that this ratio information represents.

Homework

1. Examine these two diagrams of circles and squares.

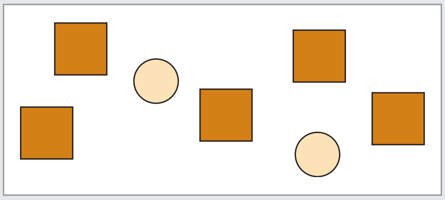


Diagram 1

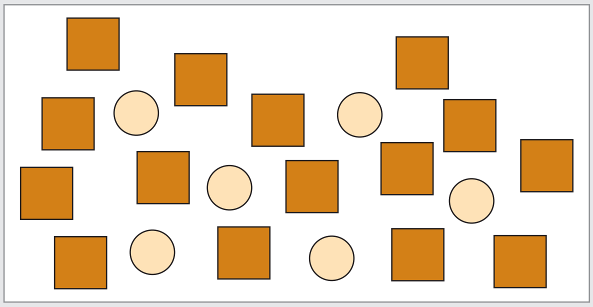


Diagram 2

a. For both Diagram 1 and Diagram 2, use subtraction to compare the number of squares to the number of circles.

b. For both Diagram 1 and Diagram 2, use division to compare the number of squares to the number of circles.

c. What information does the difference between the two quantities give you? What information does the ratio of the two quantities give you?

d. Compare your results for the two diagrams. What do you notice?

2. Give an example of a situation in which it is better to find the difference in order to compare two quantities.

3. Give an example of a situation in which it is better to find the ratio in order to compare two quantities.

LESSON 2

REPRESENTING RATIOS

GOAL

To solve problems using ratios and to represent ratios.

CONCEPT BOOK

See pages 293–295 in your *Concept Book.*

Whole Number Ratios

Whole number ratios are ratios made up of whole numbers.

Example

The ratios 20 : 16 and 5 : 4 are whole number ratios.

5 : 9 is the simplest whole number ratio for 20 : 36.

Example

1 : 1.8 is the same ratio as 5 : 9, but it is *not* a whole number ratio.

Comparisons between ratios are easier when the ratios are simplified. Whole number ratios that use smaller numbers are easier to understand.

Ratios and Fractions

A ratio can be expressed as a fraction, but a ratio and a fraction are not the same.

A *ratio* is a quantity that usually has units. A ratio expresses the relationship of one quantity to another, such as girls to boys or students to computers.

A fraction does not need to have units.

A percent is a specific type of ratio. The ratio of a number to 100 is a *percent*.

A ratio compares two quantities within one set of data or between two sets of data. When you are comparing within a set of data, you can express the relationship as a part-part comparison or as a part-whole comparison.

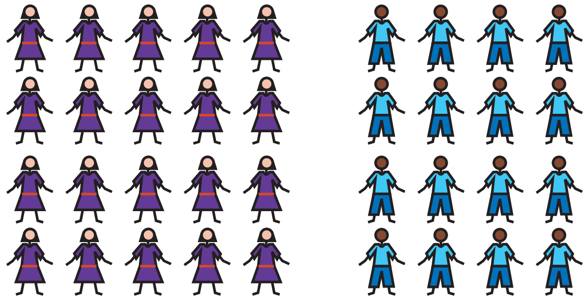
Part-Part Comparisons

A ratio that compares part of a set to another part of the same set is called a *part-part comparison*.

Example

There are 20 girls and 16 boys in a class of 36 students.

To compare the parts, divide the number of girls by the number of boys.



$$\text{number of girls} \div \text{number of boys} = 20 \div 16 = \frac{20}{16} = \frac{5}{4} = 1.25 = 125\%$$

20 ÷ 16 has been reduced, or simplified, to its lowest terms, 5 ÷ 4.

The ratio of girls to boys is 20 to 16, or 20 : 16, or simply, 5 : 4.
There are 5 girls for every 4 boys, or 1.25 girls for every boy.
The number of girls is 125% of the number of boys.

Part-Whole Comparisons

A ratio that compares part of a set to the whole set is called a *part-whole comparison*.

Example

There are 20 girls and 16 boys in a classroom. The whole set is the 36 students; one part is the 20 girls.

To compare the number of girls to the number of students, divide the number of girls by the total number of students.

$$\text{number of girls} \div \text{total number of students} = 20 \div 36 = \frac{20}{36} = \frac{5}{9} = 0.55 = 55\%$$

20 ÷ 36 has been reduced, or simplified, to its lowest terms, 5 ÷ 9.

The ratio of girls to all the students is 20 to 36, or 20 : 36.
The simplest whole number ratio is 5 : 9.
The part-whole ratio written as a decimal is 0.55, and as a percent is 55%.

Girls make up 55% of the total number of students.

Work Time

A bowl contains 28 pieces of fruit. There are 8 oranges and 20 apples.

1. Write the ratio of oranges to apples (a part-part comparison) as:

a. The simplest whole number ratio

b. A fraction in lowest terms

c. A decimal

d. A percent

2. Write the ratio of apples to all the fruit (a part-whole comparison) as:

a. The simplest whole number ratio

b. A fraction in lowest terms

c. A decimal (rounded to two decimal places)

d. A percent (rounded to the nearest whole percent)

3. Use Handout 1: *Four High Schools* and six blank index cards.

The handout contains 24 cards that describe four high schools. Each card contains information about one of the schools.

For each school, find a set of six matching cards. In each set there should be:

- A data card
- A ratio card
- A decimal card
- A fraction card
- A percent card
- A card that describes a ratio in words

Make notes about how you decide to sort the cards, and record any computations that you use.

Unit 5 HANDOUT 1 — FOUR HIGH SCHOOLS

School A There are 420 boys and 420 girls.	The ratio of boys to girls is 1 : 1.	The ratio of girls to boys is 1.25.	School B There are 360 boys and 720 girls.
The ratio of boys to girls is 4 : 5.	Two out of every three students are girls.	The ratio of girls to all students is 2 : 3.	$\frac{1}{3}$ of all students are boys.
Girls are approximately 67% of all students.	50% of the students are girls.	The ratio of girls to all students is 0.5.	Boys are approximately 44% of all students.
The ratio of girls to all students is 5 : 27.	$\frac{5}{9}$ of the students are girls.	The ratio of boys to girls is 0.5.	$\frac{22}{27}$ of all students are boys.
School D There are 880 boys and 200 girls.	One out of every two students is a girl.	There are five girls for every twenty-two boys.	There are five girls for every four boys.
Approximately 81% of all students are boys.	$\frac{1}{2}$ of the students are girls.	The ratio of boys to all students is approximately 0.8.	School C There are 360 boys and 450 girls.

- On one of the blank index cards, write the number of boys and girls in your school. Then, make a ratio card, a decimal card, a fraction card, a percent card, and a card that describes the ratio in words.

Example

Here is a matching set of cards.

MY SCHOOL There are 300 boys and 100 girls.	The ratio of boys to girls is 3 : 1.	The ratio of girls to all students is 0.25.	$\frac{3}{4}$ of all students are boys.	Girls are 25% of the student population.	One out of every four students is a girl.

Preparing for the Closing

4. Compare answers with a partner. How do your answers differ? Say why.

5. For which school was it most difficult to find the matching set of ratio, fraction, decimal, and percent cards? Say why.

6. For which school was it the easiest? Say why.

7. In a class of 12 students with 8 boys, the ratio of boys to girls is a part-part comparison and the ratio of boys to all students is a part-whole comparison.

a. Use this context to give another example of a part-whole comparison.

b. One way to describe this situation is to say that two out of three of the students are boys. Write two more ways that describe the ratio of boys to all students.

Skills

Make true statements using < and >.

a. 345 ● 354

b. 24 ● –25

c. –250 ● –251

d. –4.8 ● –5.7

e. $\frac{1}{4}$ ● $\frac{1}{3}$

f. $-\frac{1}{4}$ ● $-\frac{1}{3}$

Review and Consolidation

1. The ratio 15 : 20 is equivalent to the ratio 3 : 4. Say why.

2. Express each of these ratios as a fraction in simplest form.

   a. 14 : 35 b. 15 : 18 c. 26 : 39

3. The fraction $\frac{2}{3}$ is equivalent to the ratio 2 : 3. Say why.

4. The number 0.4 is equivalent to the ratio 2 : 5. Say why.

5. 56% is equivalent to the ratio 14 : 25. Say why.

Homework

1. In a math class, there are 16 girls and 14 boys. Write the following ratios in simplest form.

   a. What is the ratio of boys to girls?

   b. What is the ratio of girls to boys?

   c. What is the ratio of girls to the total number of students in the class?

   d. What is the ratio of boys to the total number of students in the class?

2. For each situation below, write the comparison of girls to boys in three different ways: as a ratio in simplest form, as a decimal, and as a percent.

   a. In this class, there are 14 girls and 6 boys.

   b. In this class, there are 10 girls and 16 boys.

   c. In this class, there are 12 girls and 16 boys.

   d. In this class, there are 15 girls and 10 boys.

   e. In this class, there are 16 girls and 10 boys.

UNIT RATIOS AND EQUAL RATIOS

LESSON 3

CONCEPT BOOK	GOAL
See pages 293–295, 297 in your *Concept Book.*	To identify unit ratios and equal ratios.

Unit Ratios

Unit ratios are ratios written as some number to 1. Unit ratios help you compare ratios.

Keesha wanted to determine which class has the best access to computers.

Mr. Patel's class has 24 students and 16 computers.

Ms. Wong's class has 22 students and 20 computers.

Ms. Reynolds' class has 30 students and 20 computers.

Keesha decided to compare the classes using unit ratios. She knew that unit ratios are written as *some number* to 1.

Class	Ratio	Unit Ratio	In Words
Mr. Patel's	24 : 16	$\frac{24}{16} = \frac{3}{2} = \frac{1.5}{1} = 1.5 : 1$	There are 1.5 students for every computer.
Ms. Wong's	22 : 20	$\frac{22}{20} = \frac{11}{10} = \frac{1.1}{1} = 1.1 : 1$	There are 1.1 students for every computer.
Ms. Reynolds'	30 : 20	$\frac{30}{20} = \frac{3}{2} = \frac{1.5}{1} = 1.5 : 1$	There are 1.5 students for every computer.

The unit ratio 1.5 : 1 tells you there are 1.5 students for every computer in both Mr. Patel's and Ms. Reynolds' classes.

Even though Mr. Patel's class and Ms. Reynolds' class have different numbers of students and different numbers of computers, they have an *equal ratio* of students to computers. In these two classes, students have the same access to computers.

The unit ratio 1.1 : 1 tells you there are 1.1 students for every computer in Ms. Wong's class.

In Ms. Wong's class, students have better access to computers because fewer students have to share each computer.

Of course, in reality, 1.5 or 1.1 students cannot share one computer.

Equal Ratios

Equal ratios are like equivalent fractions. You can calculate an equal ratio by representing the ratio as a fraction and then writing the fraction in simplest form.

Example

In Mr. Patel's class, the ratio of students to computers is 24 : 16.

$$24:16 = \frac{24}{16} = \frac{8 \bullet 3}{8 \bullet 2} = \frac{3}{2} = 3:2$$

In Mr. Patel's class, there are three students for every two computers.

3 : 2 is called the simplest whole number ratio that is equal to 24 : 16, because it uses the smallest possible whole numbers.

Ratios with Like Units

When comparing two quantities with units that can be changed to *like* units, you have to change the units before you simplify the numbers.

Example

To compare 10 months to 3 years, change both units to months, and then simplify by reducing the ratio to lowest terms.

$$10:36 = \frac{10}{36} = \frac{2 \bullet 5}{2 \bullet 18} = \frac{5}{18} = 5:18$$

As a unit ratio, this is $\frac{5}{18}$ to 1, which is approximately 0.27 : 1.

Work Time

1. In a math class, there are 12 boys and 18 girls.

a. Express the ratio of girls to boys as the simplest whole number ratio.

b. Explain what this ratio means.

c. Express the ratio of girls to boys as a unit ratio.

d. Explain what this ratio means.

2. In another math class, there are 18 boys and 20 girls.

a. Express the ratio of girls to boys as a unit ratio. Compare this result with the unit ratio of girls to boys you obtained in problem 1.

b. Which class is closer to having a 1 : 1 ratio of boys and girls?

c. Explain what this ratio means.

3. Chen measured his height and found he was 156 cm tall. Dwayne is 1.70 m tall. (Remember to convert each quantity to the same unit. 1 meter = 100 centimeters.)

a. Express the ratio of Chen's height to Dwayne's height as the simplest whole number ratio.

b. Express the ratio (as a decimal, rounded to two decimal places) of Chen's height to Dwayne's height as a unit ratio.

c. What percent of Dwayne's height is Chen's height? (Round to the nearest percent.)

4. In 2005, Australia's population was approximately 20 million, and the population of the United States was approximately 295,500,000.

The land area of Australia is approximately 7,680,000 square kilometers, and the land area of the United States is approximately 9,631,418 square kilometers.

a. Use a calculator to find the ratio of the population of the United States to the population of Australia. Write the answer as a unit ratio, rounded to one decimal place.

b. Use a calculator to find the ratio of the land area of the United States to the land area of Australia. Write the answer as a unit ratio, rounded to one decimal place.

c. Comment on your results for parts a and b.

d. Calculate a unit ratio (as a decimal, rounded to one decimal place) for the population density of the United States.

e. Calculate a unit ratio (as a decimal, rounded to one decimal place) for the population density of Australia.

f. Compare the population densities of the two countries. Describe the differences.

Definition

The **population density** of a country is the number of people per square kilometer.

Preparing for the Closing

5. What does it mean when two quantities are in a ratio of 1 : 1?
Give an example to support your answer.

6. What does it mean for two ratios to be equal?
Give an example to support your answer.

Skills

Solve.

a. $2 + (-3) =$

b. $2\frac{1}{2} + \left(-3\frac{1}{2}\right) =$

c. $(-50) + 25 =$

d. $25 + (-30) =$

e. $(-2.5) + (-6.5) =$

f. $(-20) + 50 =$

Review and Consolidation

1. Express each of these ratios as a decimal unit ratio, rounded to three decimal places when necessary.

a. 2 : 5

b. 7 : 8

c. 3.5 : 2.5

d. $1\frac{1}{2} : 4\frac{1}{4}$

2. Express each of these ratios in simplest whole number form. Remember to first convert both quantities to the same unit.

a. 50 cents to $2.50

b. 500 g to 12 kg

c. 3.2 km to 2400 m

3. Jamal ate half of a sandwich, and Chen ate one-third. Write the simplest whole number ratio that represents the ratio of the amount that Jamal ate to the amount that Chen ate.

Homework

1. Express each of the following numbers as a simplest whole number ratio.

a. 2.5 b. 0.8 c. 35% d. $\frac{30}{84}$

2. Express each ratio in simplest whole number form, and then complete the sentence by replacing the square with the correct value.

a. \$3 to \$9 \$3 is $\square$ of \$9

b. 2 km to 50 m 2 km is $\square$ of 50 m

c. $40\frac{1}{2}$ to $13\frac{1}{2}$ $40\frac{1}{2}$ is $\square$ of $13\frac{1}{2}$

A bag of marbles contains 7 red marbles, 18 blue marbles, and 38 orange marbles. Use this information to answer problems 3 and 4.

3. Express the number of red marbles to the number of blue marbles as:

a. A unit ratio (in decimal form, rounded to two decimal places)

b. A fraction in simplest form

c. The simplest whole number ratio

d. A percent

4. Express the number of red marbles to the total number of marbles as:

a. A unit ratio (in decimal form, rounded to two decimal places)

b. A fraction in simplest form

c. The simplest whole number ratio

d. A percent

LESSON 4

RATIO TABLES

GOAL

To solve ratio problems using ratio tables.

CONCEPT BOOK

See page 316 in your *Concept Book.*

A *ratio table* displays equal ratios.

When ordering supplies for the school store, Jamal and Dwayne will often use a ratio table to order supplies. Jamal needed to order more rulers. He knew a box of rulers costs $6.00. He wanted to order 20 boxes so he made a ratio table like this.

×20

Boxes of Rulers	1	20
Cost (dollars)	6	120

×20

Dwayne had to order more pencils. He knew that 30 boxes of pencils cost $60.00, but he only wanted to buy 5 boxes. Dwayne used this calculation.

÷ 3 ÷ 2

Boxes of Pencils	30	10	5
Cost (dollars)	60	20	10

÷ 3 ÷ 2

When Dwayne wanted to buy 9 boxes of pencils, he used this ratio table.

÷ 3 10 – 1

Boxes of Pencils	30	10	1	9
Cost (dollars)	60	20	2	18

÷ 3 20 – 2

Do you understand his thinking?

The school store also provides calculators, which cost 9 dollars each. Mrs. Carver has requested that the store keep 33 calculators on hand for her two algebra classes. Jamal and Dwayne computed the ratio table differently.

Jamal's solution:

1 + 32

Number of Calculators	1	2	4	8	16	32	33
Cost (dollars)	9	18	36	72	144	288	297

9 + 288

Dwayne's solution:

3(1) + 30

Number of Calculators	1	10	30	33
Cost (dollars)	9	90	270	297

3(9) + 270

Do you understand how each approached the problem?

Dwayne also wrote his problem like this:

$$\frac{1}{9} = \frac{10}{90} = \frac{30}{270} = \frac{33}{297}$$

While Jamal wrote his ratio table vertically $(\updownarrow)$ as well as horizontally $(\longleftrightarrow)$.

Example

This ratio table shows some of the same information as the ratio table above.

Number	Cost
1	9
8	72
32	288
33	297

The numbers in a ratio table do not need to be in order of size.

Work Time

1. Jamal's cousin Ameera manages a grocery store in the town of Lakeside. She received an order from the Monroe High School cafeteria.

   Ameera kept a copy of the school's previous order. Use the information from the previous order to create three ratio tables and help Ameera calculate the costs of the new order.

New Order

Items
5 crates of apples
29 boxes of oatmeal
17 gallons of milk

Previous Order

Items	Cost
3 crates of apples	$15
10 boxes of oatmeal	$24
20 gallons of milk	$84

   a. Create a ratio table for apples. Use it to find the cost of 5 crates of apples. Explain how you calculated the cost.

   b. Create a new ratio table for oatmeal. Use it to find the cost of 29 boxes of oatmeal. Explain how you calculated the cost.

   c. Create a new ratio table for milk. Use it to find the cost of 17 gallons of milk. Explain how you calculated the cost.

   d. Calculate the total cost of the new order (5 crates of apples, 29 boxes of oatmeal, 17 gallons of milk).

2. Use your ratio table for milk to calculate:

   a. The cost of 19 gallons of milk

   b. The cost of 32 gallons of milk

   c. The number of gallons of milk that you could buy for $588

Preparing for the Closing

3. Explain how you could use pairs of equal ratios to find a missing ratio. Give an example to support your answer.

4. a. For problems 1 and 2, what assumption did you make about the cost of each type of food?

b. Why is this important?

Skills

Solve.

a. $2x + (-3x) =$

b. $2\frac{1}{2}y + \left(-3\frac{1}{2}y\right) =$

c. $(-50a) + 25a =$

d. $25x + (-30x) =$

e. $(-2.5y) + (-6.5y) =$

f. $(-20a) + 50a =$

Review and Consolidation

1. a. Keesha wanted to invite 6 friends over to eat pizzas and watch movies. She wanted to order 4 regular, uncut pizzas to share equally with her friends.

Copy and complete this ratio table to show how much pizza there would be for each of the 7 people at Keesha's party.

Amount of Pizza	4	
Number of People	7	1

b. Chen was also having a pizza party. He invited 9 friends and wanted to order 5 regular, uncut pizzas.

Make a ratio table to show how much pizza there would be for each of the 10 people at Chen's party.

c. Which group of friends would get more pizza? Say how you know.

d. Keesha was surprised when 20 friends arrived at her house. She still wanted everyone to have the same ratio of pizza that she had originally planned.

Extend your ratio table from part a to find out how many pizzas Keesha needed to order for 20 friends.

Homework

1. Copy and complete each pair of equal ratios.

a. $\square : 20 = 3 : 4$

b. $7 : 3 = \square : 60$

c. $9.4 : 5 = \square : 100$

d. $\square : 1\frac{3}{4} = 5 : 7$

2. Use a ratio table to solve the following problem.

At Lakeside High School, the ratio of tenth-grade students to eleventh-grade students is 5 : 9.

a. There are 235 tenth-grade students. How many eleventh-grade students are there?

b. The ratio of tenth-grade to eleventh-grade students remains unchanged, and the number of eleventh-grade students increases to 720 students.

How many tenth-grade students are there now?

SOLVING PROPORTION PROBLEMS WITH RATIO TABLES

LESSON 5

CONCEPT BOOK	GOAL
See pages 315–316 in your *Concept Book*.	To use ratio tables to solve problems of proportion.

A ratio is a comparison of two quantities. When the ratio between two quantities that vary is constant, the relationship is called a *proportional relationship*. There are many proportional relationships in the world around you.

Example

Suppose you are shopping for fruit juice. 5 cans of juice cost $10.20, and 10 cans cost $20.40. You know that the ratio, the cost per can, is constant.

The cost per can to the number of cans is a proportional relationship.

$$5 : 10.20 = \frac{5}{10.2} = \frac{2 \bullet 5}{2 \bullet 10.2} = \frac{10}{20.4} = 10 : 20.40$$

5 cans cost $10.20   10 cans cost $20.40

You can represent this information in a ratio table, and then use the table to calculate missing values.

Example

You know that 5 cans of fruit juice cost $10.20.
How much will 1 can of fruit juice cost?

÷ 5   × 31

Cost (dollars)	10.20	20.40	2.04	63.24
Number of Cans	5	10	1	31

÷ 5   × 31

1 can of fruit juice costs $2.04.

Use the unit price per can, 2.04 : 1, to find the cost of any number of cans. The cost of 31 cans of juice is 31 • 2.04 = $63.24.

Work Time

1. Mrs. Simek, the art teacher at Monroe High School, wanted to purchase a new paint brush for each of her 32 students. The supplier offers brushes in packs of 4. Each pack of 4 brushes sells for $17.

Mrs. Simek asked her students to calculate the total price of the paint brushes.

a. Jamal made this ratio table to solve the problem. Describe what steps he used.

Price (dollars)	17	34	68	136
Number of Brushes	4	8	16	32

b. Rosa made this ratio table to solve the problem. Describe what steps she used.

Price (dollars)	17	272	136
Number of Brushes	4	64	32

2. Lisa made this attempt to solve the problem.

Price (dollars)	17	4.25	42.50	127.50	129.50
Number of Brushes	4	1	10	30	32

a. Identify where Lisa made a mistake, and write the correct numbers.

b. Write advice for Lisa so she will not make the same type of mistake next time.

3. Use a ratio table to find the number of paint brushes Mrs. Simek could buy for $238.

Preparing for the Closing

4. Decide whether each of the following tables is a ratio table. Say why or why not.

a.

Price (dollars)	12	30	58
Number of Brushes	4	8	16

b.

Price (dollars)	b	$4b$	$16b$
Number of Brushes	a	$4a$	$16a$

5. In your own words, say what it means for two quantities to be *in proportion*.

Skills

Solve.

a. $25 + (-36) + 11 =$

b. $4\frac{1}{3} + \left(-6\frac{1}{3}\right) + 2 =$

c. $(-40) + 100 + (-60) =$

d. $(-4.5) + (-8.5) + 13 =$

e. $135 + (-160) + 0.30 + 24.7 =$

f. $(-50) + (-50) + (-55) + 155 =$

Review and Consolidation

1. Here are the ingredients to make 30 cookies. The quantities of ingredients needed to make any number of cookies are in proportion to these quantities.

2 cups of flour

$\frac{1}{4}$ cup of milk

$\frac{1}{3}$ cup of butter

$\frac{1}{2}$ teaspoon of vanilla

3 eggs

$\frac{3}{4}$ cup of sugar

a. Dwayne wanted to make some cookies, but he has 5 eggs. What is the maximum number of cookies he could make, and the quantities of all ingredients that he would need to make that number of cookies?

b. Which measurements might Dwayne have to round up or down? Say why.

c. Dwayne also considered making batches of 20, 60, and 40 cookies. To help Dwayne, copy and complete this ratio table.

Number of Cookies	30	20	60	40
Cups of Flour				
Cups of Milk				
Cups of Butter				
Eggs				
Cups of Sugar				
Teaspoons of Vanilla				

2. a. Dwayne had a 2-pound (32-ounce) bag of flour. He looked in a cookbook and found that 1 cup of flour weighs 4 ounces.

How many cups of flour are in Dwayne's bag of flour?

b. Dwayne wanted to use the whole bag of flour to make cookies. How many cookies can he make if he buys more eggs?

c. How much of each ingredient will Dwayne need to be able to make the number of cookies you calculated in part b?

Homework

1. Look at this ratio table.

Quantity A	24	6	54
Quantity B	400	100	900

a. Explain how the values 6 : 100 and 54 : 900 were calculated.

b. What is the unit ratio, where quantity A is 1?

c. What are two other pairs of values you could add to the table?

MORE ABOUT SOLVING PROPORTION PROBLEMS

LESSON 6

CONCEPT BOOK	GOAL
See pages 296, 316–317 in your *Concept Book*.	To solve proportion problems using ratio tables, bar diagrams, and by solving equations.

In addition to ratio tables, you can use bar diagrams and equations to solve proportion problems.

Dwayne and Chen both collect model airplanes. Dwayne has two and a half times as many models as Chen. Dwayne has 15 models. How many models does Chen have?

Using a Ratio Table

The unit ratio of Dwayne's models to Chen's models is 2.5 : 1.
The simplest whole number ratio equal to this is 5 : 2.

Example

Dwayne has 15 airplanes.
To get to 15, you multiply by 6.
So, if you multiply the 1 in Chen's row by 6, 1 • 6 = 6.

×6

Dwayne	2.5	5	15
Chen	1	2	6

×6

Chen has 6 model airplanes.

Using a Bar Diagram

You can solve the same problem using a bar diagram. The bar diagram has unit squares of the same size in each bar.

Example

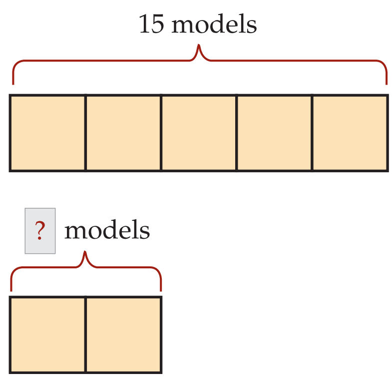
The simplest whole number ratio is 5 : 2.

Draw 5 squares for one bar for Dwayne.

Draw 2 squares in the other bar for Chen.

Since Dwayne has 15 models, each unit square must represents 3 models.

This means Chen has 6 models.



Solve by Writing and Solving an Equation

A *proportion* is an equation of two ratios.

Example

The simplest whole number ratio of Dwayne's model to Chen's model is 5 : 2.

How many models does Chen have?

Dwayne has 15 models. Let x represent the number of models that Chen has.	$15 : x = 5 : 2$
Express as fractions	$\frac{15}{x} = \frac{5}{2}$
Multiply both sides by the x	$\frac{15}{x} \bullet x = \frac{5}{2}x$
Simplify	$15 = \frac{5}{2}x$
Multiply both sides by the 2	$15 \bullet 2 = \frac{5}{2}x \bullet 2$
Simplify	$30 = 5x$
Solve	$6 = x$

Work Time

Work with a partner to solve problems 1 and 2 using the three solution methods:

- Ratio table
- Bar diagram
- Solving an equation

For each problem, one student should use a ratio table, and the other student should use a bar diagram. Then, partners should work together to solve the problem using an equation.

1. Lakeside City Hospital has a rule stating that the ratio of the nurses to patients must be as close as possible to 1 : 12.

The hospital is expanding and will have space for 672 patients.
How many nurses will the hospital need?

2. Dwayne's little brother Darell, and two of his friends, Carla and Anthony, deliver newspapers and earn tips. The total dollars in tips for Darell to Carla to Anthony are in a ratio of 2 : 5 : 11.

   Suppose Darell earns $20.80 in tips. How much do Carla and Anthony earn?

Preparing for the Closing

3. Compare each of your solutions and methods with another pair of students. Discuss your results and try to reach an agreement about your answers.

4. Explain the similarities and differences among the three methods of solving problems with proportional ratios. Which method do you prefer? Why?

Skills

What is the difference between each pair of numbers?

a. –10 and 35 b. 10 and –35 c. –20 and –65 d. 40 and 20

Hint: You might find it helpful to sketch a number line.

Review and Consolidation

1. Use a ratio table, a bar diagram, and an equation to solve this problem.

   A group of people has a ratio of left-handers to right-handers of 2 : 7.

   If there are 14 right-handers, how many left-handers are in the group?

2. Use a ratio table and an equation to solve this problem.

   Chen's mother manages a pharmacy. The total amount of money that Mrs. Lee received at the pharmacy in two months was $120,000. In that time, Mrs. Lee spent $85,000 to buy goods for the pharmacy.

   Over the next twelve months, Mrs. Lee wants to keep her ratio (money spent to money received) constant. She will be spending $95,000.

   How much money is she hoping to receive?

Homework

1. There are 216 employees in a store. The ratio of men to women is 5 : 7.

   How many men and how many women work at the store?

2. Divide the total amount of $360 into two amounts that have the ratio 7 : 5.

3. Three friends A, B, and C share a sum of money with amounts in the ratio 3 : 2 : 1. Friend A is lucky enough to receive $180.

   How much money do B and C each receive?

INTRODUCING RATES

LESSON 7

CONCEPT BOOK	GOAL
See pages 305–306 in your *Concept Book*.	To identify and represent rates, and to use them to solve problems.

You have already learned that a ratio is the comparison of two quantities using division.

A *quantity* is an amount that can be counted or measured.

Example

"35 miles" is a quantity. The amount or distance is 35 and the unit is "miles."

unit
amount

"6 hours" is a quantity. The amount is "6" and the unit is "hours."

A ratio where the units being compared are different is sometimes called a *rate*. As in all ratios, calculating a rate involves comparing two quantities by division.

Rate is a quantity formed from two other quantities by division. Each of these rate quantities is expressed with the special word *per*. The term *per* means "for each."

Example

35 miles per hour
3 students per computer
30 people per square mile
2 meters per person

A rate is a "per unit" quantity because it is stated as a certain amount of the first quantity for every unit amount of the second.

Example

15 miles per hour means 15 miles for every 1 hour.

The number associated with a rate is a single number, formed by completing a division of one number by another.

Example of a Ratio that Is Not a Rate

Example

Suppose there were 50 students and 6 teachers.

You could describe this situation as:

50 students for every 6 teachers

A student-teacher ratio of 25 to 3

These are not rates, because the descriptions use two numbers that have not been divided to form a single number.

Examples of Rates

Example

You can describe the situation of 50 students and 6 teachers as:

$8\frac{1}{3}$ students per teacher

Approximately 8 students per teacher

These quantities are rates.

Work Time

1. Describe a situation in which each of these rates could be used.

a. Beats per minute

b. Dollars per kilogram

c. Gallons per hour

Calculate each of the rates found in problems 2–7.

2. Lisa's uncle took 8 minutes to fill a 5-gallon bucket with water from a hose.

At what rate was the bucket filled?

3. Chen bought 10 pounds of potatoes for $5.50.

Find the cost of 1 pound of potatoes. Write the cost of potatoes as "cents per pound."

4. Suppose 3 dozen cans of tomato paste cost $5.76. Write the cost as a rate using "cents per can."

5. A computer operator can type 1680 words in half an hour.

What is her typing rate in "words per minute"?

6. Jamal works afternoons in a bagel shop. He earns $112.50 for 15 hours of work.

At what rate is he paid?

7. Jamal's cousin Ameera has to pay income tax at the rate of 36% per year on her annual income. Ameera knows she has paid $65,000 in taxes over the last 4 years. Ameera's annual income has not changed in that time.

a. What is the rate at which Ameera pays tax (in dollars per year)?

b. What is Ameera's annual income?

Preparing for the Closing

8. It is important to express a rate with a unit. Say why.

9. A rate is a *unit ratio*. Say why, and give some examples to support your explanation.

Skills

Solve.

a. $2 - 3 =$

b. $2\frac{1}{2} - 3\frac{1}{2} =$

c. $(-50) - 25 =$

d. $(-25) - 30 =$

e. $(-2.5) - 6.5 =$

f. $(-20) - 50 =$

Review and Consolidation

1. A car travels 400 miles in 5 hours.

a. At what average speed (rate) did the car travel?

b. At this rate, how far would the car travel in 6 hours?

c. At this rate, how long would the car take to travel 640 miles?

2. Mrs. Jackson's bank pays interest on her savings at the rate of 5% per year.
Interest is calculated and added to the balance at the end of the year.

a. A rate of 5% per year is equivalent to a ratio of 1 : 20. Say why.

b. Suppose Mrs. Jackson has $300 in the bank.
How much interest will she earn in one year?

c. Suppose the rate of interest is halved.
How much interest will she earn in one year?

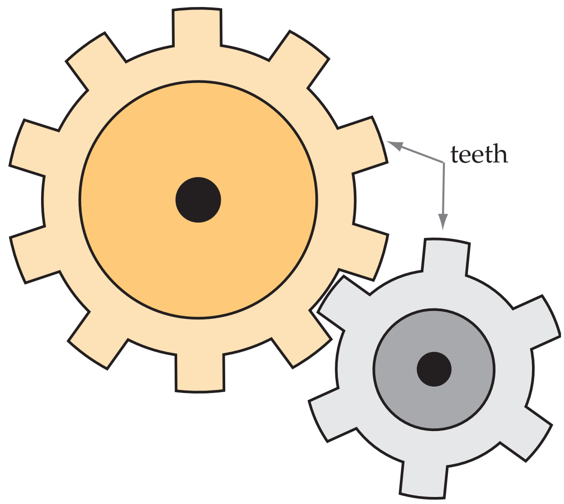
Comment

"Halved" means cut in half.

3. A machine has two gear wheels that interconnect. The larger gear wheel has 10 teeth, and the smaller gear wheel has 6 teeth.

a. What is the unit ratio of the number of teeth on the large wheel to the number of teeth on the small wheel?

b. The rate at which the larger wheel turns is 20 revolutions (complete turns) per second. At what rate is the smaller wheel turning?



Homework

1. Rosa needs to buy pretzels for the school party.

- *Crispy Pretzels* are sold in 11.5-ounce bags that cost $2.69 a bag.
- *Pretzel Twists* are sold in 9-ounce bags that cost $2.25 a bag.
- *Bargain Pretzels* cost $3.76 for one 15.8-ounce bag or $7.50 for two 15.8-ounce bags.

All three brands taste about the same. Rosa is having trouble comparing prices.

Crispy Pretzels	11.5-ounce bag	$2.69 per bag
Pretzel Twists	9-ounce bag	$2.25 per bag
Bargain Pretzels	15.8-ounce bag	$3.76 per bag or $7.50 for two bags

a. List the pretzels from least expensive to most expensive. Explain how you made your decisions. Include calculations to support your reasoning.

b. Which pretzels do you think Rosa should buy? Why?

LESSON 8

REVIEWING RATIO AND ARITHMETIC

GOAL

To review using ratio tables, bar diagrams, and equations to solve ratio and rate problems.

CONCEPT BOOK

See pages 291–297, 305–306, 315–316 in your *Concept Book*.

Comparing Numbers

Comparing two numbers by subtraction, $a - b$, gives a new number $(a - b)$, called the *difference*.

Comparing two numbers, a to b, by division gives a new number, the *ratio*, $\frac{a}{b}$, $b \neq 0$.

Ratios and Rates

A ratio is a comparison between two quantities. It can also show the relationship between quantities measured with the same units, or units that can be converted to the same unit.

Rate is a quantity formed by dividing one quantity by another quantity.

Expressing Ratios

Ratios can be represented in many ways:

- as a division, $a \div b$
- as a fraction, $\frac{a}{b}$
- as a decimal
- in ratio notation, $a : b$
- as a percent, %

Unit ratios are those written as a ratio of some number to 1.

Whole number ratios are those in which the numbers in the ratio are both whole numbers.

Ratios are often simplified by reducing them to their lowest terms. Reducing a ratio makes it easier to interpret the particular ratio. This process is similar to simplifying fractions.

To find *equal ratios*, multiply or divide both numbers of one ratio by the same nonzero number.

When the ratio of two quantities is 1, then the two quantities are the same.

Solving Problems

Ratio tables and bar diagrams are useful tools for solving proportion problems in which you need to find the unknown value of a quantity.

A proportional relationship can also be represented using a formula. This formula can be used to find an unknown quantity.

Work Time

1. Lisa had a hair ribbon 58 cm long. She cut 12 cm off the ribbon so she could have two ribbons, one shorter than the other.

   Express each of the following ratios as the simplest whole number ratio possible.

   a. The ratio of the shorter ribbon to the longer ribbon

   b. The ratio of the longer ribbon to the original ribbon

2. Express the ratios in problem 1 as:

   a. Unit ratios (using fractions)

   b. Percents (rounded to the nearest whole percent)

3. Find the unknown values in this ratio table. Express your answers as integers, proper fractions, or mixed numbers.

Quantity A	12	1		4	8		100
Quantity B	14		1			100	

4. Country A has a population of approximately 50 million people and an area of 1.5 million square kilometers.

   a. Find the population density of this country using the unit ratio "people per square kilometer."

   b. The population density of a country is a rate. Say why.

   c. Country B has a population of approximately 12 million people and exactly the same population density as country A.

   What is the area of country B in square kilometers?

5. Three farmers (X, Y, and Z) buy seed for their farms and share it in the ratio 3 : 7 : 8. If they buy 144 kg of seed, how many kilograms of seed does each farmer receive? Show the problem solution using a ratio table, a bar diagram, and an equation.

Preparing for the Closing

6. Discuss your responses to the Work Time problems with another student.

Try to reach an agreement about your decisions and reasoning.

Skills

Solve.

a. 16.5 – (–18.5) =

b. (–123.4) – (–45) =

c. (–50.6) – (–25.4) =

d. 30 – (–25) =

e. (–2.8) – (–7.4) =

f. (–20) – (–137) =

Review and Consolidation

Read each of the following statements and decide whether it is *always true*, *sometimes true*, or *never true*.

Justify your answer for each statement. Give examples, using numbers, to support your reasoning.

1. If you compare two numbers using division, you will get the same result that you would get if you compared the two numbers using subtraction.

2. You find a ratio by dividing one number by another number (except zero).

3. As a ratio of two quantities gets closer to equaling 1, the two quantities get closer to being the same.

4. A percent can be thought of as a ratio.

5. If two quantities have units that are different, you can find the ratio of the quantities by converting one unit to the other.

6. A ratio table is a collection of equal ratios.

7. To find an unknown number using a ratio table, use the operation of multiplication.

8. A ratio of whole numbers can be expressed as a unit ratio.

9. Any unit ratio can be converted to a whole number ratio.

10. A rate is also a ratio.

Homework

1. Sketch a bar diagram that shows the ratio 4 : 11.

2. Express each of these ratios as the simplest possible whole number ratio.

a. 0.55

b. 1.2

c. 34%

d. 4.8 : 1

3. Johnson's Pet Shop has 12 kittens and 27 puppies for sale.

a. Find the simplest whole number ratio of puppies to kittens, and write a sentence that explains your result.

b. Find the unit ratio of puppies to kittens, and write a sentence that explains your result.

c. Mr. Johnson wants to keep the ratio of puppies to kittens exactly the same. He buys 9 more puppies.

How many more kittens does he need?

4. Keesha's mother drives 80 miles in 2 hours.

a. At what average speed is she traveling?

b. Explain why this average speed is a rate.

LESSON 9

ENLARGING AND REDUCING

GOAL

To investigate and generalize enlargement and reduction of objects.

CONCEPT BOOK

See page 274 in your *Concept Book*.

Ratios and proportionality help you understand the relationship between quantities. Using ratios and proportionality can help you predict unknown quantities.

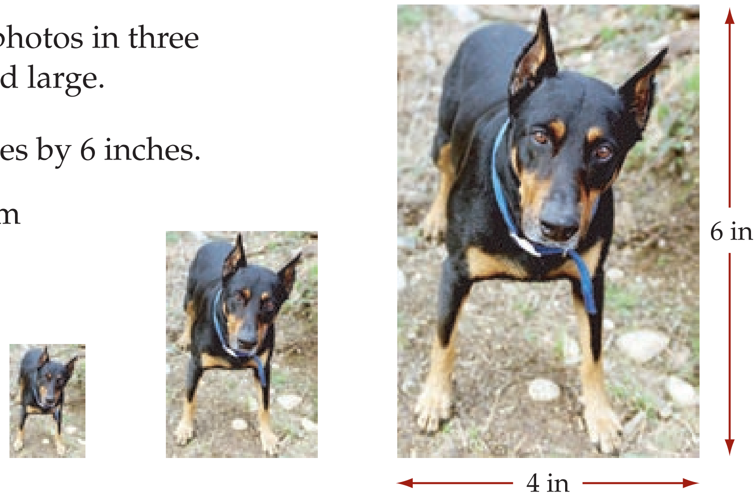
Example

A photo shop can print photos in three sizes: small, medium, and large.

The large photo is 4 inches by 6 inches.

The height of the medium photo is 3 inches

The height of the small photo 1.5 inches.



The ratio of height to width of each photo remains constant (the same).

Since you know the height and the width of the large photo, you can divide to get the ratio of the photo height to the photo width.

$$\frac{\text{height}}{\text{width}} = \frac{6\text{ in}}{4\text{ in}} = 1.5$$

You can use this ratio to find the widths of the other two photos.

Each of the other pictures should have a ratio of 1.5. You can make a ratio table.

		Large	Medium	Small	
Height (cm)	h	6	3	1.5	h
Width (cm)	w	4	?	?	$1.5h$

If you want a picture that is 4.5 inches in height, you could add the 3- and the 1.5-heights.

3 + 1.5

		Large	Medium	Small	New	
Height (cm)	h	6	3	1.5	4.5	h
Width (cm)	w	4	2	1		$1.5h$

2 + 1

Or, you could multiply the 1.5-height by 3.

× 3

		Large	Medium	Small	New	
Height (cm)	h	6	3	1.5	4.5	h
Width (cm)	w	4	2	1		$1.5h$

× 3

In general, when you enlarge or reduce a shape, you multiply the linear dimensions by a *scale factor*. In the example above, the ratio of length to height is 1.5 on each picture.

The scale factor between the picture with a height of 1.5 and the picture with a height of 4.5 is 3 times (or 300%).

Work Time

1. An original photo has a height of 3 cm and a width of 5 cm. The ratio table shows the measures of some enlarged and reduced copies of the photo.

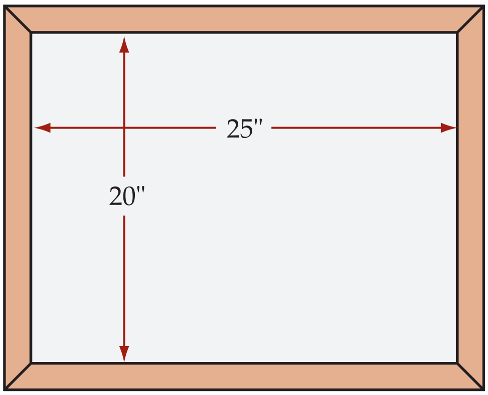
Copy this table and complete the missing values.

Height (cm)	h	3	1.8		9	
Width (cm)	w	5		4		10

2. Jamal has a photograph that measures 5 inches by 7 inches. Suppose he wants to enlarge the photo and put it into a frame.

Which of the frames below should he use?

Comment
"Enlarge" means to make bigger.

A


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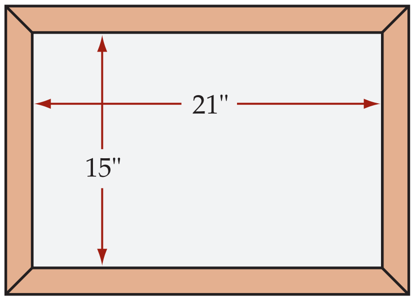
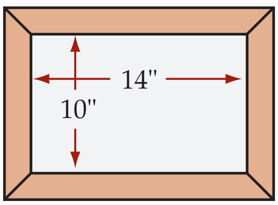
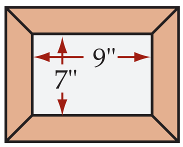
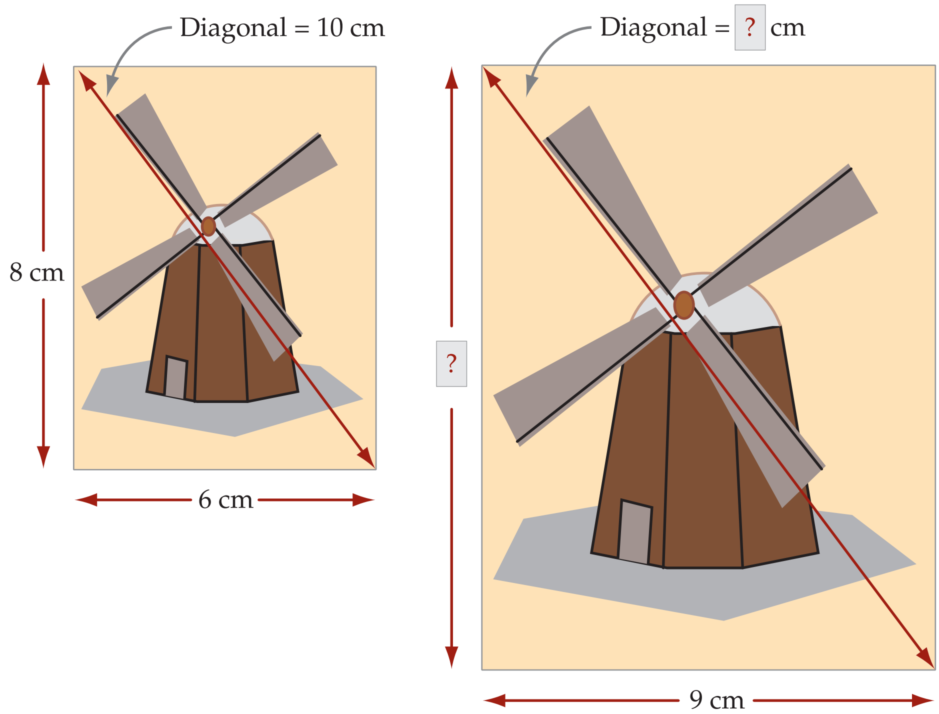


C


D


3. The picture of the windmill on the right is an enlargement of the picture on the left. (The pictures are not drawn to scale. Use the measurements that are given.)



a. Using a ratio table or an equation, calculate the measures of the unknown dimensions—the height and the diagonal of the larger picture.

b. Write the scale factor of the smaller picture to the larger picture. Write the scale factor of the larger picture to the smaller picture. Comment on these results.

c. Using graph paper, draw the enlarged rectangle on the grid. Make sure that you draw an accurate rectangle with the correct height and diagonal length. (You do not need to draw the windmill.)

d. Discuss the reasons for any errors you may have made in your drawing.

e. What mistake has been made in the following statement? Explain.

When the picture is enlarged, the height will be 11 cm, because the width is 3 cm longer. The length of the diagonal will be 13 cm.

Preparing for the Closing

4. Look again at the pictures in the beginning of this lesson. Even though all three of the pictures are different sizes, their shapes look the same. Say why.

5. All the lengths of the photos in problem 1 are in proportion to each other. Say what this means, and justify your answer.

6. If you measured the pictures in problem 3 using inches instead of centimeters, you would calculate the same ratio. Say why.

Skills

Solve.

a. Give three pairs of negative numbers with a total difference equaling 50 for each pair.

b. Give three pairs of numbers, each pair consisting of a positive and a negative number, and the total sum of each pair equal to 40.

Review and Consolidation

1. A photograph has a width of 8 cm and a height of 12 cm.

a. Calculate the unit ratio of width to height.

b. What is the simplest whole number ratio of height to width?

2. A photo shop wants to reduce the photograph from problem 1 so that the photo's height is $9\frac{1}{2}$ cm. Use a ratio to calculate the width of the reduced picture.

3. Which of the following measurements could represent an enlarged or reduced version of the photo in problem 1?

Ⓐ 8 cm wide by 10 cm high

Ⓑ 5 cm wide by 7.5 cm high

Ⓒ 13 cm wide by 17 cm high

Ⓓ 12 cm wide by 18 cm high

Homework

1. You measure a picture and find that the picture's ratio of height to width is 1.25. Suppose the picture is 8 cm wide. What is its height?

2. Some measurements of enlarged and reduced versions of a photograph are listed in the table below. Copy the table. Fill in the missing values in the ratio table.

Width (inches)	8	4	3	
Height (inches)		5		20

3. Dwayne wants to send an enlarged copy of his school picture to his grandparents. If the picture he has measures 3 inches wide by 4 inches high, and he wants to send a copy to his grandparents that is 12 inches wide, how high would the copy be?

LESSON 10

SCALE FACTOR AND RATIO

GOAL

To apply a scale factor to a quantity.

CONCEPT BOOK

See pages 274 and 320 in your *Concept Book.*

When you enlarge or reduce a photograph, the ratio of height to width remains the same; each copy is proportional. In Lesson 9, you saw that photographs of the dog could be different sizes, but that the photos all had the same ratio of the height to the width, 1.5 : 1. These photos were similar figures.

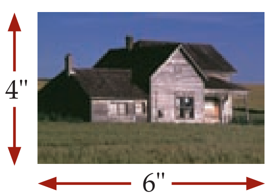
Plane figures and solids that are mathematically *similar* to each other have the same shape, but not necessarily the same size.

When you calculate new sizes for enlarged and reduced photographs, you use a scale factor. A *scale factor* is the ratio of the length of one side of a figure to the corresponding side of a similar figure. In general, when you enlarge or reduce a photograph, you multiply the photograph's height and width by a constant scale factor to get the height and width of the new photo. On photocopy machines, the scale factor is often written as a percent.

Example

Suppose the scale factor is 2. The measured lengths of the enlarged version are *twice* those of the original, or 2 • 100% = 200% of the original measurements.

For a photo 4 × 6 inches, the enlarged photo is 8 × 12 inches.



Example

If the scale factor is $\frac{1}{2}$, the linear dimensions of the reduced version are *half* those of the original, or $\frac{1}{2}$ • 100% = 50% of the original measurements.

For a photo 4 × 6 inches, the reduced photo is 2 × 3 inches.

Work Time

Chen's sister, Amy, wanted to make a flyer for her band, *The Purples*. She asked Chen to help. "I have a photo of a guitar that would look great on the flyer," said Amy, "but it's too big. Can we make it smaller?"

"We could reduce it on the copy machine," suggested Chen. "How big is your photo?"

"Both the photo and the flyer are 6 inches wide by 8 inches high. I want the reduced photo to fit in a corner of the flyer and to be exactly 3 inches high."

"You know that the ratio of height to width remains constant when a photograph is enlarged or reduced," said Chen. "If you want the photo to be 3 inches high, then the ratio of the reduced photo's height to the original photo's height will be 3 : 8 or $\frac{3 \text{ inches}}{8 \text{ inches}} = 0.375$. The measured lengths of the reduced photo will be 37.5% of the original size."

"Wow! So the reduced photograph will be $0.375 \bullet 6$ inches = 2.25 inches wide."

"That's it. We need to reduce the photo on this copy machine to 37.5% of its original size."

Use Handout 2: *Student Band Poster*.

The handout contains the poster that Amy and Chen discussed. Now they want to reduce the poster to a size that will fit in a display area.

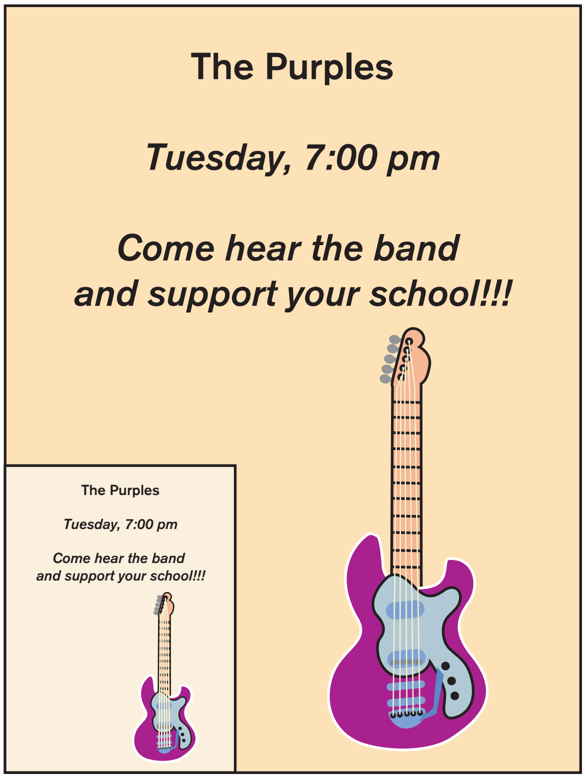
1. a. Measure the poster using a ruler.

b. Calculate the scale factor of the reduction by comparing the height of the original poster to the height of the reduced poster.

c. Use the widths to check your answer.

2. Calculate the height-to-width proportions for the original and the reduced poster.

What do you notice?



3. Suppose you made other comparisons between the reduced photograph and the original version (for example, the length of the guitar in each poster).

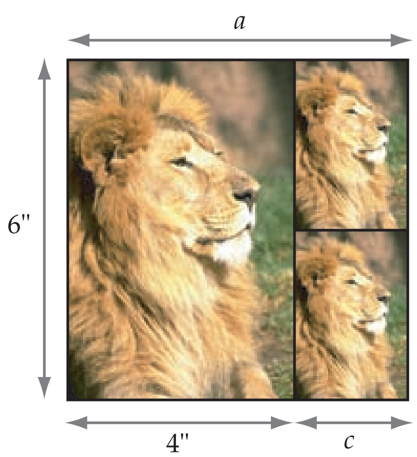
How do you think those ratios would be related to the scale factor you computed in problem 1?

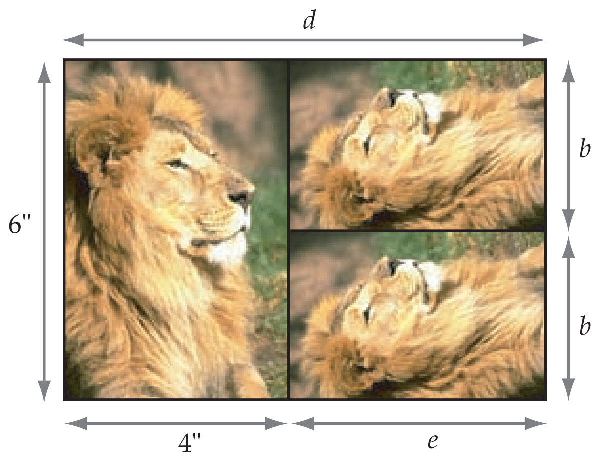
4. Using the table below, check your answer to problem 2 by making at least three different measurements of the original and the reduced versions of the flyer.
Calculate the ratio between them.

Measurement	Original Version	Reduced Version	Ratio (reduced to original)
Length of guitar			

5. A photographer wants to print a photo, along with two smaller, proportional copies of the photo, on the same piece of paper. Here are two ways he can do this.

Calculate the missing dimensions that are represented by letters.





Preparing for the Closing

6. Define *scale factor* in your own words.

7. You can multiply the lengths in a figure by a scale factor to reduce or enlarge that figure without changing its shape.

a. What is the result when the scale factor is greater than 1?

b. What is the result when the scale factor is less than 1?

c. What does it mean for the scale factor to be equal to 1?

8. In problem 1, you found the ratio (scale factor) between the two pictures. In problem 2, you found the ratio of height to width within each picture.

Explain in writing how these results are related.

Skills

Solve.

a. $17.9 - (-25.9) =$

b. $(-1400.4) - (-45) =$

c. $(-50.5) - (-25.4) =$

d. $17.9 + 25.9 =$

e. $(-1400.4) + 45 =$

f. $(-50.5) + 25.4 =$

Review and Consolidation

1. A model car is constructed using a 1 : 20 scale. This model has a scale factor of $\frac{1}{20}$ in relation to the original car.

a. Write the scale factor $\frac{1}{20}$ as a percent.

b. If the real car is 12 feet long, how long will the model be?

2. Lisa wants to enlarge a photo that has a width of 8 cm and a height of 12 cm so that the photo will have a width of 20 cm.

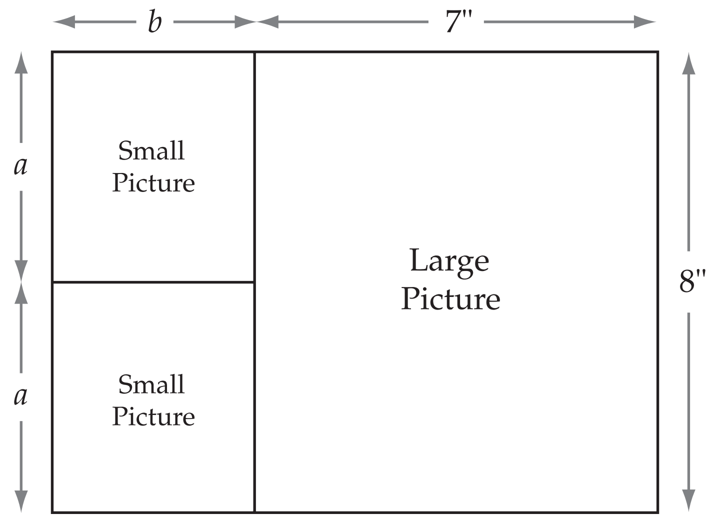
a. What is the scale factor of the enlargement?

b. What will the height of the new photo be?

3. A photographer wants to print three copies of a photograph on the same sheet of paper, one larger version and two smaller ones. She decides to arrange them in the way shown by the diagram.

a. What will the ratio be between the width of the larger copy and the width of the smaller copies?

b. What will the missing measurements (a, b) be?



Homework

1. Suppose Amy and Chen use a copy machine to enlarge their original flyer (6 inches wide by 8 inches high) to 175% of the original measurements.

a. Write the scale factor as a decimal.

b. What will be the width of the enlarged version?

2. Keesha is also making a flyer for the show. She makes an original copy that is 8 inches by 11.5 inches, and then makes a smaller quarter-sheet version that is 4 inches by 5.75 inches.

a. What is the scale factor of the reduced version?

b. What is the size-reduced version as a percent of the original version?

LESSON 11

THE SIMILARITY RATIO

CONCEPT BOOK	GOAL
See pages 255–257 in your *Concept Book*.	To identify the similarity ratio for similar polygons and use it to solve problems.

In everyday use, the word *similar* means “things that are alike in a general way.” In mathematics, the word *similar* has a precise meaning.

Plane figures and solids that are mathematically *similar* to each other have the same shape, but not necessarily the same size.

The mathematical phrase “is similar to” describes a particular type of relationship that exists between two or more shapes. The symbol ~ means *is similar to*.

Example

Here are two similar quadrilaterals.

In similar quadrilaterals, the pairs of *corresponding angles* are of equal measure.

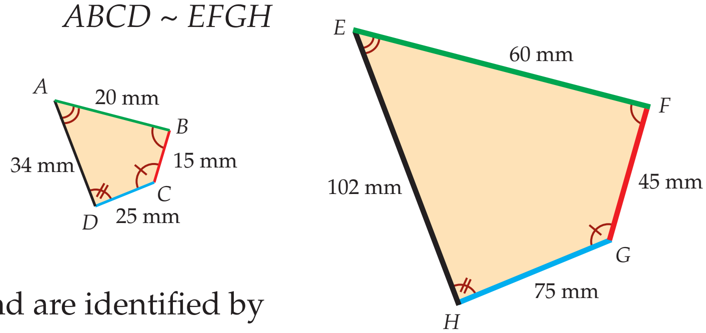
$ABCD \sim EFGH$

$\angle A = \angle E$

$\angle B = \angle F$

$\angle C = \angle G$

$\angle D = \angle H$



Equal angles are *congruent* and are identified by congruence marks: |, ||, and, ⌒.

In similar quadrilaterals, every pair of *corresponding sides* has lengths in the same ratio.

$\overline{AB} : \overline{EF} = 20 : 60 = 1 : 3$

$\overline{BC} : \overline{FG} = 15 : 45 = 1 : 3$

$\overline{CD} : \overline{GH} = 25 : 75 = 1 : 3$

$\overline{DA} : \overline{HE} = 34 : 102 = 1 : 3$

Similar two-dimensional figures must meet both of these conditions of similarity:

- The measures of corresponding angles are equal.
- The ratios of the lengths of every pair of corresponding sides are equal.

Any two (or more) figures that satisfy these two conditions are similar.

The *scale factor* is the ratio of any pair of corresponding lengths in two similar figures. This ratio is also called the *similarity ratio*. It is often represented as *k*.

Triangles

Triangles are special figures because you do not need to check that both conditions of similarity are true. If one condition is true, the other condition is also true. In other words, two triangles that have equal angles always have proportional sides, and two triangles that have proportional sides always have equal angles.

Order

The order in which you compare figures is important. The resulting similarity ratio, k, is determined by which figure is first and which figure is second in the comparison.

Example

If $ABCD \sim EFGH$, then the similarity ratio is 1 : 3, or $k = \frac{1}{3}$.

Quadrilateral $ABCD$ is one-third the size of quadrilateral $EFGH$.

If $EFGH \sim ABCD$, then the similarity ratio is 3 : 1, or $k = \frac{3}{1}$.

Quadrilateral $EFGH$ is three times the size of quadrilateral $ABCD$.

If two figures are similar, then all of their corresponding measurements (length, width, diagonal, and perimeter) are proportional.

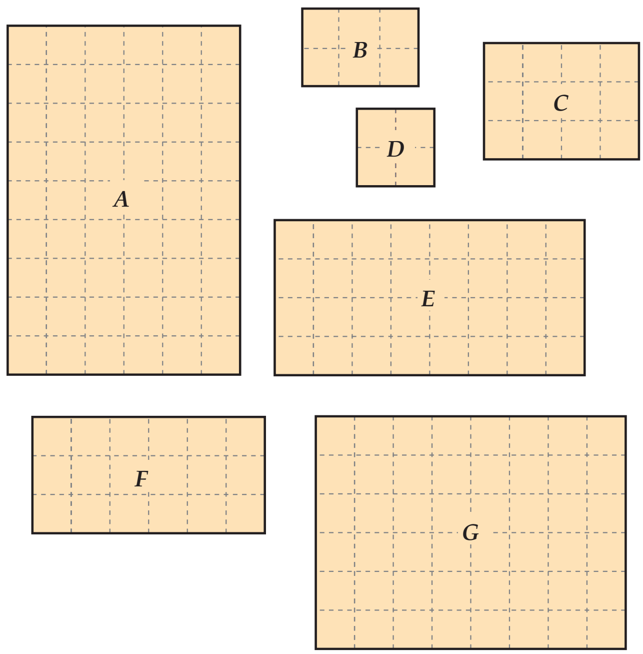
Work Time

1. Some of these rectangles are similar.

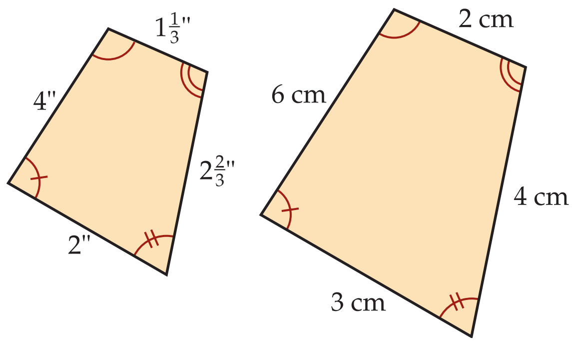
   a. Compare the lengths and widths of the rectangles, and decide which pairs of rectangles fulfill both conditions of similarity. Write your answers using the symbol ~.

   b. Give the similarity ratios for each pair of similar rectangles.

   c. All the rectangles have equal angles, but only some of the rectangles are similar. Say why.



2. Examine these two quadrilaterals.

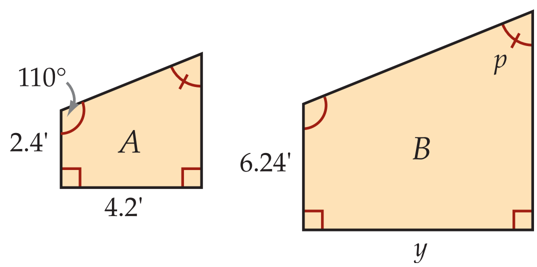


a. Use your calculator to verify that the ratio is the same between each pair of corresponding sides of the quadrilaterals.

b. The two quadrilaterals are similar. Say why.

3. Examine these two quadrilaterals.

Quadrilateral A ~ Quadrilateral B.



a. What is the measure of $\angle p$?

> **Hint:** Think about the sum of the interior angles of a quadrilateral or the sum of the interior angles of a triangle.

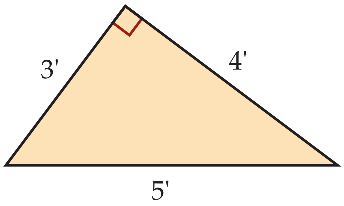
b. What is the similarity ratio when you compare quadrilateral B to quadrilateral A? (Express your answer as the simplest whole number ratio.)

c. Use the similarity ratio to find the length of the side labeled y.

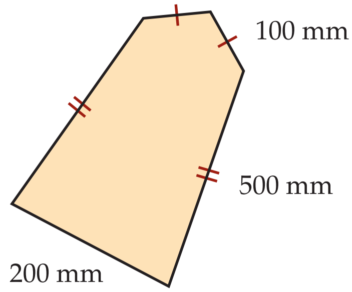
d. Choose a scale factor of your own, and use it to sketch a third quadrilateral that is similar to quadrilateral A. Write the scale factor that you used. Write angle and side length measures on your sketch where possible. Is this third quadrilateral also similar to quadrilateral B?

4. Use the similarity ratios $k_1 = \frac{1}{5}$ and $k_2 = \frac{3}{2}$ to sketch two figures that are similar to the triangle. Do the same for the polygon (a pentagon).

a.



b.



Preparing for the Closing

5. Why is understanding the concept of correspondence important to understanding the concept of similarity?

6. In problem 4, when you used the similarity ratio of $\frac{1}{5}$ the new figures were smaller than the originals.

In contrast, when you used the similarity ratio of $\frac{3}{2}$ the new figures were larger than the originals.

Write an explanation for these results.

7. Similar figures have the same shape.

Explain how the equal measures of the corresponding angles and the equal ratios of the lengths of corresponding sides result in similar figures have the same shape.

8. You cannot be sure that two polygons (other than triangles) are similar if you only know that all pairs of corresponding angles are of equal measure. Say why.

9. You cannot be sure that two polygons (other than triangles) are similar if you only know that all pairs of corresponding sides have the same similarity ratio. Say why.

Skills

Solve.

a. $50 + (-25) =$

b. $(-6.4) + (-10.7) =$

c. $17\frac{1}{4} + \left(-12\frac{3}{4}\right) =$

d. $50 - (+25) =$

e. $(-6.4) - 10.7 =$

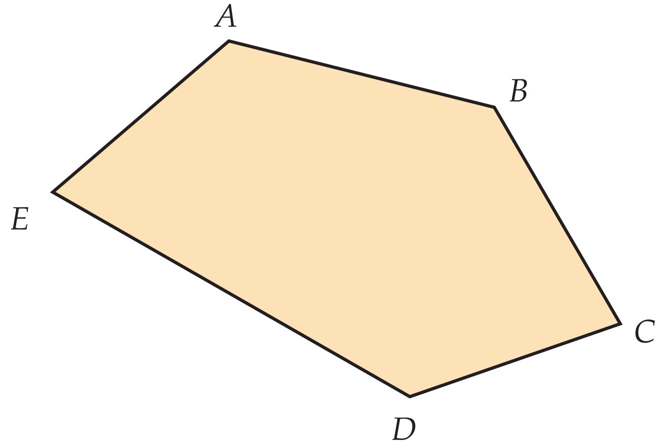
f. $17\frac{1}{4} - 12\frac{3}{4} =$

Review and Consolidation

You will need a ruler and a protractor.

1. Measure all the side lengths and angle measures for pentagon *ABCDE*.

Record all the measurements in millimeters.



2. Sketch a second pentagon, $A'B'C'D'E'$, with side lengths in the ratio 2 : 1 to those of pentagon *ABCDE*, but with angles that are different.

Your pentagon does not have to be accurately constructed, but try to make the side measures as close as possible to the correct measures.

3. Sketch a third pentagon, $A''B''C''D''E''$, with angles that are equal to the corresponding angles in pentagon *ABCDE*, but with side lengths that are not all in the same ratio to the corresponding sides in the original pentagon.

Your pentagon does not have to be accurately constructed, but try to make the angle measures as close as possible to the correct measures.

4. Sketch a fourth pentagon, $A'''B'''C'''D'''E'''$, similar to pentagon *ABCDE*, with a similarity ratio of 2.5.

5. Other students will have sketched their own pentagons for problems 2, 3, and 4.

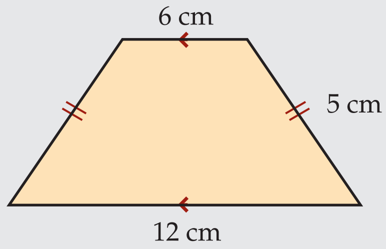
Which of their pentagons is most likely to be the same shape and size as yours? Say why.

Homework

CONCEPT BOOK

See pages 227 for definition of isosceles trapezoid.

1. Examine this isosceles trapezoid.



Sketch two figures that are similar to the trapezoid using each of the similarity ratios below. Write the new side measures on each diagram.

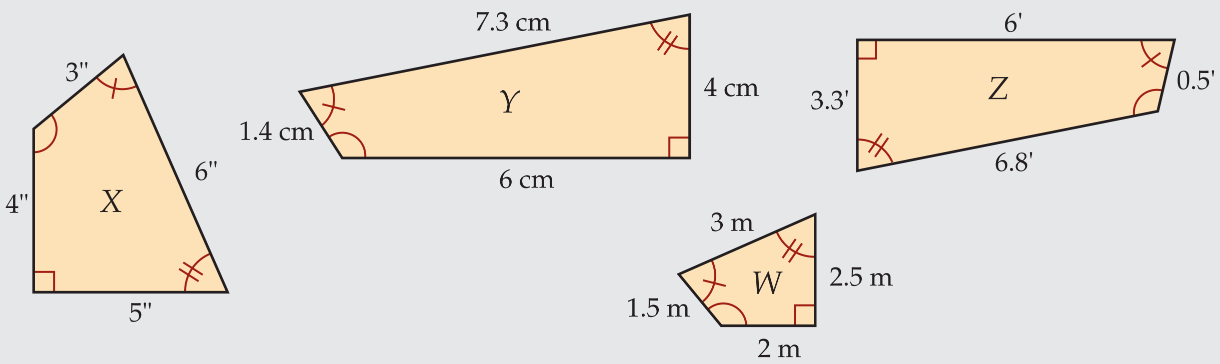
a. $k = \frac{1}{5}$

b. $k = \frac{3}{2}$

2. To determine whether two rectangles are similar, you need to consider only one of the similarity conditions.

Which one? Why?

3. Examine this set of quadrilaterals.



a. Only one of the quadrilaterals *Y*, *Z*, or *W* is similar to quadrilateral *X*.

Say which quadrilateral is similar, and explain your choice.

b. Write a reason why each of the other quadrilaterals is not similar to quadrilateral *X*.

SIMILAR TRIANGLES

LESSON 12

CONCEPT BOOK	GOAL
See pages 261–266 in your *Concept Book*.	To identify similar triangles and to use their properties to solve problems.

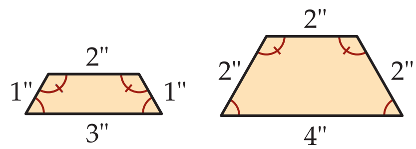
Like other similar figures, similar triangles have the same shape but different sizes, as expressed by the two conditions of similarity:

- The measures of corresponding angles are equal.
- The ratios of the lengths of every pair of corresponding sides are equal and constant by a similarity ratio, k.

When you worked with polygons, if only one of the conditions was true, the polygons were not necessarily similar.

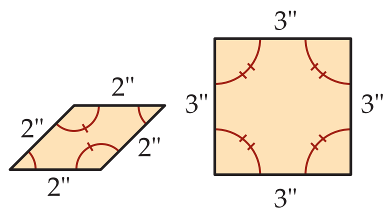
Example

In these quadrilaterals, the first similarity condition (equal angles) is true, but the second condition (proportional sides) is not true. The figures are not similar.



Example

In these quadrilaterals, the second similarity condition (proportional sides) is true, but the first condition (equal angles) is not true. The figures are not similar.

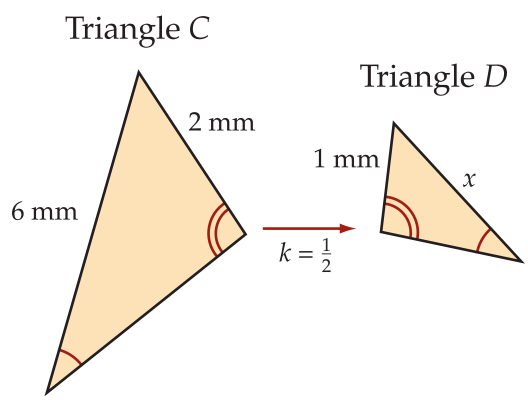


Triangles are special polygons because you do not need to check that both conditions of similarity are true. If one condition is true, the other condition is also true. In other words, two triangles that have equal angles automatically have proportional sides, and two triangles that have proportional sides automatically have equal angles.

Because the sum of the interior angles of any triangle is 180°, then triangles that have two angles that are equal in measure must also have third angles that are equal in measure.

In other words, two triangles with two angles equal in measure are similar.

If you know that two triangles are similar, you can calculate the lengths of the unknown sides using the similarity ratio. You can do this by setting up a ratio table or by using an equation.



Using Ratio Tables

One way to find side x is to use the similarity ratio *within* the triangles to set up a ratio table.

Example

×3

Triangle *C*	2	6
Triangle *D*	1	x

×3

The ratio table shows that x must be 3 mm.

Another way is to set up a ratio table using corresponding sides *between* the triangles.

Example

÷2

Side 1 in triangle *C*	6	**in triangle *D***	x
Side 2 in triangle *C*	2	**in triangle *D***	1

÷2

Once again, you can see that the result is $x = 3$ mm.

Using Equations

One way to find side x is to use the similarity ratio *between* the triangles in the equation.

Example

Because the similarity ratio of triangle D to triangle C is 1 : 2, then $1 : 2 = x : 6$.

Expressed as equivalent fractions this is $\frac{1}{2} = \frac{x}{6}$

Multiplying both sides by 6 $\frac{1}{2} \bullet 6 = \frac{x}{6} \bullet 6$

Simplifying $3 = x$

Another way is to set up a ratio table using corresponding sides *within* the triangles.

Example

Because the similarity ratio of the two known sides in triangle C is 6 : 2, then $x : 1 = 6 : 2$.

Expressed as equivalent fractions this is $\frac{x}{1} = \frac{6}{2}$

Simplifying $x = 3$

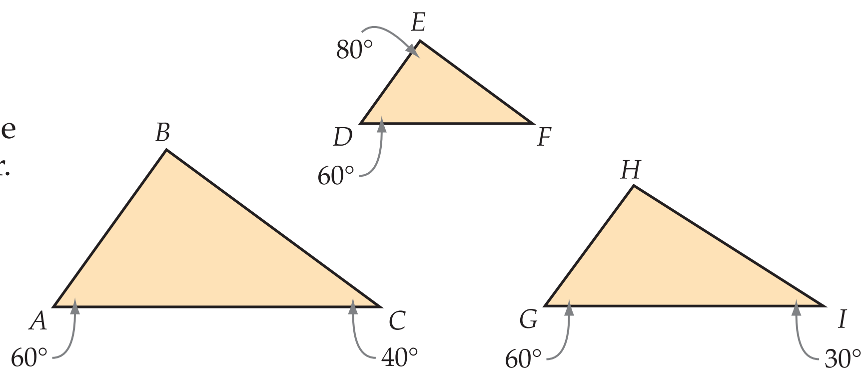
Work Time

1. One pair of triangles in this set of three triangles is similar.

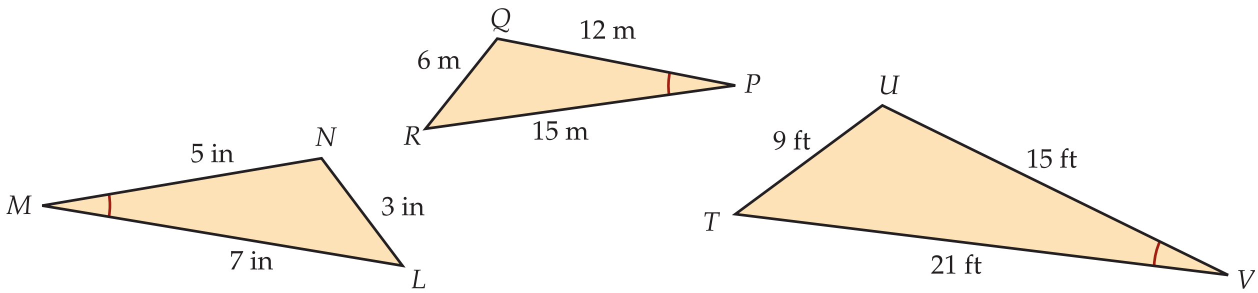
a. Identify the pair of similar triangles.

b. Say how you know these two triangles are similar.

c. Find the measure of the third angle in each of the similar triangles.



2. One pair of triangles in this set of three triangles is similar.



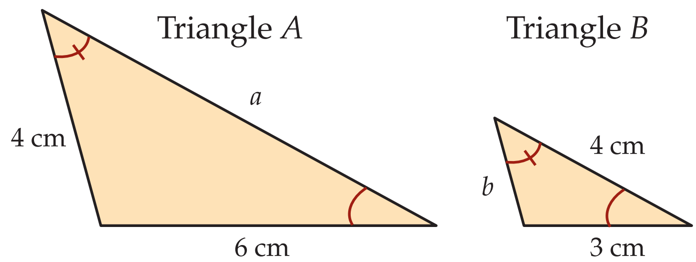
a. Identify the pair of similar triangles.

b. Say how you know they are similar.

c. Write three equal ratios in a ratio table using the three pairs of corresponding sides.

d. Use your ratio table from part c to find the scale factor between triangles *TVU* and *LMN*.

3. Triangle *A* and triangle *B* are similar. There is a similarity ratio that scales the dimensions of *A* to those of *B*.



a. Write three equal ratios in a ratio table using the three pairs of corresponding sides. Use this information to find the scale factor between the triangles.

b. Use your ratio table to help you calculate the values of *a* and *b*.

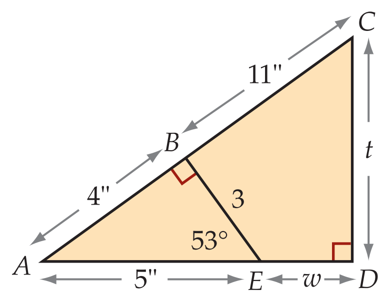
c. Calculate the values of *a* and *b* by solving an equation.

d. Explain two methods you can use to calculate the values of *a* and *b* using the similarity ratio within the triangles.

4. a. Triangles *ABE* and *ADC* below are similar. Say why.

b. Find the measure of $\angle ACD$.

c. Find the measures of side lengths *t* and *w*.

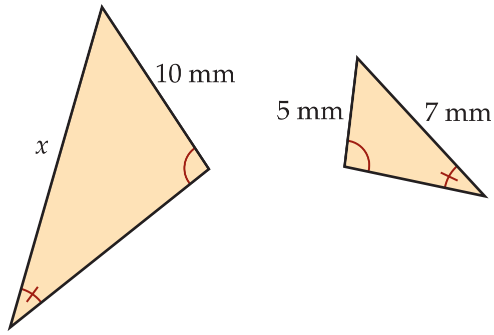


Preparing for the Closing

5. Review the Work Time problems with a partner. Correct any mistakes you made.

6. Dwayne and Chen had the same reason for concluding that the two triangles shown must be similar.

What is the reason?



7. Rosa and Lisa were asked to find the unknown length x in one of two similar triangles.

Read what each girl wrote. Explain who used ratios *within* triangles and who used ratios *between* triangles.

Rosa's Method	Lisa's Method
$x : 7 = 10 : 5$ $\frac{x}{7} = \frac{10}{5}$ $\frac{x}{7} = 2$ $\frac{x}{7} \bullet 7 = 2 \bullet 7$ $x = 14$ mm	$\frac{x}{10} = \frac{7}{5}$ $\frac{x}{10} = 1.4$ $\frac{x}{10} \bullet 10 = 1.4 \bullet 10$ $x = 14$ mm

8. An important property of ratios is: "If $a : b = c : d$, then $a : c = b : d$."

Say how comparisons *between* similar triangles and *within* similar triangles show that this property is true. Give a numerical example to support your explanation.

Skills

Solve.

a. $(-60x) + 30x =$

b. $60x - 30x =$

c. $60x + (-30x) =$

d. $25a + (-20a) =$

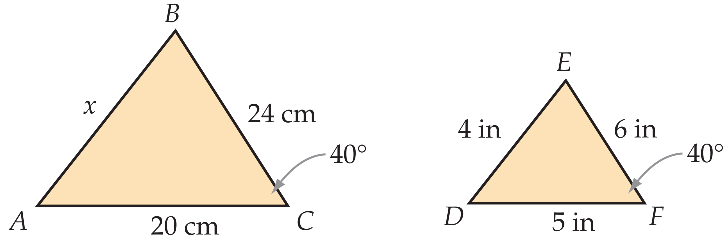
e. $(-75x) - (+25x) =$

f. $(-56.9a) - 12.5a =$

Review and Consolidation

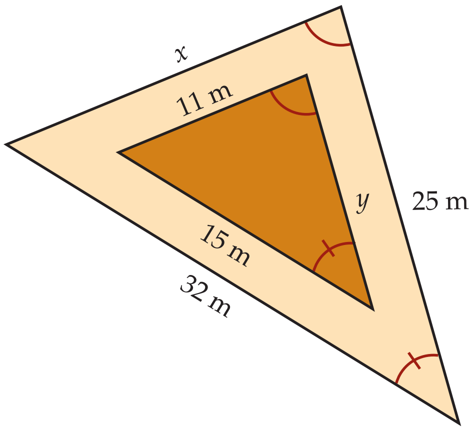
1. Triangles *ABC* and *DEF* are similar.

Find the measure of $\overline{AB}$.

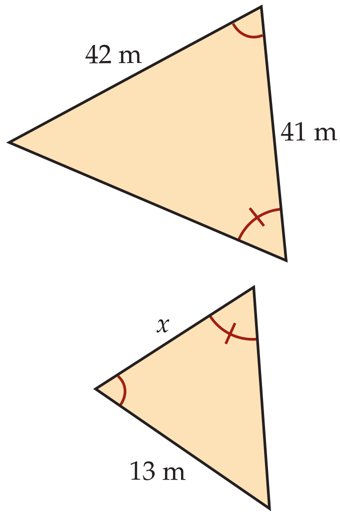


2. The two triangles in part a are similar. The two triangles in part b are similar. Find the unknown side lengths of the similar triangles using two different methods. Give answers to two decimal places where appropriate.

a.



b.

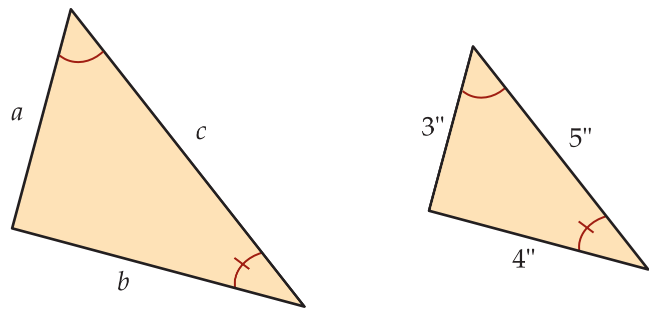


3. a. Triangle *X* is similar to triangle *Y*. Say how you know.

b. Express the similarity ratio between the triangles in three different ways.

c. Use the similarity ratio within the triangles to calculate the ratio.

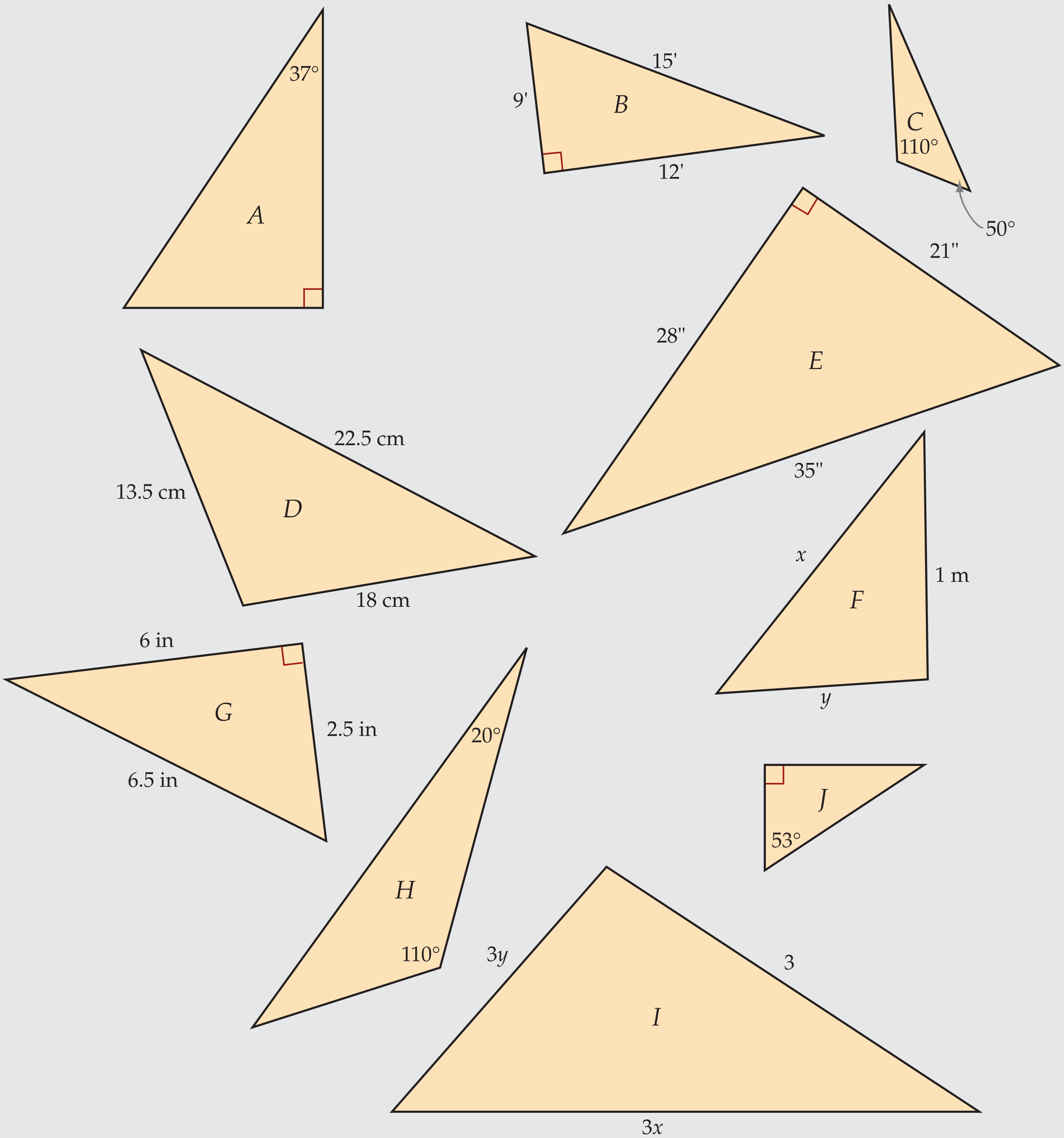
d. It is not possible to calculate the lengths *a*, *b*, or *c*. Say why.



Triangle *X* Triangle *Y*

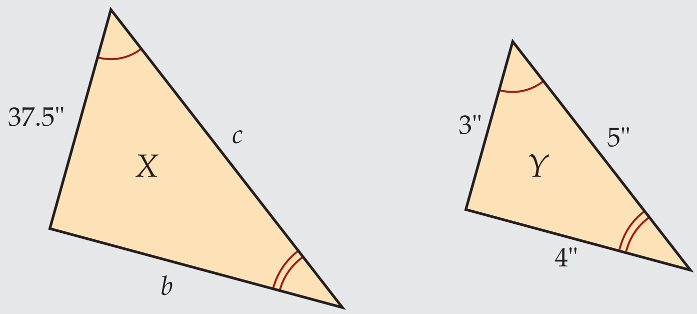
Homework

1. Here is a set of triangles. Note that they have not been drawn using an accurate scale. Find all the pairs of similar triangles.



2. Look at triangle F in problem 1. If x is 1.5 m and y is 0.8 m, then find the unknown lengths in the triangle that is similar to triangle F.

3. Triangles X and Y are similar. The ratio m scales the dimensions of X to those of Y.



a. Express the ratio m in three different ways.

b. Find the value of the ratio $\frac{b}{c}$.

c. Use a ratio table to show how to find the values of b and c.

SIMILAR TRIANGLE APPLICATIONS

LESSON **13**

CONCEPT BOOK

See pages 267–268 in your *Concept Book.*

GOAL

To use similar right triangles to solve problems of indirect measurement.

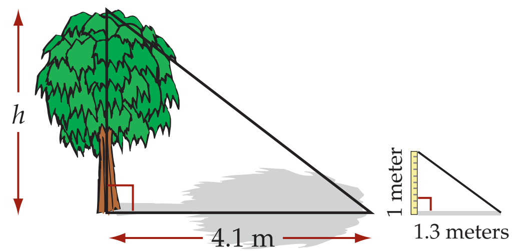
You can solve everyday, practical problems by knowing the acute angles and side lengths in a right triangle.

Height Measurements

You can use similar right triangles to calculate the height of things that are too tall to be directly measured.

Example

The height of the tree and its shadow form a right triangle. The meter stick and its shadow form a right triangle. These two figures are similar right triangles.



Use the lengths of the shadows of the tree and the meter stick to calculate the height, h, of the tree. Set up a proportional relationship to calculate h.

(The shadows must be measured on the same day and at the same time.)

Distance Measurements

You can use similar right triangles to calculate distances that are too great to measure directly.

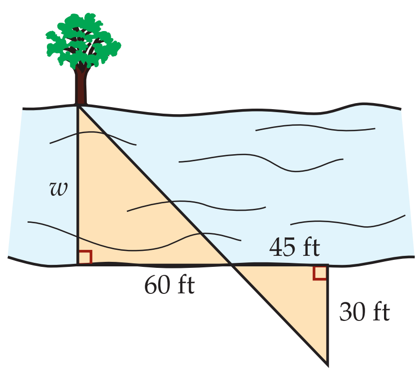
Example

You want to find the width (w) of a river. The two triangles in the figure are similar. Why?

You know: $\dfrac{30}{45} = \dfrac{w}{60}$

$$30 \bullet 60 = 45w$$
$$1800 = 45w$$
$$40 = w$$

The width of the river is 40 feet.



Putting Mathematics to Work

Work Time

Problems 1 through 4 use similar right triangles to determine unknown measurements.

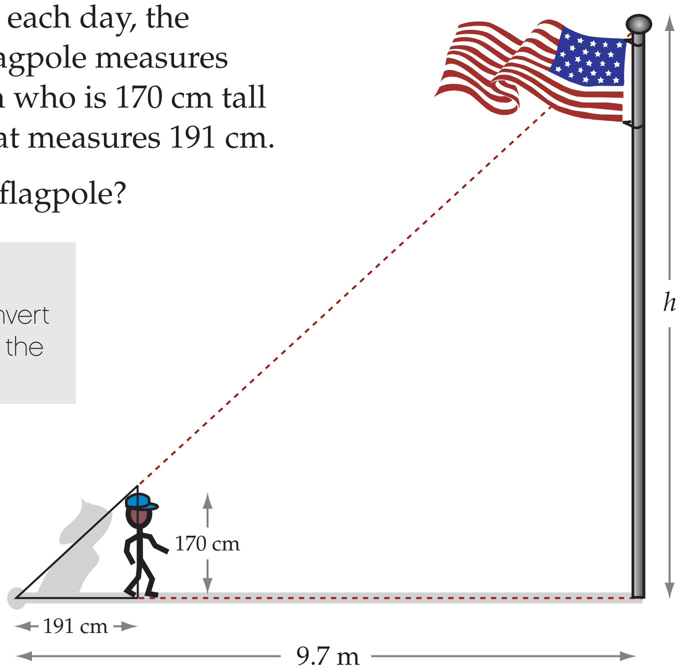
With a partner, solve each problem in two ways. One of you should use ratios *within* the triangles, and one of you should use ratios *between* the triangles. Compare answers as you work through the problems.

Imagine you and your partner are flying off for a tropical island holiday!

1. At the airport, there is a flagpole. At a certain time each day, the shadow of the flagpole measures 9.7 m, and a man who is 170 cm tall has a shadow that measures 191 cm.

How high is the flagpole?

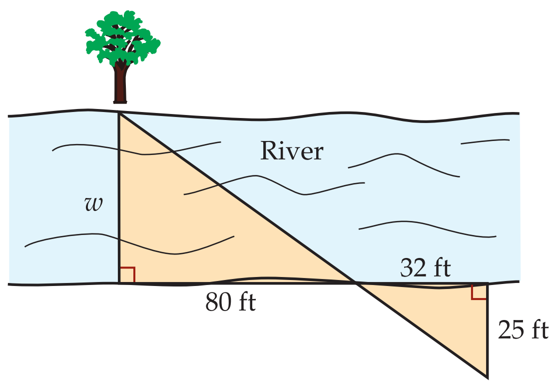
> **Comment**
> Remember to convert measurements to the same units.



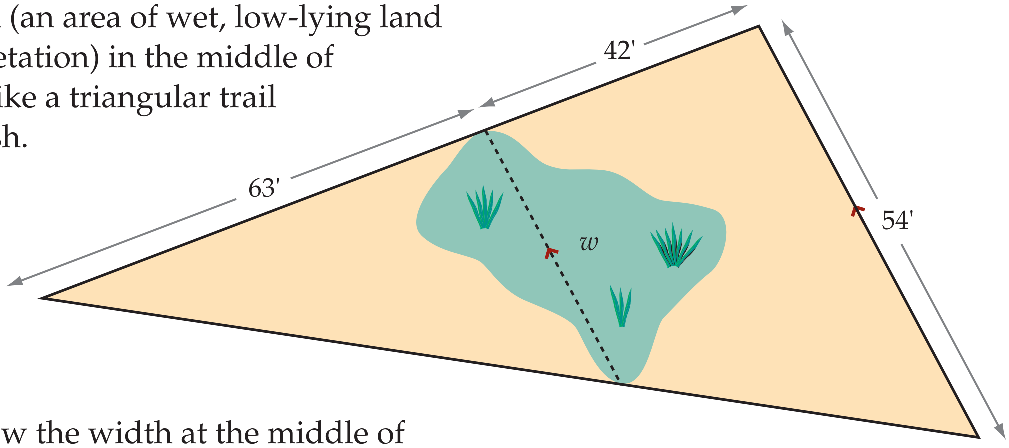
2. There is a wide river on the island that has no bridge. It is home to several alligators. You want to know how far it is across the river. You decide the safest way to measure the width of the river is to use math.

The diagram shows the measurements that you make. You know the right triangles are similar.

How wide is the river?



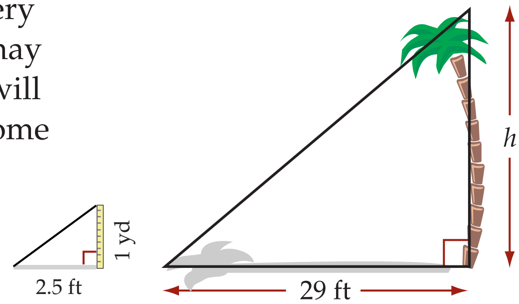
3. There is a marsh (an area of wet, low-lying land with grassy vegetation) in the middle of an island. You hike a triangular trail around the marsh.



You want to know the width at the middle of the marsh. To find this measurement, you need to calculate the length of an imaginary line parallel to one of the lengths of your hike. As you hike, you make the measurements shown in the diagram above.

How long is side *w*?

4. Directly outside your hotel window, there is a very tall palm tree. You have heard that a hurricane may be coming, and you are concerned that the tree will fall down and smash your window. You make some direct measurements to help you determine whether you are in danger.



A stick measuring 1 yard has a shadow of 2.5 ft at the same time the tree has a shadow of 29 ft. The distance between the base of the tree and the hotel is 12 yards.

Are you safe?

Preparing for the Closing

5. Compare all of your answers with those of your partner. Did you get the same answers? Say why or why not.

6. Make up two more problems about a tropical island holiday. Think of at least one problem that involves triangles other than similar right triangles. Give your problems to your partner to solve. Compare and discuss your solutions.

Skills

Simplify.

a. $(-50x) + (-10.7x)$

b. $5\frac{1}{4}x - 17\frac{3}{4}x$

c. $3x - 2a$

d. $(-60a) - 30a$

e. $(-60a) - 30$

f. $(-60a) - 30b$

Review and Consolidation

1. Rosa wants to estimate the height of a tall building. The building casts a shadow 32 m long at the same time of day that a 40-cm ruler casts a shadow 25 cm long.

Which of these pairs of equivalent ratios can be used to calculate the height (in m) of the building? Choose the correct answer.

Ⓐ $\frac{h}{3200} = \frac{40}{25}$

Ⓑ $\frac{h}{3200} = \frac{25}{40}$

Ⓒ $\frac{h}{0.4} = \frac{32}{25}$

2. Draw a labeled diagram and find the height of the building in problem 1.

3. Sketch a labeled diagram to help solve this problem.

Dwayne leans a 3-meter ladder against a wall. The ladder, the ground, and the wall form a right triangle.

The foot of the ladder is 1.2 m from the base of the wall, and the ladder reaches 2.75 m up the wall.

A 27-cm stick is also leaning against the wall and makes exactly the same angles with the ground and the wall as the ladder. The stick, the ground, and the wall form a right triangle.

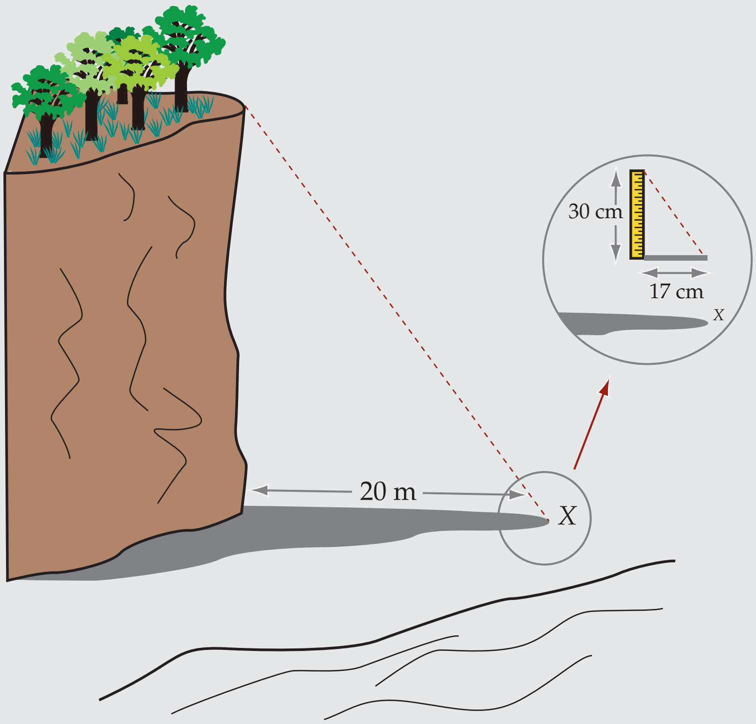
a. How far up the wall does the stick reach?

b. How far out from the base of the wall is the bottom of the stick?

Homework

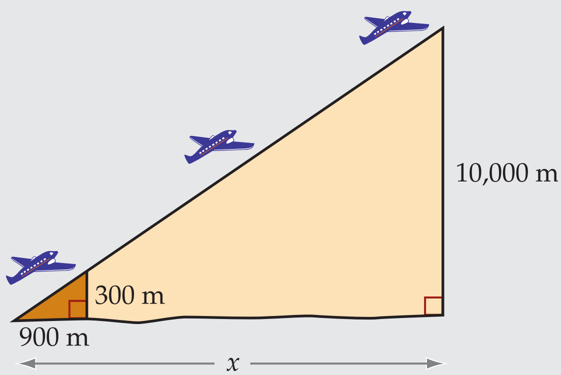
1. A hiker passes a vertical cliff, the shadow of which is 20 meters long. The hiker stops and positions a 30-cm ruler on the ground nearby, holding it vertically. The ruler casts a shadow of 17 cm.

How high is the cliff?



2. When the plane below has traveled a horizontal distance of 900 m, it is 300 m above the ground.

Determine the horizontal distance the plane will have traveled when it is 10,000 m above the ground.



LESSON 14

REVIEWING RATIO AND GEOMETRY

GOAL

To review the concepts of ratio, scale factor, similarity ratio, and similar figures.

CONCEPT BOOK

See pages 262–268 in your *Concept Book.*

Scale Factor

The *scale factor* is the ratio $\frac{a}{b}$ $(b \neq 0)$, where a is a linear measure of the original figure, and b is a linear measure of the enlarged or reduced figure.

Scale Factor	less than 1 < 1	equal to 1 $= 1$	greater than 1 > 1
	reduction (smaller)	**congruent** (same)	**enlargement** (bigger)

Conditions of Similarity in Polygons

Two or more two-dimensional objects are mathematically similar if they satisfy these two conditions:

- The measures of corresponding angles are equal.
- The ratios of the lengths of every pair of corresponding sides are equal, fixed by a scale factor (or *similarity ratio*), k.

If you know that two polygons are similar then you can calculate the lengths of unknown sides using the similarity ratio. A helpful way to do this is to use a ratio table or an equation.

Conditions of Similarity in Triangles

Triangles are special polygons. If one of the similarity conditions is true then the other condition is true.

- Two triangles with equal corresponding angles automatically have sides in proportion.
- Two triangles with corresponding sides in proportion automatically have equal corresponding angles.

You can find unknown sides in similar triangles using either the similarity ratio *between* the two triangles, or the similarity ratio *within* each triangle.

Work Time

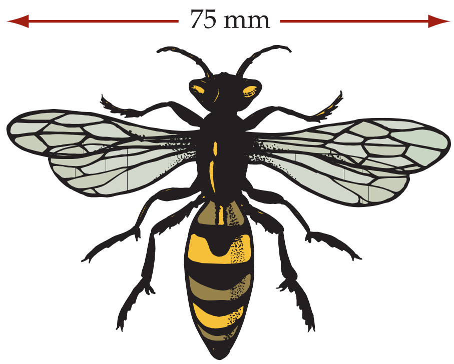
1. The scale on a map is 1 inch equals 50,000 feet.

The measured distance on the map between two towns is 2 inches.

What is the actual distance between the two towns, in miles?

Hint: There are 5280 feet in a mile.

2. Chen has a picture of a hornet that is 75 mm wide.



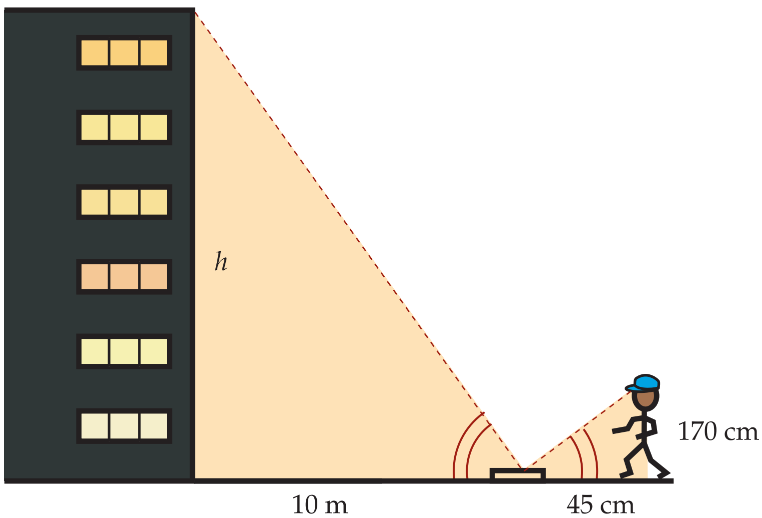
a. The average length of a hornet's wingspan is 25 mm.

What will Chen have to do to his picture to make it the size of a real hornet?

b. Chen is making a poster for his science project. He plans to increase the picture by a scale factor of 5.

What will be the width of the picture for the poster?

3. Jamal is curious to know the height of his apartment building. He decides to use similar right triangles to calculate the height of the building. Jamal places a mirror on the ground 10 m from the base of the building and backs up until he can just see the top of the building in the mirror. This happens when Jamal is a horizontal distance of 45 cm from the mirror. Jamal is 170 cm tall at eye level.



Using Jamal as one side of a right triangle, calculate the height of the building.

Preparing for the Closing

4. Discuss your solutions for the Work Time problems with another student. Try to resolve any differences and reach an agreement.

5. Be prepared to explain your ideas during the Closing.

Skills

Solve.

a. (–7) + (–7) + (–7) =

b. 3 • (–7) =

c. (–5) + (–5) + (–5) + (–5) =

d. 4 • (–5) =

e. (–23) + (–23) + (–23) =

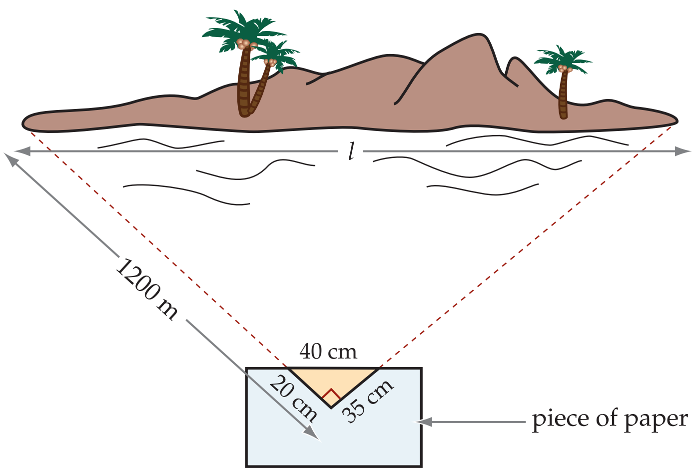
f. 3 • (–23) =

Review and Consolidation

1. From a viewpoint off the shore, the length of an island can be calculated using similar triangles.

On a piece of paper, a surveyor draws line segments of lengths 20 cm and 35 cm, joined together to form a right angle and pointed at both ends of the island. The surveyor completes the right triangle by drawing a segment parallel to the horizon. It is 40 cm.

The surveyor knows that he is 1200 meters away from the left corner of the island. How long is the island?



Homework

Assessing Your Work

1. Review your work so far in this unit, and select a piece that you believe is the best example of your understanding of the mathematics presented.

   The piece of work that you select could show:

   - Your use of ratio tables
   - Your understanding of the concepts of ratio and rate or ratio and similarity
   - Your ability to solve word problems involving proportional relationships

   Your piece of work should show more than one concept.

   When choosing your piece of work, show that you have:

   - Used the concept accurately to solve the problem
   - Represented the concept in multiple ways (numbers, symbols, diagrams, or words)
   - Explained your solution and the concept well to the reader

2. Write a brief explanation of why you chose this piece and how it demonstrates your knowledge and ability.

LESSON 15

PROGRESS CHECK

GOAL

To review ratio tables, scale factors, similarity ratios, rates, and similar figures.

CONCEPT BOOK

See pages 255–264, 291–306, 316–320 in your *Concept Book*.

Work Time

1. Copy and complete the ratio table below.

Quantity 1 x	1	3		9	12		0.25	x
Quantity 2 y		9	18	27		100		$?x$

2. A piece of wood 8 meters long is cut into 2 pieces with a ratio of 6 : 4.

What is the length of each piece of wood?

3. Keesha needs to buy peaches to make a pie. At store A, she can buy 7 peaches for $4.00. At store B, she can buy 5 peaches for $2.50.

Where will she get the better deal, at store A or store B? Why?

4. An artist wants to reduce a picture that is 85 cm wide and 35 cm high. He wants the reduced picture to be 23 cm high.

How wide will the picture be? (Round your answer to the nearest centimeter.)

5. At Monroe High School, there is a flagpole. The shadow of the flagpole at a certain time of day is 10.5 m. At the same time, Jamal's shadow is 202 cm. Jamal is 165 cm tall.

How high is the flagpole in meters? (Round your answer to two decimal places.)

Preparing for the Closing

6. Compare your answers to problems 1–5 with another student.

Did you get the same answers? Say why or why not.

7. List the mathematical concepts you need to understand to answer problems 1–5.

Skills

Solve.

a. 5 • 10 =
b. 5 • (–10) =
c. (–5) • 10 =
d. (–5) • (–10) =
e. 50 • (–125) =
f. (–50) • (–125) =

Review and Consolidation

1. For each situation below, write the comparison of boys to girls as the simplest whole number ratio, as a decimal (rounded to two decimal places), as a fraction, and as a percent (rounded to the nearest whole percent).

	Whole Number Ratio	Decimal	Fraction	Percent
14 Girls 6 Boys				
10 Girls 16 Boys				
16 Girls 10 Boys				

2. Each classroom in Lakeside High School uses 62.5 square yards of carpet.

a. How many square yards of carpet are needed for two classrooms? Three classrooms? Four classrooms?

b. Make a ratio table to represent this situation.

c. Extend your ratio table to determine the number of square yards of carpet needed for twelve classrooms. Show at least two ways you can find this answer.

d. The principal, Mr. Peterson, orders 1250 square yards of carpet. How many classrooms can be carpeted?

Homework

1. Dwayne and Chen wanted to reduce a drawing that had a width of 8 inches and a height of 11 inches so that it would fit onto a flyer they were preparing. The drawing had to be no more than 5 inches wide.

   a. First, Dwayne tried to calculate the new dimensions using a scale factor of 1.5. His result did not work. Say why.

   b. Chen tried a scale factor of 1. His result did not work. Say why.

   c. Dwayne then tried a scale factor of 0.8. Did he get useful results with this scale factor?

   d. Dwayne and Chen clearly need your help. Find the scale factor that will reduce the width of the drawing to 5 inches. Calculate what the height of the drawing will be using this scale factor.

UNIT PRICE

CONCEPT BOOK

See pages 297 and 307 in your *Concept Book.*

GOAL

To relate unit price to proportions.

You can use ratios to represent prices.

Unit price is a rate that tells you the price per unit of measure.

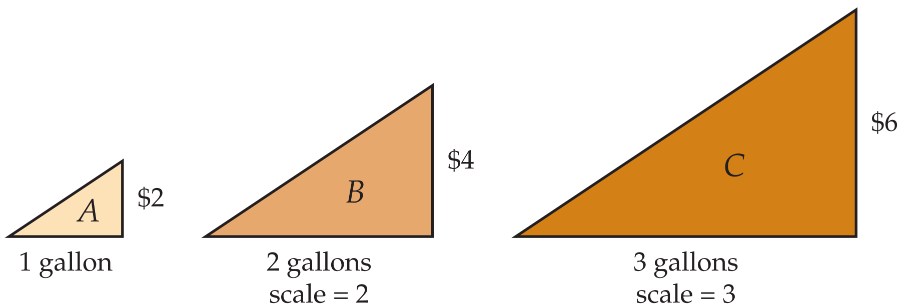
Example

If 1 gallon of gas costs \$2, then the unit price of gas is \$2 per gallon.

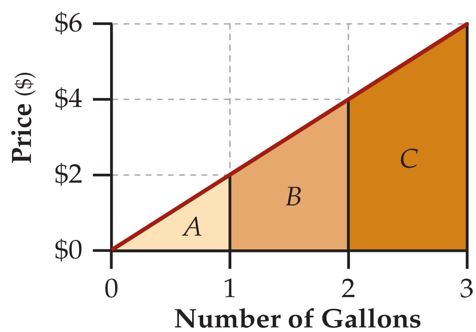
You can represent the relationship, or ratio, between cost and quantity using similar triangles.

Example

Triangle *A* represents the unit price of gas: \$2 per 1 gallon. To represent some other costs and quantities of gas, use similar triangles *B* and *C*.



You can place these triangles on top of each other with one vertex at (0, 0) on the graph. The line formed on the graph (the hypotenuse of the triangle) represents the cost of various quantities of gas.



Example

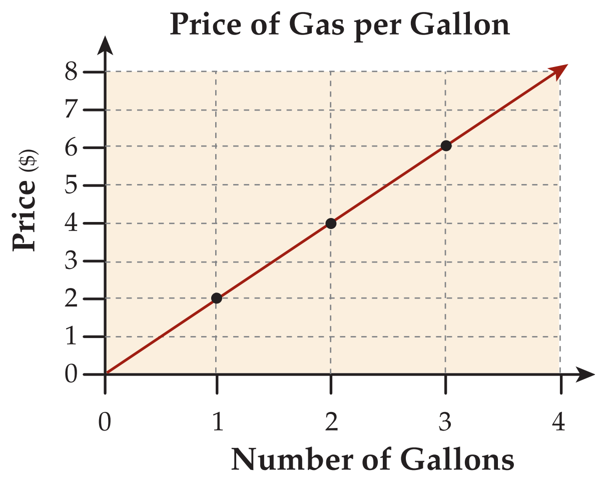
This graph shows the price of various quantities of gas.

It shows the same information as the triangles in the previous example.

Since you know the rate, or price per gallon, you can calculate the price for any amount of gas. Suppose you bought 2.29 gallons of gas. The cost would be $4.58.

Multiply the number of gallons by the rate (the price per gallon).

2.29 gal • $2 = $4.58



You can see the approximate price on the graph.

Example

Use the unit price and a ratio table to calculate the exact cost of 2.29 gallons.

From the table, you can see that the constant ratio between each pair of quantities is 2 : 1.

× 2.29

Number of Gallons	1	2	3	2.29
Price (dollars)	2	4	6	4.58

× 2.29

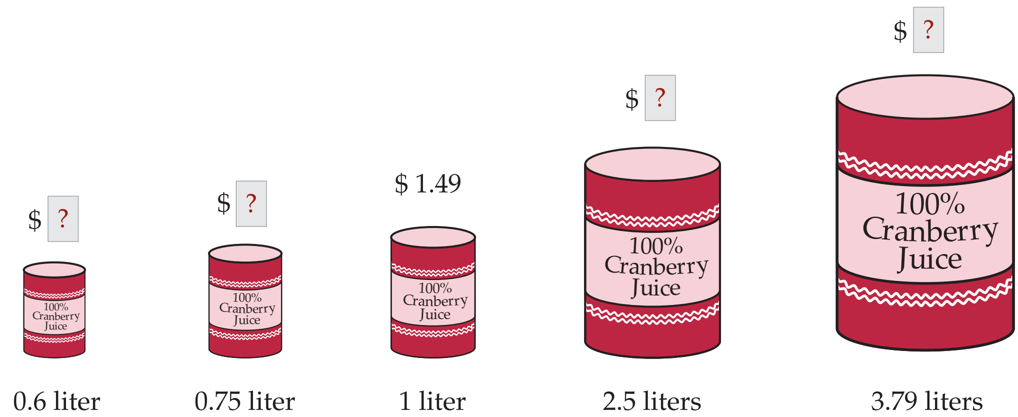
$$\frac{2}{1} = \frac{4}{2} = \frac{6}{3} = \frac{4.58}{2.29}$$

This constant ratio is called the *constant of proportionality*. In this example, the constant of proportionality is 2, which is the same as the unit ratio.

(Notice that a constant of proportionality never has a unit of measure!)

Work Time

1. The ratio of the cost of a can to its volume is the same for all the cans. The price of one can is given.



a. Write estimates for the missing prices.

b. What is the unit price per liter?

c. Using a ratio table and a calculator, find the exact price of each can.

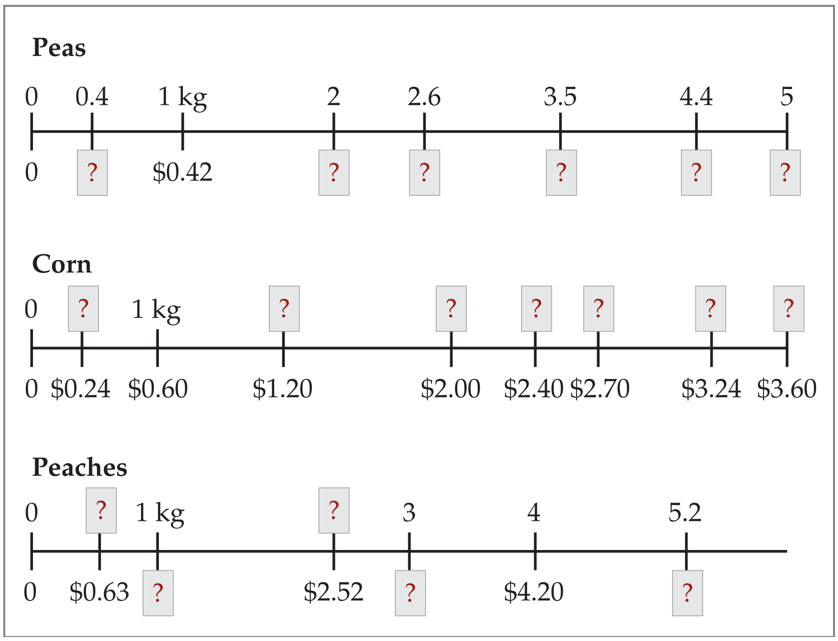
d. What is the constant of proportionality in this situation?

e. Sketch and label a set of five similar triangles to represent the proportional relationship between volume (in liters) and price (in dollars) for the cans of cranberry juice.

2. a. How would you find the missing prices and weights of peas, corn, and peaches shown in these diagrams?

Is there one method you could use for all?

b. Copy the diagrams, and fill in the missing prices and weights.



3. a. Sketch and label a graph of price (in dollars) per quantity (in kilograms) to represent the relationship for different quantities of peaches.

   b. Use your calculations from part b of problem 2 to write the unit price of peaches (in dollars per kilogram).

   c. What is the quantity of peaches you can buy for one dollar?

   d. What is the constant of proportionality between price and the number of peaches you can buy?

   e. What is the constant of proportionality between the number of peaches you can buy and the price?

Preparing for the Closing

4. Explain the relationship between the triangle representation and graph representation of price per quantity.

5. Explain the relationship between unit price and the graph representation of price per quantity.

6. a. If the unit price of peaches increases, what happens to the quantity you can buy for one dollar?

   b. If the unit price is cut in half, what happens to the quantity of peaches you can buy for one dollar?

7. The scale factor (or similarity ratio) between a pair of similar figures is also a constant of proportionality. Say why.

Skills

Solve.

a. $56 \div (-7) =$

b. $(-56) \div 7 =$

c. $(-56) \div (-7) =$

d. $(-630) \div 9 =$

e. $(-630) \div (-90) =$

f. $630 \div (-90) =$

Review and Consolidation

Problems 1 and 2 refer to this table, which shows the rates charged by the Rapid Delivery Service company. Rapid Delivery Service charges by the weight of the package and the distance it needs to travel. The company has divided the distance into different zones. (Increasing zone numbers means increasing distances.)

Zone	Economy Rate (per 1-lb package)	One-Day Rate (per 1-lb package)	Two-Day Rate (per 1-lb package)
1	\$0.50	\$0.75	\$0.50
2	\$0.50	\$1.50	\$0.75
3	\$0.75	\$2.25	\$1.00
4	\$1.25	\$3.00	\$2.00

1. The packages listed below were sent using the Economy Rate.

a. Copy and complete this table.

Weight of Package Sent to Zone 1	w	5 lb	10 lb	15 lb	20 lb
Cost	c				

b. Sketch and label a graph to represent the relationship between the weight of the packages and the cost for the Economy Rate to Zone 1.

c. Keesha's friend Zoe lives in Zone 1. Keesha sends her a package at the Economy Rate. The package weighs 3 pounds. How much will shipping the package cost Keesha?

d. Keesha sends another package to a different friend in Zone 1 using the Two-Day Rate. The cost to send the package is \$5.50. How much did the package weigh?

e. The delivery service uses a minimum weight of 1 pound. What is the cost of a 1-pound package to Zone 3 at the One-Day Rate?

f. How much will Keesha pay to send a 27-pound package to Zoe (in Zone 1) at the Two-Day Rate?

g. Keesha sends a 27-pound package to Zoe (in Zone 1) at the One-Day Rate. Compare this cost with the cost of the package sent at the Two-Day Rate.

2. Consider the table of Rapid Delivery Service charges again.

a. For which rate (Economy, One-Day, or Two-Day) does the cost increase by a constant amount for each increase in zone?

b. Sketch and label a graph to represent the relationship in part a.

c. How much would Keesha be charged to send a 12-pound package to her friend Lizzie, who lives in Zone 4, at the One-Day Rate?

Homework

1. At New York Super Market, 2 pounds of sliced turkey cost $11.50.

At Gina's Deli, $\frac{1}{4}$ pound of turkey costs $3.25.

a. What is the unit price of turkey at each store? Say how you know.

b. At which store is turkey more expensive? Say how you know.

c. Suppose New York Super Market changed the unit price of its sliced turkey to the unit price of Gina's Deli. What is the new price of 2 pounds of turkey at New York Super Market?

2. At Great Grocery, olives cost $5 per pound. 20 olives weigh 1 pound.

a. What is the unit price in terms of pounds?

b. What is the unit price in terms of individual olives?

c. What is the cost of 2.5 pounds of olives?

d. How many individual olives can be purchased for $1.00?

e. How many individual olives can be purchased for $1.25?

UNIT CONVERSION

LESSON 17

CONCEPT BOOK	GOAL
See pages 307–308 in your *Concept Book*.	To use ratios to convert units of measure, and to learn that the conversion factor is a constant of proportionality.

Suppose you made a measurement using feet as the unit, and you want to convert the measurement into miles. To convert units, you have to start with a known rate.

Example

There are 5280 feet in one mile. 5280 feet = 1 mile.

Using the inverse property of multiplication, $\frac{a}{a} = 1$, for any number $a \neq 0$, you know $\frac{1 \text{ mile}}{1 \text{ mile}} = 1$.

You can substitute 5280 feet for 1 mile to get $\frac{5280 \text{ feet}}{1 \text{ mile}} = 1$.

This is a ratio of quantities and it is a rate: 1 = 5280 feet/mile.
It is read as "5280 feet per mile."

5280 feet per one mile is a conversion factor. It is also a unit ratio.

A *conversion factor* is a unit ratio and also a constant of proportionality.

Example

How many feet are there in 5 miles?

Using a Ratio Table

There are 26,400 feet in 5 miles.

		×5 →	
Miles m	1	5	m
Feet f	5280	26,400	5280 m
		×5 →	

Using an Equation

Multiply the number of miles, 5, by the conversion factor, the number of feet per mile. Notice how the miles "cancel out" to result in 26,400 feet.

$$5 \cancel{\text{ miles}} \bullet \frac{5280 \text{ feet}}{\cancel{\text{mile}}} = 26{,}400 \text{ feet}$$

Converting Money

Example

Suppose you travel to Europe, where euros are used instead of dollars. When you exchange $20 at the bank, you get 16 euros.

Use a ratio table to find the *conversion rate* (the exchange rate) for euros to dollars. Write unit ratios.

÷ 20 ÷ 0.8

Euros	16	0.8	1
Dollars	20	1	1.25

÷ 20 ÷ 0.8

There are 1.25 dollars per euro.

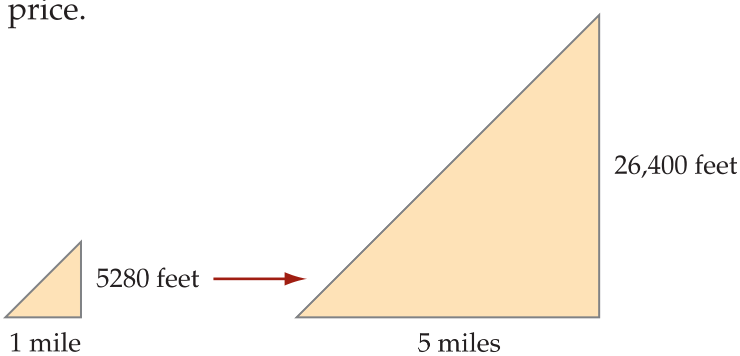
The exchange rate for euros to dollars is $\frac{\$1.25}{\text{euro}}$.

There are 0.8 euros per dollar.

The exchange rate for dollars to euros is $\frac{0.8 \text{ euros}}{\text{dollar}}$.

Example

You can represent unit conversion using similar right triangles, just as you did with unit price.



Work Time

1. Suppose a bank will exchange $15 for 165 yen.

a. Make a ratio table. Calculate the conversion rates for changing from dollars to yen and from yen to dollars.

b. Use your ratio table to find the value of 100 yen in dollars.

Comment

In Japan, they use yen for money.

2. Rosa's uncle works with an electrician. They work 8 hours every weekday (Monday through Friday). After working 14 days, her uncle gets paid $1120.

a. Use a ratio table to calculate her uncle's hourly pay rate.

b. Calculate his hourly pay rate using an equation with units. Show your calculations.

c. Use your result from part b to calculate how much Rosa's uncle would earn for 5 work days.

3. Chen can run 200 yards in 1 minute. At this rate, how many *feet* can he cover in 10.5 seconds?

Preparing for the Closing

4. a. Define "unit of measure" in your own words. Explain how the word "unit" can be used in a different way.

b. Define "conversion rate" in your own words.

5. When you multiply a quantity by a conversion factor, the value of the quantity does not change. Say why.

6. Explain two ways of knowing which conversion rate to use when converting from one unit to another.

7. A conversion factor is a constant of proportionality between two quantities with different units. Say why.

Skills

Solve.

a. 5 • 10 • (–5) =

b. 5 • 10 • (–5) • (–5) =

c. 5 • 10 • (–5) • (–5) • (–5) =

d. (–8) • (–9) • 10 =

e. (–8) • (–9) • 10 • (–1) =

f. (–8) • (–9) • 10 • (–1) • (–2) =

Review and Consolidation

1. Driving at a constant speed, Mrs. Tanaka travels 3080 feet on the freeway in 30 seconds.

a. At this rate, how many feet would Mrs. Tanaka travel in 60 seconds?

b. At this rate, how many feet would Mrs. Tanaka travel in 75 seconds?

2. In this problem, "USD" means *American dollars*, and "AUD" means *Australian dollars*.

a. Suppose it costs USD 0.72 to buy AUD 1.
What is the conversion rate for Australian dollars to American dollars?

b. How many American dollars can be bought with AUD 50?

c. What is the conversion rate for American dollars to Australian dollars?

d. How many Australian dollars can be bought with USD 50?

3. Ameera's car uses 19.6 gallons of gas to travel 382.2 miles.

a. At what rate, in gallons per mile, does Ameera's car use gasoline?

b. How much gas would Ameera use to drive 235 miles?
(Round your answer to two decimal places.)

Homework

1. A recipe for 12 muffins calls for 2 cups of flour, 2 eggs, and $\frac{1}{3}$ cup of milk.
Dwayne has 3 cups of flour, 4 eggs, and 3 cups of milk.

Recipe for 12 Muffins
2 cups flour 2 eggs $\frac{1}{3}$ cup of milk

Dwayne has:
3 cups flour 4 eggs 3 cups of milk

a. What is the greatest number of muffins he can make?

b. If he wants to bake 42 muffins, what does Dwayne need to buy at the store?

UNIT ANALYSIS

LESSON
18

CONCEPT BOOK	GOAL
See pages 287–289 in your *Concept Book.*	To solve problems using unit analysis.

Suppose that an automobile travels 120 miles on 6 gallons of gasoline. You can write two rates to represent this situation.

$$\frac{120}{6}\ \frac{\textit{miles}}{\textit{gallon}} \quad \text{or} \quad \frac{6}{120}\ \frac{\textit{gallons}}{\textit{mile}}$$

Recall that the unit of a rate quantity is a ratio of the units of the quantities that are divided. You write "miles per gallon" (not "miles per gallons") and "gallons per mile" (not "gallons per miles").

Suppose you want to use the rate of gas consumption above to calculate the number of miles you could travel on a tank of 30 gallons.

You can use the method called "unit analysis" to solve this problem. Using unit analysis, you focus on the units involved.

1. **Write the units of the given quantities.**

   The given units are $\frac{\textit{miles}}{\textit{gallon}}$ *and gallons.*

2. **Write the unit of the quantity you need to calculate.**

   The answer to the problem has the unit miles.

3. **Decide on the operation needed to obtain the required unit from the given units.**

   The unit "miles" is obtained by multiplying the unit "miles per gallon" by the unit "gallons."

   $$\frac{\textit{miles}}{\cancel{\textit{gallon}}} \bullet \cancel{\textit{gallons}} = \textit{miles}$$

Comment

The units **gallon** and **gallons** are cancelled as though they were fractions with digits, just as you do when converting units. (It does not matter that one word is singular and one is plural—as long as the units are counting the same quantity, they can be cancelled.)

4. **Repeat the operation(s) using the numbers given in the problem.**

   $$\frac{120}{6} \bullet 30 = \frac{20}{1} \bullet 30 = 600 \text{ (miles)}$$

Example

Jamal is preparing to paint his bedroom. A 4-liter can of paint costs $55. An area of 16 square meters can be painted with one liter of this paint. Jamal wants to know the cost per square meter of painting his room.

Using unit analysis to solve the problem:

1. The given rates are "the unit cost of the paint" and "the area of wall covered per liter of paint."

   The units of these rates are $\frac{\$}{\textit{liter}}$ and $\frac{\textit{square meters}}{\textit{liter}}$.

2. Jamal needs to find the rate that has the unit $\frac{\$}{\textit{square meter}}$.

3. To get the required unit you need to divide the given units, because the unit *square meter* must be in the denominator.

$$\frac{\left(\frac{\$}{\textit{liter}}\right)}{\left(\frac{\textit{square meters}}{\textit{liter}}\right)} = \frac{\$}{\cancel{\textit{liters}}} \bullet \frac{\cancel{\textit{liters}}}{\textit{square meter}} = \frac{\$}{\textit{square meter}}$$

4. Repeat the calculation using the given numbers:

$$\frac{55}{4} \bullet \frac{1}{16} = 0.86 \frac{\$}{\textit{square meter}}$$

It costs Jamal 86 cents for every square meter of wall he paints.

Work Time

1. Read through the problem below. Starting with the three quantities *balloons*, *breaths*, and *minutes*, write the units for all the rate quantities that are involved in this situation.

> Lisa is blowing up balloons for her birthday party. She wants to inflate 30 balloons. After ten minutes she has inflated 12 balloons. Lisa used 20 breaths to inflate each balloon.

2. What unit results from each multiplication?

a. $\dfrac{miles}{hour} \bullet hours$

b. $\dfrac{miles}{hour} \bullet \dfrac{gallons}{mile}$

c. $\dfrac{\$}{day} \bullet \dfrac{days}{year} \bullet years$

3. A drop of oil forms a circle of radius 2 centimeters on the surface of a pool of water.

As the oil spreads out, the radius of the circle increases at the rate of 0.7 centimeters per second. The area of the circle is increasing 12.6 square centimeters for every centimeter of increase in the radius.

Use unit analysis to determine the rate at which the area of the circle is increasing in square centimeters per second.

4. Rosa, Dwayne, and Chen are going to take a ride in a hot air balloon, but first they do some research about hot air ballooning. This is what they found out.

A hot air balloon has an envelope (the balloon part), a burner, and a basket for people to ride in.

A powerful fan is used to inflate the balloon with cold air. Then the burner is used to heat the air and make the balloon stand up. Warmer air is less dense than cooler air, so the balloon becomes buoyant and lifts off the ground.

Here are some facts:

- The pilot expects the balloon to reach a maximum vertical height of about 2000 ft in 10 minutes.
- The air outside gets colder by about 3.5°F for each 1000 feet that the balloon rises.
- The radius of the balloon at maximum inflation is 20 feet.
- The balloon is fully inflated with cold air in 15 minutes.
- Heating the air prior to lift off takes 2 minutes.
- Propane (fuel used to heat the air before and during the flight) is carried in 3 tanks that hold 20 gallons each.

- Average fuel consumption is about 15 gallons an hour.
- When fully inflated, the volume of the envelope is 100,000 cubic feet.
- Each cubic foot of hot air can lift about 0.015 pound.
- Rosa weighs 110 pounds, Dwayne weighs 143 pounds, Chen weighs 154 pounds, and the pilot weighs 187 pounds.
- The envelope weighs 250 pounds, the basket weighs 140 pounds, and the burner weighs 50 pounds. One propane tank weighs 135 pounds; so the 3 propane tanks weigh 405 pounds total.

a. Write the unit(s) for the rate at which the volume of the envelope increases during inflation.

b. Write the unit(s) for the rate at which the radius increases during inflation.

c. Write the unit(s) for the weight that can be lifted for each cubic foot of hot air.

d. Write the unit(s) for the average rate of fuel consumption during the flight.

e. Write the unit(s) for the rate at which the temperature decreases as the balloon rises.

f. Write the unit(s) for the speed at which the balloon rises to its maximum height.

g. Use unit analysis to calculate the time that the balloon has been inflating when the radius of the balloon is 6 feet.

h. Use unit analysis to calculate the time (to the nearest minute) that the balloon has been flying (not including inflation time) when 2.8 gallons of propane have been used.

i. Chen is a little concerned that the fully-inflated balloon, with all four of them in it, will not be able to lift off. Use unit analysis to find the answer for Chen.

Preparing for the Closing

5. Sometimes it is necessary to multiply units to obtain the unit required to answer a problem, and sometimes it is necessary to divide the units.

Explain how you know when it is appropriate to multiply or divide, and give an example of each.

6. You can complete unit conversion problems using unit analysis. Say why, and give an example.

Skills

Solve.

a. $125 \div 5 \bullet 2 =$ b. $125 \div (-5) \bullet 2 =$ c. $(-125) \div (-5) \bullet 2 =$ d. $(-125) \div (-5) \bullet -2 =$

e. $130 \bullet 4 \div 10 =$ f. $-130 \bullet 4 \div 10 =$ g. $-130 \bullet -4 \div 10 =$ h. $-130 \bullet -4 \div -10 =$

Review and Consolidation

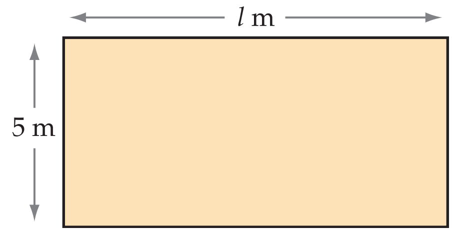
1. Write the operation that you would need to achieve the unit $\frac{gallons}{mile}$ beginning with these given units.

a. *gallons* and *miles*

b. $\frac{miles}{gallon}$

c. $\frac{gallons}{hour}$ and $\frac{miles}{hour}$

2. This rectangle has a a width of 5 meters and a length, *l*, also in meters.



Use unit analysis to calculate the rate (in square meters per minute) at which the area is increasing if the length is increasing at the rate of 0.5 meters per minute.

Homework

1. What unit results from multiplying these units?

a. $\frac{cm}{sec} \bullet seconds$ b. $\frac{pounds}{gallon} \bullet \frac{gallons}{day}$ c. $\frac{cm}{sec} \bullet \frac{seconds}{day} \bullet \frac{m}{cm}$

2. One of the unit calculations in problem 1 involves a unit conversion. Say which one, and explain how you know.

3. Complete the operations with the given units to find the required unit.

a. Given: $\frac{liters}{person}$ and $\frac{people}{day}$

Required: $\frac{liters}{day}$

b. Given: $\frac{gallons}{\$}$ and $\frac{gallons}{mile}$

Required: $\frac{\$}{mile}$

4. Complete these problems using the unit analysis from problem 3 part a.

a. The organizers of a two-day rock festival want to provide enough water for all the people attending the festival. They expect a total of 12,500 people to attend each day of the festival over the two days. They estimate that each person will drink 2 liters of water each day.

How many liter bottles of water do they expect to distribute per day?

b. The organizers can buy a 12,000 liter tank of water for $8000. How much will they need to spend per person (per day) on water?

5. Complete this problem using the unit analysis from problem 3 part b.

On a recent drive, Mr. Tanaka noticed that he used 5 gallons of gasoline to travel 110 miles. Before he set out, he paid $20.40 for 12 gallons of gasoline.

What was the cost of gas for every mile that was traveled?

REPRESENTING PROPORTIONAL RELATIONSHIPS

LESSON **19**

CONCEPT BOOK	GOAL
See pages 315–319 in your *Concept Book*.	To learn how proportional relationships can be represented in a table, a formula, and a graph.

When the ratio of two quantities is constant, the quantities are proportional.

You can represent a *proportional relationship* as a ratio table.

Example

Suppose the unit price for tomatoes is \$0.80 per pound, or 0.8 : 1.

Cost (dollars)	c	0.80	1.60	2.40	3.20	4.00
Weight (pounds)	w	1	2	3	4	5

The constant ratio of cost to weight is shown by dividing any value of c by its corresponding w value.

$$\frac{c}{w} = \frac{0.80}{1} = \frac{1.60}{2} = \frac{2.40}{3} = \frac{3.20}{4} = \frac{4.00}{5}$$

The constant of proportionality is 0.80.

Suppose you want to know the cost of any number of pounds of tomatoes, w? The ratio table shows that the cost is $0.8w$.

$\times\, w$

Cost (dollars)	c	0.80	1.60	2.40	3.20	4.00	$0.8w$
Weight (pounds)	w	1	2	3	4	5	w

$\times\, w$

You can use the constant of proportionality to write a formula to show the proportional relationship between cost and weight: $c = 0.8w$.

Example

This formula represents the cost in terms of the weight. You can use it to find the cost for any weight of tomatoes.

price per pound

$$c = 0.8w$$

total cost of tomatoes weight of tomatoes

What is c when w is 200? (What is the cost of 200 pounds of tomatoes?)

$$c = 0.8 \bullet 200 = 160$$

The cost of 200 pounds of tomatoes is $160.

Example

$c = 0.8w$

You can also use this formula to solve inverse problems.

What is w when c is 50? (How many pounds of tomatoes can you buy for $50?)

$$50 = 0.8w$$

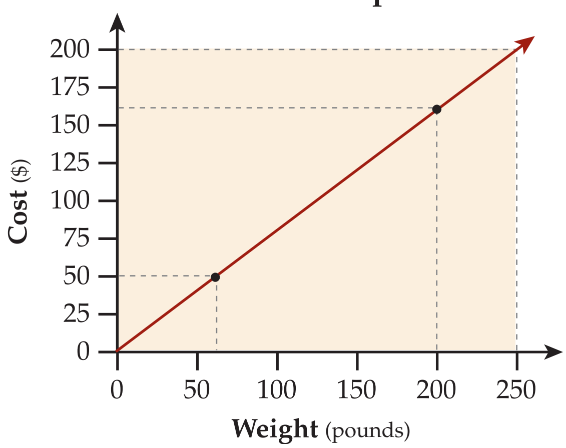
$$\frac{50}{0.8} = \frac{0.8\,w}{0.8}$$

$$w = 62.5$$

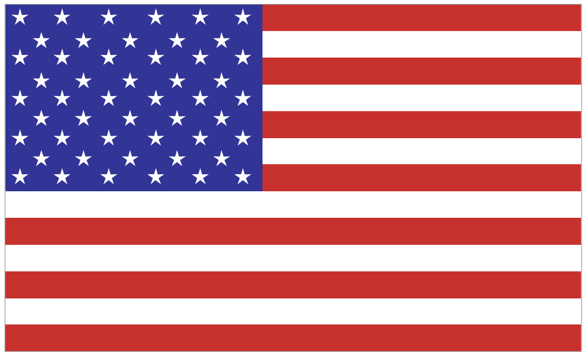
For $50, you can buy 62.5 pounds of tomatoes.

You can show this relationship on a graph of cost (in dollars) and weight (in pounds).

Cost of Tomatoes per Pound



Work Time



Dwayne's aunt works for a company called U.S. Flags Are US. The company manufactures and sells flags. Dwayne's aunt prepares all of the stars.

Sometimes, U.S. Flags Are US receives an order for a large quantity of flags. Dwayne's aunt wants to determine a rule so she knows how many stars to prepare.

Comment

There are 50 stars on the U.S. flag.

1. Dwayne's aunt begins by starting a ratio table showing the unit ratio for the two quantities—number of flags and number of stars.

   Make up three more corresponding pairs of values for the two quantities (number of flags and number of stars) that she could include in this table. Complete the table.

Number of Flags	f	1				f
Number of Stars	s	50				

2. Use your ratio table to find the constant of proportionality in the relationship between the two quantities. Explain how you found it.

3. Use the ratio table to calculate the number of stars Dwayne's aunt will need in order to make f flags.

4. Let f stand for the number of flags and s stand for the number of stars. Use the constant of proportionality to write a formula that represents the relationship between f and s (s = ?).

5. Sketch a graph of the relationship between the number of stars, s, and the number of flags, f.

   Hint: Mark the horizontal axis for 0 to 10 flags. Your graph will be easier to read if you make the length of a unit on the vertical axis smaller than on the horizontal axis.

6. Use your formula to find the number of stars Dwayne's aunt needs in order to make 206 U.S. flags.

7. Use your formula to find the number of flags Dwayne's aunt can make if she has prepared 350 stars.

Preparing for the Closing

8. Look at the graph you made for problem 5. Your graph should be separate points that are not joined by a line. Say why.

9. Explain the role of the constant of proportionality in the formula that you found in problem 4.

10. Look at your answer for problem 6. You could have solved this problem by using a ratio table, but you could not have solved this problem by using your graph. Say why.

11. Look at your answer for problem 7. You could have solved this problem using a graph or a ratio table. Explain how you could use each of these methods.

Skills

Solve.

a. 6.66 • (–2) ÷ 2 =

b. (–78.4) ÷ 2 • (–3) =

c. (–25) • 305 ÷ 5 =

d. (–15) ÷ (–300) • (–600) =

Review and Consolidation

Dwayne's aunt now thinks about the relationship between the number of stripes (S) she will need in order to make a certain number of U.S. flags (f). The U.S. flag has 13 stripes.

1. Name the quantities that are proportional to each other.

2. Write the unit ratio for the number of stripes to the number of flags.

3. Express your answer to problem 2 as a rate.

4. What is the constant of proportionality in this relationship?

5. Write a formula for the number of stripes, S, in terms of the number of flags, f.

6. Sketch a graph of the number of stripes and the number of flags. Put the number of stripes on the vertical axis and mark the number of flags from 0 to 10 on the horizontal axis.

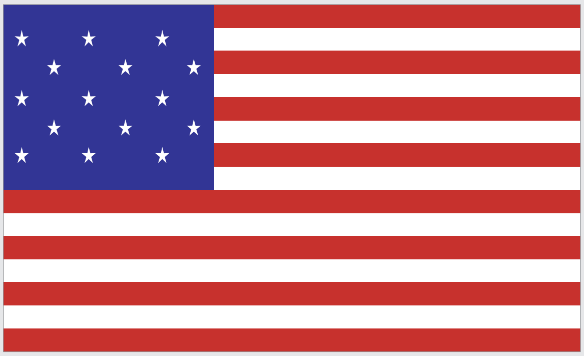
7. If Dwayne's aunt needs to make 50 flags, how many stripes does she need? Show how you calculated your answer.

8. How many flags can Dwayne's aunt make if she has prepared 585 stripes? Show how you calculated your answer.

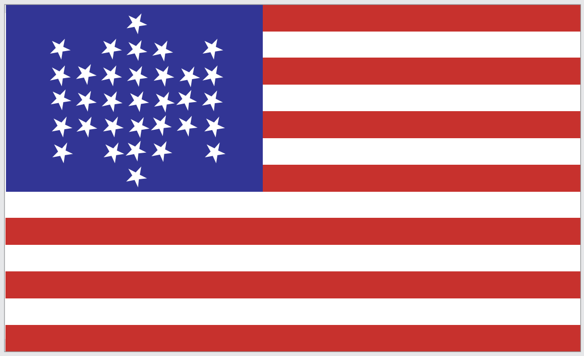
Homework

1. The flags below are the "Star Spangled Banner" and the "Fort Sumter Flag."

Star Spangled Banner



Fort Sumter Flag



Write each of the following ratios as simplest whole number ratios:

a. Stars to stripes on the Star Spangled Banner

b. Stars to stripes on the Fort Sumter Flag

c. Stars to stripes on the current U.S. flag

d. Stars on the Star Spangled Banner to stars on the Fort Sumter Flag

e. Stars on the Star Spangled Banner to stars on the Fort Sumter Flag to stars on the current U.S. flag

2. For the Star Spangled Banner:

a. Write a formula for the number of stars in terms of the number of flags (in the form of $s = ?f$).

b. Write a formula for the number of stripes in terms of the number of flags (in the form of $S = ?f$).

3. Do the same for the Fort Sumter Flag.

a. Write a formula for the number of stars in terms of the number of flags.

b. Write a formula for the number of stripes in terms of the number of flags.

LESSON 20

IDENTIFYING PROPORTIONAL RELATIONSHIPS

GOAL	CONCEPT BOOK
To identify proportional relationships.	See pages 315–319 in your *Concept Book.*

You have learned that one quantity is proportional to another if the two quantities have a constant ratio. The quantities are in a *proportional relationship*.

Example

Suppose a certain brand of cheese costs $5 per kilogram.

This ratio table shows that the price of cheese, p, is in proportion to the number of kilograms (n) that you buy.

Price (dollars)	p	5	10	15	$5w$
Weight of Cheese (kg)	w	1	2	3	w

You can verify that this constant ratio $p : w$ is $5 : 1$ or 5. Divide each value of p by the corresponding value of w:

$$\frac{10}{2} = \frac{15}{3} = \frac{5}{1} = 5$$

You can use the constant ratio, 5, (the constant of proportionality) to write a formula:

$$p = 5w$$

price of cheese constant of proportionality

This formula shows that p is a *constant multiple* of w. This means that no matter which values of w you use, all values of p are multiples of 5.

In summary, you know that y is proportional to x if there is some constant of proportionality, k, such that the following statements are true:

constant of proportionality

$$\frac{y}{x} = k \text{ and } y = kx$$

Here are three good questions to ask when you have a relationship between two quantities, such as x and y:

- Is y proportional to x?
- How do you know that y is proportional to x?
- If y is proportional to x, what is the constant of proportionality, k?

During Work Time, you will practice asking these questions.

Work Time

1. Each table below is a ratio table because there is a constant of proportionality, k, between each pair of values.

For each table, calculate the constant of proportionality using $\frac{y}{x} = k$. (You may want to use a calculator.)

a.

x	y
3	5.25
7	12.25
12	21
18	31.5
25	43.75

b.

x	y
3	2.4
7	5.6
12	9.6
18	14.4
25	20

c. Write a formula in the form of $y = kx$ for parts a and b.

2. Two of these tables of values are ratio tables and two are not.

A

x	1	3	6		15
y	1.06	1.2	1.8	2.4	3.0

B

x	0.5	1.0	1.5	2.0	2.5
y	13	26	39		65

C

x	1	2	3	4	5
y	3.5	7	10.5		16.5

D

x	2	5	7	9	
y	4.4	11	15.4	19.8	26.4

a. Which two tables are ratio tables?
(Meaning that they represent a proportional relationship.)

b. Write a formula in the form of $y = kx$ for the two proportional relationships.

c. Use your formulas to find the missing values in the tables that represent proportional relationships.

3. Use the grids on Handout 3: *Graphing Proportional Relationships* to draw labeled graphs that represent the four relationships in problem 2.

Preparing for the Closing

4. Say how you can determine whether two quantities are proportional to each other. Use your results from Work Time to support your answer.

5. A relationship can have two variables and still not be proportional. Say why.

6. Look again at the graphs you drew in problem 3. What do you observe when you compare the graphs that represent proportional relationships to the graphs that represent non-proportional relationships?

7. In proportional relationships, when one variable equals zero, the other variable must also equal zero. Say why, using what you know about the tables, formulas, and graphs of proportional relationships to support your answer.

8. Sometimes, a proportional relationship has meaning for only some values. Say why, and give an example to support your reasoning.

Skills

Complete these statements.

a. Adding 12 gives the same result as subtracting ________.

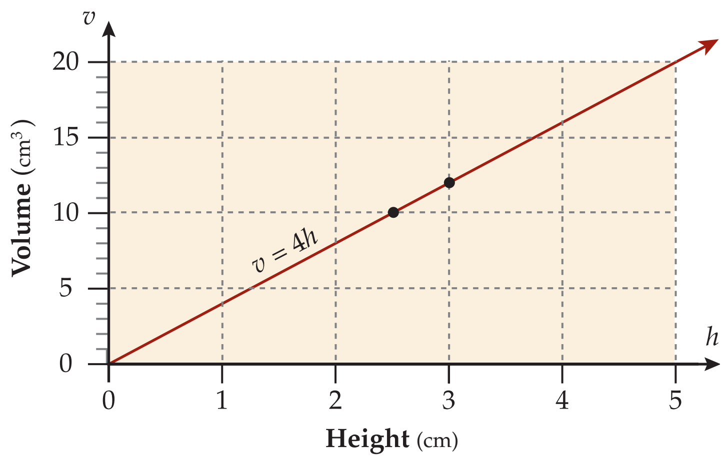
b. Subtracting 15 gives the same result as adding ________.

c. A negative number multiplied by a negative number equals ________.

d. A positive number divided by a negative number equals ________.

Review and Consolidation

1. The graph below represents the relationship between the volume of sand in a container and the height of the sand.



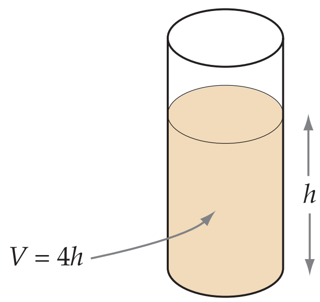
a. What does the point (3, 12) on the graph represent?

b. What does the point (2.5, 10) on the graph represent?

c. In this situation, the volume of sand is proportional to the height of the sand. Say why.

d. What is the constant of proportionality, and what does it represent in this situation?

e. Suppose you adjust the graph so that the total volume of the container is 100 cm^3. What would be the value for h?



CONCEPT BOOK

See pages 252–253 for more information on the volume of a cylinder.

Homework

1. The owners of Nifty Mart advertised that they would not change their unit price of gas for one week.

   Keesha wanted to check that they kept their promise, so she recorded some of the volumes of gas bought and the prices paid by five customers at Nifty Mart over five days. Unfortunately, she lost some of the information!

   Here is the data she recorded.

Day of the Week	Volume of Gas (gallons)	Price ($)
Monday	58	127.60
Tuesday	7	15.40
Wednesday	11.4	
Thursday		41.80
Friday		33.00

   a. Does it seem like the owners of Nifty Mart have kept their promise? Say how you know.

   b. Assuming the owners kept their promise, find the missing values in the table for Wednesday, Thursday, and Friday.

   c. Write a formula that will allow Keesha to calculate the price (p) of any number of gallons of gas (n) at Nifty Mart for this particular week.

   d. Sketch a graph of the relationship between price and number of gallons.

GRAPHING PROPORTIONAL RELATIONSHIPS

LESSON **21**

CONCEPT BOOK	GOAL
See pages 318–319 in your *Concept Book.*	To interpret graphs of proportional relationships.

One quantity is proportional to another if the two quantities have a constant ratio. The quantities are in a *proportional relationship*. Because the quantities can have different values, you can call them *variables*.

You know that y is proportional to x if there exists some constant of proportionality, k, such that the following statements are true:

$$\frac{y}{x} = k \text{ and } y = \overset{\text{constant of proportionality}}{\overset{\downarrow}{k}}x$$

Many problems involve situations in which one quantity is proportional to another. The situations may seem different, but they all use formulas such as $y = kx$, although the letters used to represent the quantities may be different and the constant of proportionality, k, may be replaced by a given number.

Example

Suppose the unit price for tomatoes at Fruit World is \$0.60 per pound. This relationship is shown on the ratio table as the unit ratio 0.6 : 1.

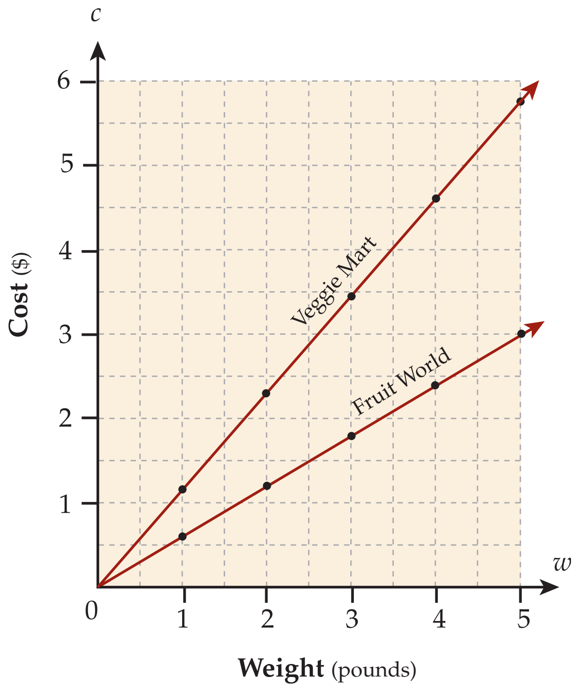
Weight (pounds) w	1	2	3	4	5
Cost (dollars) c	0.60	1.20	1.80	2.40	3.00

At Veggie Mart, the unit price for tomatoes is \$1.15 per pound. The unit ratio is 1.15 : 1.

Weight (pounds) w	1	2	3	4	5
Cost (dollars) c	1.15	2.30	3.45	4.60	5.75

The formula for the cost of tomatoes at Fruit World is $c = 0.6w$.
The formula for the cost of tomatoes at Veggie Mart is $c = 1.15w$.

This graph represents both of these proportional relationships:

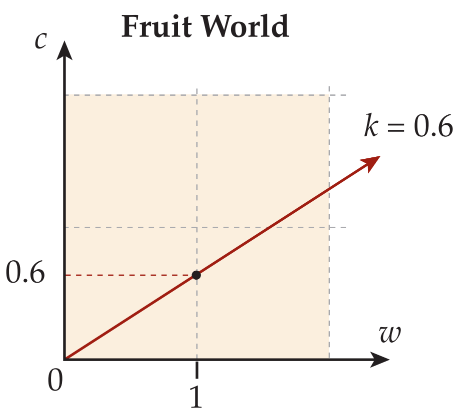


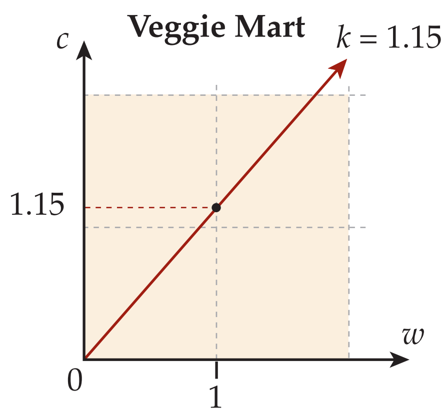
When the weight in pounds is equal to 1, the cost in dollars is equal to the unit price. This value is equivalent to the constant of proportionality.

On the Fruit World graph, when $w = 1$, $c = 0.6$, so $k = 0.6$.

On the Veggie Mart graph, when $w = 1$, $c = 1.15$, so $k = 1.15$.

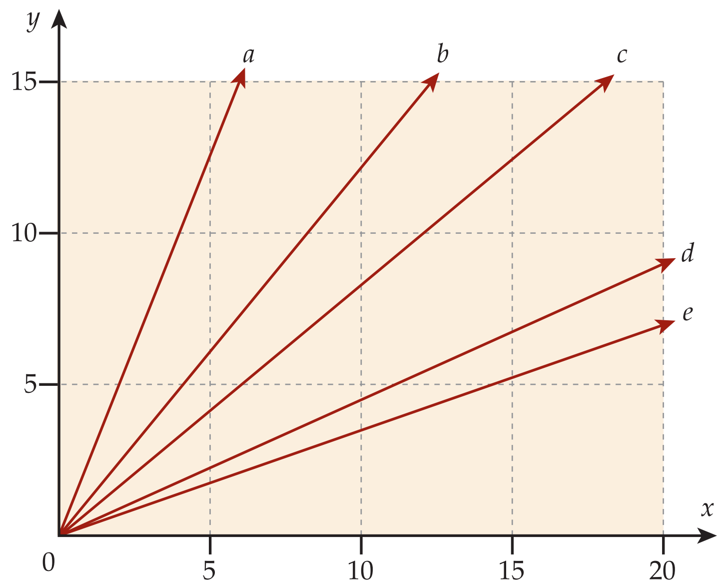
You can see this relationship in more detail on these zoomed-in graphs.





Work Time

1. The graphs of five proportional relationships are shown on the coordinate grid.



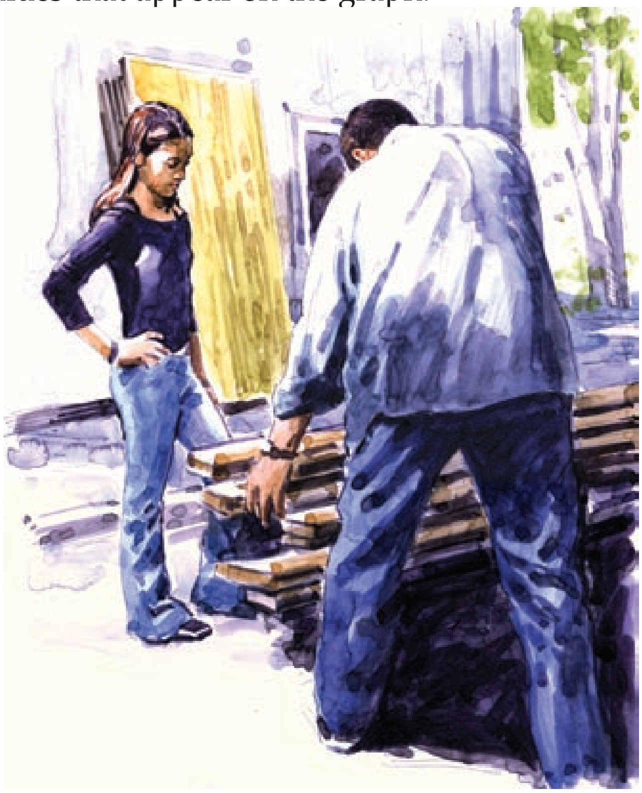
a. Approximate, as closely as possible, the constant of proportionality for each one.

b. The graph of a proportional relationship with a constant of 1 would lie between two of these lines. Say why.

c. Choose one of the lines and write a formula for it. Use x and y to name the variables.

d. Choose a different line and make a table for it, using five pairs of values that appear on the graph.

2. Mr. Valdez bought lumber to remodel his restaurant. He made a stack of 5 pieces of plywood. Rosa measured the height of the stack and found that it was 4 inches.



Mr. Valdez also made a stack of 5 two-by-fours. Rosa measured the height of this stack as well, and found that it was 8 inches.

In both cases, there is a proportional relationship between the height of the stack and the amount of lumber.

a. Find the constant of proportionality for each situation.

b. Write a formula for the height, h, in terms of the number of pieces of lumber, n, for both the plywood and the two-by-fours.

c. On grid paper, plot a graph for each relationship on the same set of coordinate axes.

Preparing for the Closing

3. How is the constant of proportionality represented on the graph of a proportional relationship?

4. Compare the graphs that you made for part c of problem 2 with the constants of proportionality that you calculated in part a.

How does a graph with a value of $k < 1$ compare to a graph with a value of $k > 1$?

5. Explain how you could use your graphs to find the number of pieces of plywood and the number of two-by-fours that measure a given height.

Skills

Solve.

a. $20[1 + (-26)] =$

b. $5(40 - 20) =$

c. $8(5 - 10) =$

d. $20[(-1) + 26] =$

e. $5[(-40) + 20] =$

f. $-8[(-5) + 10] =$

Review and Consolidation

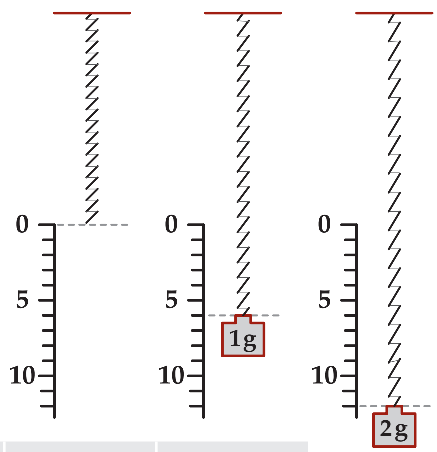
1. Imagine a hanging spring that is stretched by attaching weights to the bottom.

The length of the spring increases by 6 cm when a 1-gram weight is attached to it.

The length of the spring increases by 12 cm when a 2-gram weight is attached to it.

The length of the spring increases by 18 cm when a 3-gram weight is attached to it, and so on.

In this table, x represents the weight (in grams), and y represents the increase in length (in centimeters) when each weight is attached to the spring.



x (grams)	0	1	2	3	4	5
y (cm)	0	6	12	18	24	30

a. Is y proportional to x?

b. Write the formula that represents the relationship of y to x.

c. Is $\frac{y}{x}$ the same for every (x, y) pair in the table?

d. Is k equal to $\frac{y}{x}$ in your formula?

2. Repeat parts a through d for a second spring that increases in length by 4 cm for every 1-gram weight that is attached to it.

3. Plot graphs to represent the proportional relationships in problems 1 and 2. Sketch both of your graphs on the same set of axes. Which graph is steeper? Say why.

Homework

1. Consider the following statement:

   In a pair of reciprocal numbers, one is smaller than the other and one is larger than the other.

   This statement is *sometimes true*. Say why.

2. Consider the following statement:

   In a pair of reciprocal numbers, one is larger than 1 and one is smaller than 1.

   Is this statement *always true*, *sometimes true*, or *never true*? Justify your answer.

3. When you divide the larger number by the smaller number, is the quotient greater than 1 or less than 1? Say why.

> **Comment**
>
> Remember, the quotient is the result of a division.

4. When you divide the smaller number by the larger number, is the quotient larger than 1 or smaller than 1? Say why.

5. When you divide a number by itself, are the quotient and its reciprocal both equal to 1? Say why or why not.

6. As a number decreases from 1 toward 0, what happens to its reciprocal? Say why.

7. As a number increases from near 0 toward 1, what happens to its reciprocal? Say why.

LESSON 22

FORMULAS AND PROPORTIONAL RELATIONSHIPS

GOAL	CONCEPT BOOK
To use two different formulas to represent proportional relationships between quantities.	See pages 318–321 in your *Concept Book.*

You can write two formulas for any proportional relationship.

Example

Lisa says, "Our car gets 26 miles per gallon."

- This tells you that Lisa's car travels 26 miles for each gallon of gas it uses.
- It also tells you that the car uses gas at the rate of 1 gallon per 26 miles.

Distance (miles) d	**Volume of Gas** (gallons) g
26	1
52	2
78	3
104	4

The table shows the relationship between two quantities: distance (in miles) traveled, d, and volume of gas used (in gallons), g.

You already know that if you calculate the constant of proportionality, you can find a formula that represents the number of miles traveled in terms of the number of gallons of gas used.

$$k_1 = \frac{d}{g} = \frac{26}{1} = 26$$

The formula for d in terms of g: $d = 26g$

To calculate the number of miles you could travel using 35 gallons of gas, use the formula:

$$d = 26 \bullet 35 = 910 \text{ miles}$$

The second formula represents the number of gallons of gas used in terms of the number of miles traveled.

Example

The constant of proportionality for the number of gallons of gas used to the number of miles traveled is:

$$k_2 = \frac{g}{d} = \frac{1}{26}$$

The formula for g in terms of d:

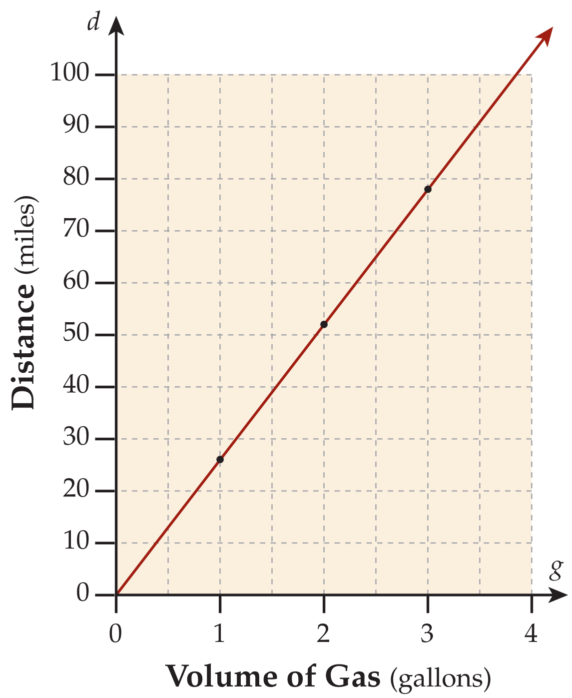
$$g = \frac{1}{26}d$$

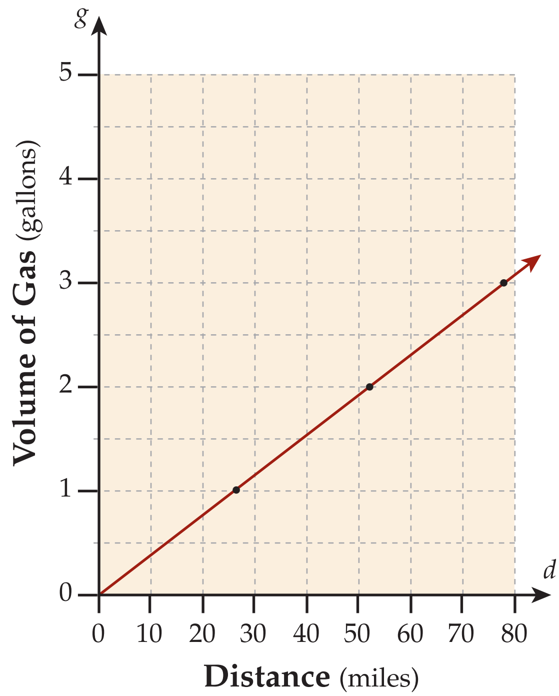
Notice that the two constants of proportionality (k_1 and k_2) have reciprocal values.

$$k_1 = \frac{d}{g} = \frac{26}{1} = 26 \qquad k_2 = \frac{g}{d} = \frac{1}{26}$$

Both formulas accurately represent the relationship between gallons of gas and miles traveled. Which formula you use depends on the question you would like to answer.

These relationships can be graphed as:

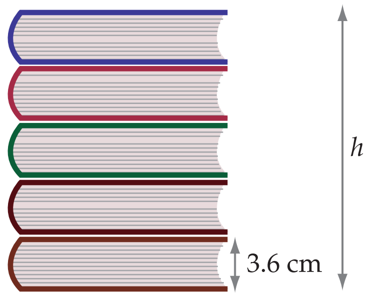




Work Time

1. Suppose you make a stack of identical books.

Here is a picture showing the height of the stack and the thickness (3.6 cm) of a single book.



a. Create a ratio table that relates the height of the stack with stacks of 1, 2, 5, and 10 books. Let h represent the height of the stack in centimeters and n represent the number of books in the stack.

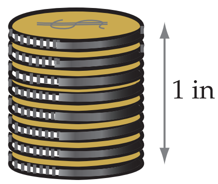
b. Find the constant of proportionality and write a formula for the height of the stack in terms of the number of books.

c. Use your formula to find the height of the stack of 13 of these books.

d. Write a formula for the number of books in terms of the height. (Your formula will start with $n = \ldots$)

e. Use your formula from part d to find the number of books when the height is 54 cm.

2. The number of coins in a stack of identical coins is proportional to the height of the stack. To find the number of coins in a stack, measure the height of the stack instead of counting the coins.



Suppose a stack of 8 coins is exactly 1 inch high.

a. Write a formula for the number of coins in terms of the height of the stack of coins. (Choose your own letters to stand for the two quantities.)

b. Use your formula from part a to find the number of coins when the height is 12.5 in.

c. Write a formula for the height of the stack in terms of the number of coins.

d. Use your formula from part c to find the height of the stack when there are 43 coins.

e. Eight identical coins of a different type make a stack that measures 1.5 in. If you found a formula for the number of coins in terms of the height of these coins, it would be different from the formula you found in part a. Say why.

Preparing for the Closing

3. On pages 106–107 you found $k_1 = 26$ and $k_2 = \frac{1}{26}$.

These constants of proportionality, k_1 and k_2, are reciprocals. Say why.

4. Use your results for Work Time problems 1 and 2 to verify your response to problem 3.

Skills

Solve.

a. $10(a + 2a) =$

b. $10(a - 2a) =$

c. $-10(a - 2a) =$

d. $20[a + (-a)] =$

e. $53(x - 2x) =$

f. $36[(-4y) + y] =$

Review and Consolidation

1. A stack of 8 identical coins is 1.6 cm high. Show that this information satisfies the formula $n = 5h$, where n is the number of coins and h is the height of the stack in centimeters.

2. If you double the number of coins in a stack of identical coins, what happens to the height?

3. Make a table that relates the values of n and h for the formula $n = 5h$. Include entries for $h = 1$ to $h = 15$.

4. How many coins would be in a stack 80 cm high?

5. The formula $n = 5h$ gives the number of coins in terms of the height of the stack. Write a formula for h in terms of n.

6. Suppose there is another type of coin for which the formula for n in terms of h is $n = 4h$. Is this coin thicker or thinner than the first type (formula of $n = 5h$)? Say why.

Homework

A sign in a store reads, "100 blank CDs for just \$20."

Let p represent the cost of the CDs (in dollars) and n represent the number of CDs.

1. Find the constant of proportionality, k_1, for the ratio of the price to the number of CDs.
2. Write a formula for the price in terms of the number of CDs. (Your formula will be in the form of $p = k_1 n$.)
3. Use your formula from problem 2 to calculate the price of 700 CDs.
4. Find the constant of proportionality, k_2, for the ratio of the number of CDs to the price.
5. Write a formula for the number of CDs in terms of the price. (Your formula will be in the form of $n = k_2 p$.)
6. Use your formula from problem 5 to calculate the number of CDs you can buy for \$60.
7. What is the relationship between the constants of proportionality k_1 and k_2?

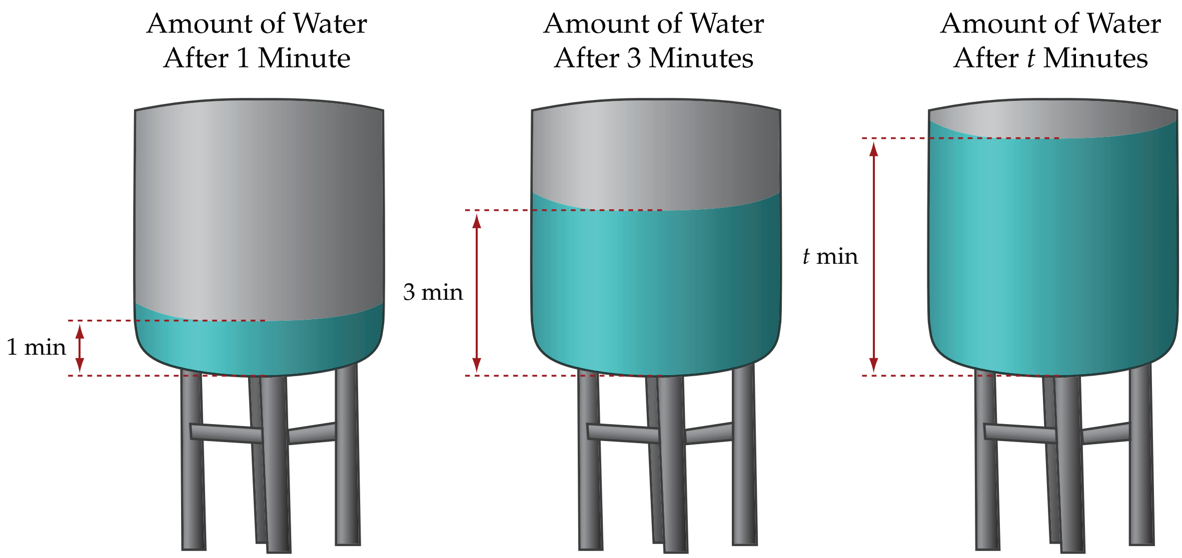
INTRODUCING FUNCTIONS

CONCEPT BOOK	GOAL
See pages 311–313, 370–371 in your *Concept Book.*	To learn that proportional relationships can be defined as functions.

A proportional relationship between two variables x and y can be represented as $y = kx$, where k represents the constant of proportionality.

Example

If you fill a water tank at a rate of 20 gallons per minute, the quantity of water in the tank changes with the time, t, taken to add the water. If in t minutes, g gallons of water is added, once you determine the value of t, the time, you can determine the value of g, the gallons of water.



You can express g in terms of t because there is a proportional relationship between the quantities time and the number of gallons of water.

The constant of proportionality is $k = \frac{g}{t} = 20$, so $g = 20t$.

You can use this equation to determine how many gallons, g, will be in the tank at t seconds. Mathematicians say, "g is a function of t."

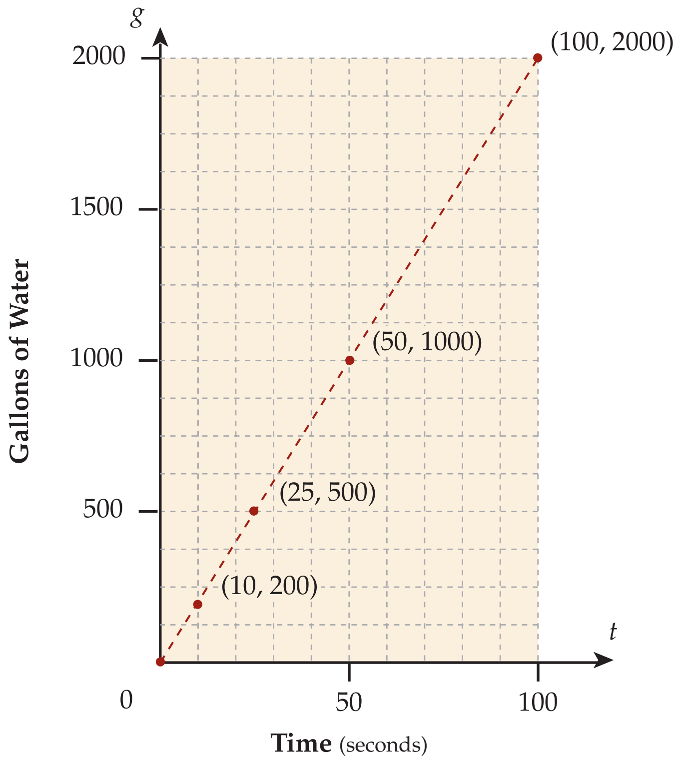
Using the function you can calculate values of g that correspond to values of t.

Time (in seconds) t	0	1	10	11	100
Gallons of Water g	0	20	200	220	2000

The correspondence between values of t and values of g can be represented using an arrow diagram and words.

$t \longrightarrow \boxed{\text{multiply by 20}} \longrightarrow g$

The function can also be represented on a graph.



Notice that for any value of t only one value of g is possible. This means that the function never has two g-values for the same t-value.

Work Time

1. y is proportional to x.

a. When $x = 28$, $y = 7$. Express y as an equation in terms of x.

b. y is a function of x. What does that mean?

c. Calculate the value of y for $x = 16$.

d. Calculate the value of y for $x = 0.5$.

2. a. Use an arrow and words to represent each of these three functions.

$$y = x \qquad y = \frac{9}{7}x \qquad y = 0.97x$$

b. Copy and complete these three tables of values—one for each of the three functions above. Use each function to calculate the y-values corresponding to the given x-values.

Comment
You may use a calculator.

x	0	10	15	20	25	100
y						

x	0	10	15	20	25	100
y						

x	0	10	15	20	25	100
y						

3. Rosa's aunt lives in an apartment on the fourth floor. To get exercise, Rosa always uses the staircase when she visits her aunt.

Rosa is curious to know the vertical height (in meters) that she reaches when she climbs the stairs. She measures the vertical height between some pairs of stairs and finds that it is always 20 centimeters.

a. Rosa thinks that the vertical height, h, is a function of the number of steps, n, that she climbs.

Do you agree with Rosa? Say why or why not. Support your conclusion using an arrow and words, a table, a graph, and a formula.

b. Rosa does not think that the number of steps, n, is a function of the vertical height, h.

Do you agree with Rosa? Say why or why not. Support your conclusion using a formula and an arrow and words.

Preparing for the Closing

4. Define the word *function* as a mathematician would use it.

5. This is the table of values from page 112 that shows a proportional relationship.

Time (in seconds) t	0	1	10	11	100
Gallons of Water g	0	20	200	220	2000

a. Show how to determine g when $t = 200$ using the table as a ratio table.

b. Show how to determine g when $t = 200$ using the table as a function.

6. Use your answer for problem 5 to help you write an explanation for these statements.

For a proportional relationship ($y = kx$):

a. When you think of a table of values as a ratio table, you work between pairs of values to determine values of y.

b. When you think of a table of values as a function, you work within pairs of values to determine values of y.

Skills

Solve.

a. $10(30 + 20) =$ b. $10(2a + a) =$ c. $10(a + b) =$

d. $10(30 - 20) =$ e. $10(2a - a) =$ f. $10(a - b) =$

Review and Consolidation

1. a. Use an arrow and words to represent each of these three functions.

$$y = 50x \qquad y = \frac{5}{6}x \qquad y = 0.08x$$

b. Copy and complete these three tables of values—one for each of the three functions above. Use each function to calculate the y-values corresponding to the given x-values.

Comment
You may use a calculator.

x	1	0.5	10	35	367	10,200
y						

x	1	0.5	10	35	367	10,200
y						

x	1	0.5	10	35	367	10,200
y						

Homework

1. Let d represent the distance traveled by a car moving at a constant speed of 45 miles per hour for h hours.

   a. Write the formula that expresses d as a function of h.

   b. Express the function in part a using an arrow and words.

   c. Write the formula that expresses h as a function of d.

   d. Express the function in part c using an arrow and words

   e. Copy and complete this table using the appropriate function.

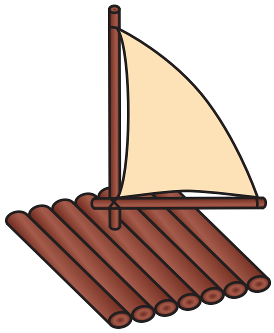
Time (hours) h	0.5	1.5	3			6.1	
Distance (miles) d				202.5	252		450

INVERSELY PROPORTIONAL RELATIONSHIPS

LESSON **24**

CONCEPT BOOK	GOAL
See pages 324–325, 370–371 in your *Concept Book*.	To learn about inversely proportional relationships and that inversely proportional relationships are functions.

A group of students is planning to enter a raft-building competition.



The grand prize for the best raft is \$600. The amount of money that each student could win depends on the number of people in the group. Two people would each win \$300, three people would each win \$200, and so on.

This table of values shows the relationship between the number of people, x, in the group and the amount of money, y, won by each.

Number of People	x	1	2	3	4	5	6
Money Won (\$)	y	600	300	200	150	120	100

The table shows how the values of y change as the values of x change.

When x is doubled, y is halved; when x is tripled, y is one-third; and so on.

Also, the product of each pair of values is 600.

$$xy = 600$$

In a situation like this, when the product of each ordered pair is constant, then the relationship is *inversely proportional*.

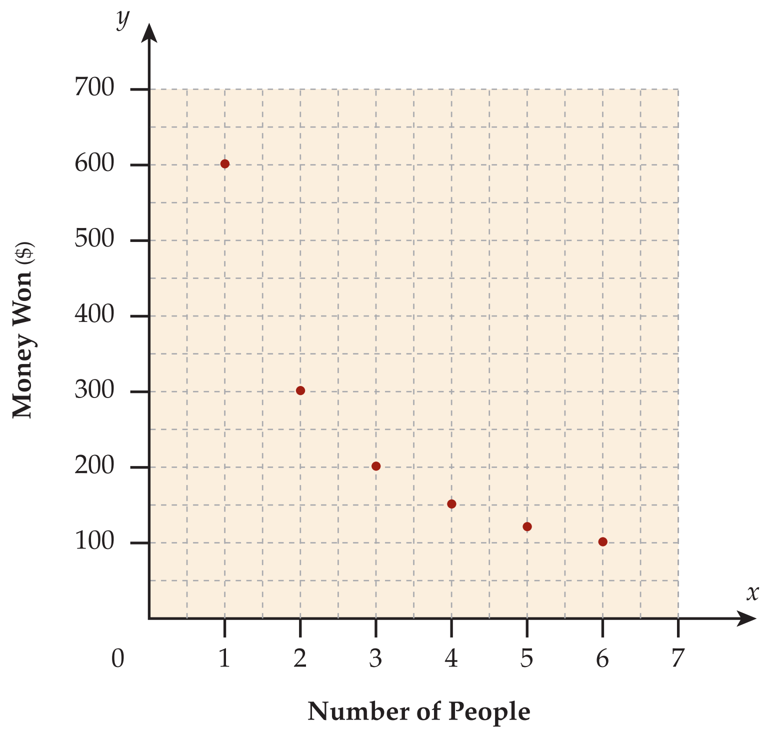
In an inversely proportional relationship:

- $xy = k$ (where k is the constant of proportionality)
- $y = \dfrac{k}{x}$
- y is a function of x

In the example, the function is $y = \dfrac{600}{x}$.

To determine values of y, the function is "600 divided by x," or "600 multiplied by the reciprocal of x."

Here is a graph of this function. Notice that it is not a straight line!



Work Time

1. y is inversely proportional to x.

a. When $x = 28$, $y = 7$. Express y as an equation in terms of x.

b. y is a function of x. Explain what that means.

c. Calculate the value of y for $x = 16$ and $x = 0.5$.

2. Given this table of values:

x	0.5	2.2		80
y	6	26.4	42	

a. Identify whether the relationship is proportional or inversely proportional, and say how you know.

b. Write the formula that expresses y as a function of x in each case.

c. Use the appropriate function to determine the missing values.

3. Given this table of values:

x	3.6		86.4	156
y	0.3	5.5	7.2	

a. Identify whether the relationship is proportional or inversely proportional, and say how you know.

b. Write the formula that expresses y as a function of x in each case.

c. Use the appropriate function to determine the missing values.

4. Given this table of values:

x	0.2	10		120
y	60		0.25	0.1

a. Identify whether the relationship is proportional or inversely proportional, and say how you know.

b. Write the formula that expresses y as a function of x in each case.

c. Use the appropriate function to determine the missing values.

5. Let w be the width and l be the length of a rectangle with an area of 36 cm^2.

a. Make a table of values and list ten ordered pairs for the relationship between width and length.

b. The relationship between the variables w and l is an inverse proportion. Say why.

c. Write the equation that expresses l as a function of w.

d. Use the function to find l when $w = 90$ cm.

e. Use the function to find l when $w = 2.7$ cm.

Preparing for the Closing

6. How can you tell from a table of values whether y is proportional to x or inversely proportional? How does the constant of proportionality show up in the table?

7. How can you tell from a formula whether y is proportional to x or inversely proportional? How does the constant of proportionality show up in the formula?

8. How do proportional functions differ from inversely proportional functions?

Skills

Solve.

a. $-10(30 + 20) =$

b. $-10(2a + a) =$

c. $-10(a + b) =$

d. $-10(30 - 20) =$

e. $-10(2a - a) =$

f. $-10(a - b) =$

Review and Consolidation

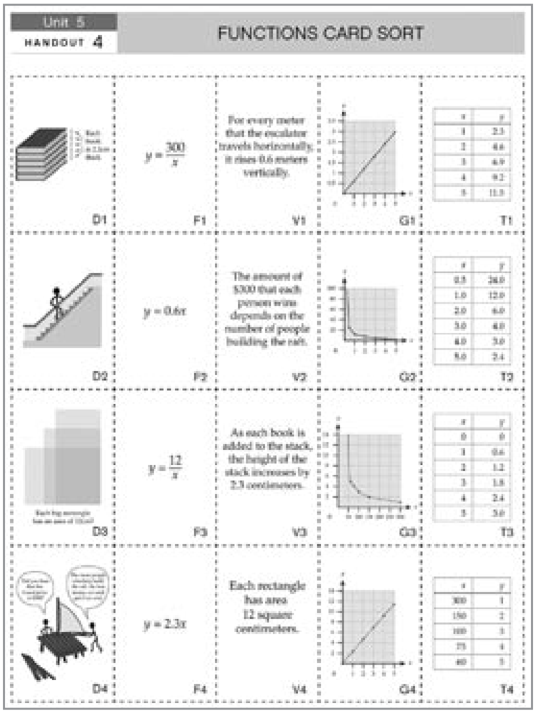
1. Your teacher will give you five cards for each of the following representations.

- Pictures
- Tables
- Verbal descriptions
- Graphs
- Formulas

Your job is to sort the cards into equivalent sets.

That means you have to find the picture, table, verbal description, graph, and formula that represent the same function.

After you have sorted the cards, you will be asked to glue them together as a group. Then, you need to sort the groups of representations into two groups—the ones that are proportional and the ones that are inversely proportional.



Homework

1. State whether each of these equations is proportional or inversely proportional. (k is a nonzero constant.)

a. $d = 55t$ b. $xy = 36$ c. $y = \dfrac{k}{x}$ d. $t = 3.3n$ e. $\dfrac{y}{x} = k$

2. Write a formula for each of the following situations. Identify the constant of proportionality in each case.

a. The time, t, in hours that it takes a car to travel a distance of 150 miles to your favorite camp site at the speed of r miles per hour.

b. The cost, c, in dollars of n pounds of walnuts selling for $0.69 a pound.

c. The length, l, in meters and width, w, in meters of a rectangular swimming pool with an area of 345 square meters.

d. The circumference, C, of circle as a function of its radius, r.

CONCEPT BOOK

See page 239 for circumference of a circle.

LESSON 25

DISTANCE, RATIO, AND PROPORTIONALITY

GOAL

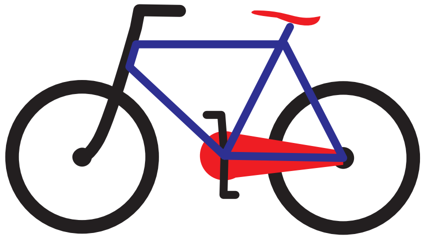
To apply ratio and proportionality to solve a problem.

CONCEPT BOOK

See pages 315–321 in your *Concept Book.*

Putting Mathematics to Work

Jamal saved his money and bought himself a new bicycle.



He wanted to come up with a formula that told him how far he traveled just by counting the number of times the pedals turn. His bicycle has three gears. He knew that for the same number of turns of the pedals, he will travel the least distance in low gear and the greatest distance in high gear.

Jamal conducted an experiment to gather some data about how far the bike traveled on a level (horizontal) path when he was using three different gears—high, medium, and low. He started at a certain place and used a tape measure to determine the distances he traveled for various numbers of turns of the pedals.

These are his measurements.

Low Gear

Number of Complete Turns of the Pedal	3	5	10
Distance Traveled (feet)	20' 6"	34' 2"	68' 4"

Medium Gear

Number of Complete Turns of the Pedal	2	4	7
Distance Traveled (feet)	20' 6"	41'	71' 9"

High Gear

Number of Complete Turns of the Pedal	1	5	12
Distance Traveled (feet)	20' 6"	102' 6"	246'

You will write three formulas for Jamal, clearly explaining your work, so that he will know how to calculate distance based on the number of times his pedals turn for each of the three gears.

Work Time

1. Jamal thinks the two variables are in proportional relationships in each of the three situations, but he does not know for sure. Write an explanation that makes this clear for Jamal.

2. What is the ratio of distances covered for one complete turn of the pedals for high gear to medium gear to low gear ?

3. At what rate (feet per pedal turn) is Jamal pedaling when he is:
   a. In high gear?
   b. In medium gear?
   c. In low gear?

4. Write three formulas (one for each of the three gears) that will tell Jamal how far he will travel (d feet) if the pedals turn t times.

   Hint: First, convert the units of the measurements in the table of values to feet, rather than feet and inches. Express each of these measurements as a fraction in lowest terms.

   Express the constants of proportionality as fractions in lowest terms.

5. When he rode from his home to school one morning in high gear, Jamal counted that his pedals turned exactly 500 times. How far did Jamal bike to get to school?

   Use a formula from problem 4 to calculate your answer, and express it in miles, rounded to one decimal place.

   Hint: There are 5280 feet in a mile.

6. How many times will his pedals turn on the journey to school if he keeps his bike in low gear all the way?
   a. Show how to find the answer using a ratio table.
   b. Show how to find the answer using a formula.

7. Show that you can estimate solutions to problems 5 and 6 using a graph.

8. One morning, Jamal decided that he would pedal 100 times in high gear, 100 times in medium gear, and 50 times in low gear. Will he get to school?

Preparing for the Closing

9. Compare all of your methods and solutions with at least one other student. Try to reach an agreement about your results. If your solution methods are different, discuss the differences and similarities, and make sure you understand the strategies your partner used.

10. Prepare explanations of your solutions for the Closing.

Skills

Solve.

a. $(25 + 25) \bullet -2 =$

b. $-2(25 + 25) =$

c. $(140 \div -7) - 2 =$

d. $(140 \div -7) + -2 =$

e. $300 \bullet 135 \div 5 =$

f. $300 \bullet -135 \div -5 =$

Review and Consolidation

1. Write three formulas (one for each of the three gears) that tell Jamal how many times the pedals will turn (t) if he travels d feet.

Hint: First, convert the units of the measurements in the table of values to feet, rather than feet and inches. Express each of these measurements as a fraction in lowest terms.

Express the constants of proportionality as fractions in lowest terms.

2. Rosa lives 0.5 mile away from Jamal along a flat, straight road.

Plot graphs for each of the three formulas that you found for problem 1 and use them to estimate how many complete turns Jamal's pedals will make on the ride to Rosa's house if he makes three separate journeys—one in each gear.

Homework

1. A 4" by 6" photo is enlarged by a scale factor of 5.5. What are the dimensions of the enlarged photo?

2. Is it better to buy a 0.75-kg can of tomatoes for $1.95 or a 1-kg can of tomatoes for $2.75?

3. Here are two tables of values. Which table represents a proportional relationship? Say why.

Table A

x	3	10
y	15	22

Table B

x	3	10
y	15	50

4. For the proportional relationship that you identified in problem 3:
   a. Find the constant of proportionality.
   b. Write the formula that represents the relationship.
   c. Plot a graph using these x-values: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10.

LESSON 26

GEOMETRY, RATIO, AND PROPORTIONALITY

GOAL

To use ratio and proportion to solve geometric problems.

CONCEPT BOOK

See pages 262–264, 320 in your *Concept Book*.

In this lesson, you will investigate the relationship between two quantities:

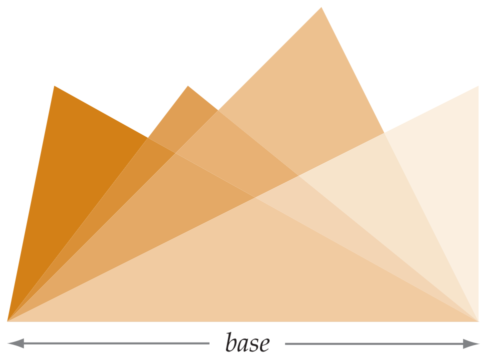
- The area of a triangle with a given base
- The height of the triangle

CONCEPT BOOK

See pages 242–243 to review area of a triangle.

Work Time

This diagram illustrates four triangles with the same base.



1. Imagine several different triangles with a 12-cm base.

a. What is the area of a triangle with a base of 12 cm and a height of 1 centimeter?

b. What is the area of a triangle with a base of 12 cm and a height of 2 centimeters?

c. What is the area of a triangle with a base of 12 cm and a height of 2.5 centimeters?

d. Make a table with these three pairs of values for height and area.

e. In your own words, describe the area of any triangle with a base of 12 cm as a relationship of its height.

f. Represent this relationship with a formula, using A to represent the area and h to represent the height.

In problems 2–7, you will work in groups to investigate how the area of a triangle with a given base changes in relation to the height of the triangle. Your teacher will give each group a different length in centimeters to use as the given base.

To get started on your work together:

- Investigate the area of a small number of triangles with your given base by sketching triangles with the same base and varying heights and then finding the areas of the triangles you have drawn.
- Give a verbal description of the area of a triangle with this base as a relationship to its height.

In these problems, you will:

- Make a table to organize the data from your triangles.
- Write a formula that describes the relationship between the varying height, h, of the triangles and the area, A, of the triangles.
- Sketch a graph to represent this relationship.

2. You need to organize your data about the height and area of triangles with your given base. Do this by copying the table below and filling in the missing values.

Height (cm)	1	2.5	3	3.5	10	100	200	h (any height)
Area (cm^2)								

3. Calculate the constant of proportionality that will allow you to write a formula for the area of a triangle, A, in terms of height, h, for all triangles that have your base.

4. Represent the relationship of A in terms of h as:

a. A formula

b. A graph

5. What does the constant of proportionality represent in this situation?

6. Using each of these representations, how can you tell that the area is proportional to the height of a triangle with a given base?

a. A table

b. A formula

c. A graph

7. a. Make a table like the one below, and record your findings in the first row of the table.

Group	Given Base (cm)	Formula

b. Record the findings of other groups in the other rows of the table.

Preparing for the Closing

8. Say how you know whether y is proportional to x in the following representations:

a. A graph b. A formula c. A table

9. If the base of a triangle is constant, how many variables determine the area of the triangle?

10. A triangle with a base of 13.6 centimeters and a height in centimeters represented by h has an area in square centimeters represented by A. Express A in terms of h: that is, write a relationship that represents A in relation to h.

Skills

Solve.

a. $(a + a) \bullet -2 =$

b. $-2(a + a) =$

c. $(4a \div -a) - 2 =$

d. $(4a \div -a) + -2 =$

e. $300a \bullet 135a \div 5a =$

f. $300a \bullet -135a \div -5a =$

Review and Consolidation

These problems use a triangle with the following properties:

- base = 14 centimeters
- height (in centimeters) = h
- Area (in square centimeters) = A

1. Express A in terms of h: that is, write a formula that represents A in relation to h.

2. Find the value of A for each of the following values of h.

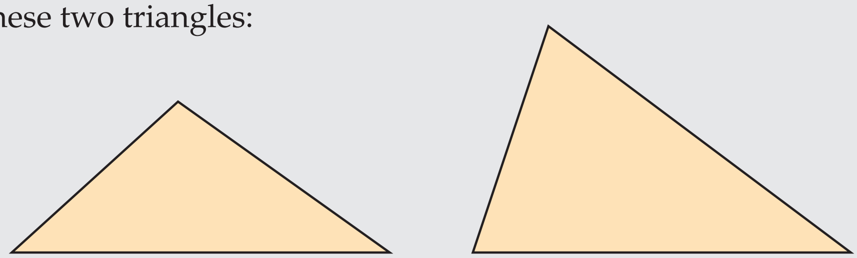
a. $h = 2.5$ cm b. $h = 7.5$ cm c. $h = 12.5$ cm

3. Find the value of h for each of the following values of A.

a. $A = 98\text{ cm}^2$ b. $A = 0.5\text{ cm}^2$ c. $A = 153\text{ cm}^2$ d. $A = 44\text{ cm}^2$

Homework

1. Compare these two triangles:



5 cm 5 cm

Notice that they both have the same base, 5 cm.

a. The height of the triangle on the right is 3 cm.
What is the area of this triangle?

b. The area of the triangle on the left is 5 cm^2.
What is the height of this triangle?

c. Write a formula that describes the relationship between area, A, and height, h, in triangles with a fixed base of 5 cm.

d. What is the formula for the area of a triangle of any base, the dimensions of which are measured in the same units?

LESSON

27 THE UNIT IN REVIEW

GOAL

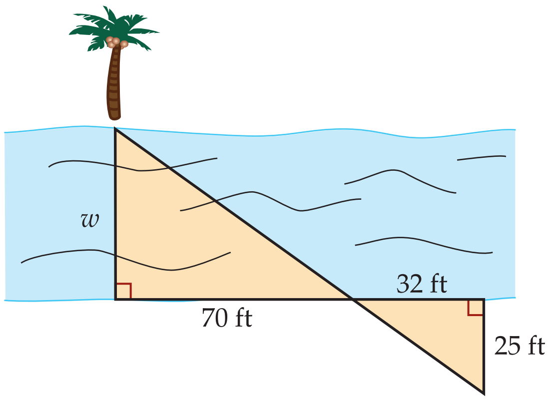
To review the concepts studied in the unit.

CONCEPT BOOK

See pages 255–264, 291–313, 315–320 in your *Concept Book*.

Work Time

1. The diagram below shows the measurements made on one side of a river. How wide is the river?



2. Determine the unit price for each purchase, rounded to the nearest cent. Use this information to decide which is the better buy in each case. Explain each choice.

a. 1.5 kg of bananas for \$4.50 or 2.5 kg of bananas for \$7.00

b. 5 m of cloth for \$15.85 or 6 m of cloth for \$18.55

3. A steep stairway rises 1.5 meters in the first six steps. There are another 20 steps to the top. How high is the stairway?

4. If, in ΔABC and ΔXYZ, $\angle B = \angle Y$ and $\angle C = \angle Z$, then the triangles are similar. Say why.

5. If a computer enlarges a 4 by 2-inch photograph to 6 by 4 inches, then both images (large and small) will not be in proportion. Say why.

6. At the Fantastic Fruit Juice factory, a machine fills bottles with juice at the rate of 15 bottles per minute.

a. Copy and complete this ratio table to represent the situation.

Time (minutes) t	**Number of Bottles** b
1	
	30
	40
10	

b. Make a graph of the number of bottles per minute that shows the proportional relationship.

c. Write two formulas: one for the number of bottles in terms of time, and one for time in terms of the number of bottles.

d. Use an equation and a graph to determine the number of bottles the machine can fill in 5 minutes.

Preparing for the Closing

7. Check all your answers to the Work Time problems with another student. Reach an agreement when your answers differ.

8. Write a short paragraph that summarizes your understanding of these key concepts:

- Unit ratio
- Similarity ratio
- Rate
- Constant of proportionality

How are these concepts alike? How are they different?

Skills

The following table gives the monthly rainfall in centimeters (to the nearest cm) over a 12-month period in a certain district in the United States.

Month	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Rainfall (cm)	12	12	9	8	8	7	6	12	10	14	16	13

Graph this data.

Review and Consolidation

1. Late in the afternoon, two buildings cast shadows of lengths 23 meters and 15 meters. The taller of the two buildings is 65 meters high. How high is the other building?

2. The side lengths in quadrilateral *ABCD* are all double the side lengths of another quadrilateral, *WXYZ*. You cannot use this information to say that the two quadrilaterals are similar. Say why.

Homework

Your homework today is to prepare for the End-of-Unit Assessment.

- Use the *Concept Book* to review the main ideas from this unit.
- Find mistakes you have made in your work during the unit, and write out explanations and corrections.

COMPREHENSIVE REVIEW

UNITS **1–5**

1. The number π is not a rational number. Say why.

2. Express the following comparisons as both a whole number ratio and a fraction.

a. Forty-four carrots to sixty-six tomatoes

b. The height of a 150-cm girl to the height of a 162-cm boy

c. The length of a 12-inch ruler to a 1-yard ruler

3. Rewrite each expression using the conventions for multiplication and division.

a. $a \div 3$ b. $4 \div n$ c. $5 \bullet 9 \bullet a$

4. Write these numbers in order from least to greatest. Use the < symbol.

0.945 0.954 0.0945 0.9045 0.0954

5. Dwayne decides to empty his swimming pool in order to clean the interior surface. He pumps the water out at a rate of 25 gallons per minute.

If Dwayne's pool holds 14,625 gallons, how long will it take for him to empty the pool?

6. Calculate the value of each expression.

a. $15 \bullet 2 \div 10$ b. $7(2 \div 10)$ c. $\dfrac{9+3}{9-3}$ d. $(4 \bullet 2) \div (10 \bullet 4)$

7. a. Calculate $(-5) \bullet (+3) - (-12)$.

b. Calculate $-5 \bullet \left[\left(+3\right) - \left(-12\right)\right]$.

c. The same numbers and the same operations appear in both parts a and b. Explain why the answers are different.

8. Calculate the slant height of a cone with perpendicular height of 12 centimeters and a radius of 5 centimeters.

9. a. Only one number between 14 and 18 has just two factors.

What number is it?

b. Only one number between 14 and 18 has an odd number of factors.

What number is it?

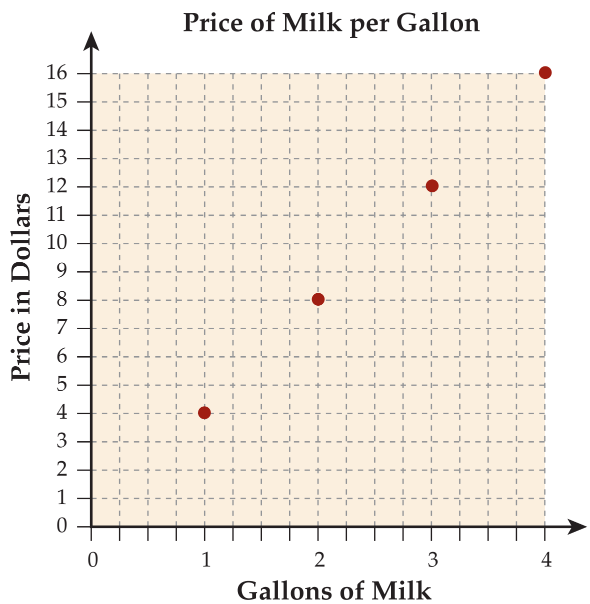
10. a. $4 \bullet \frac{2}{8} =$ b. $3 \bullet \frac{4}{6} =$ c. $7 \bullet \frac{1}{7} =$

11. Use this graph for the following problems.

a. What is the unit price of milk per gallon?

b. How much do 3 gallons of milk cost?

c. You have $8. How many gallons of milk can you buy?



Page numbers in red are found in the *Concept Book*.

Number Sense

Gr. 3 NS: 2.7	Determine the unit cost when given the total cost and number of units. 75–80; 307
Gr. 6 NS: 1.0	Students compare and order positive and negative fractions, decimals, and mixed numbers. Students solve problems involving fractions, ratios, proportions, and percentages: 6–10, 16–20, 34–37, 72–74; 293–295, 305–307
Gr. 6 NS: 1.2	Interpret and use ratios in different contexts (e.g., batting averages, miles per hour) to show the relative sizes of two quantities, using appropriate notations (a/b, a to b, $a : b$). 1–20; 291–295, 297
Gr. 6 NS: 1.3	Use proportions to solve problems (e.g., determine the value of N if $4/7 = N/21$, find the length of a side of a polygon similar to a known polygon). Use cross-multiplication as a method for solving such problems, understanding it as the multiplication of both sides of an equation by a multiplicative inverse. 38–71, 126–132; 291–295, 297

Algebra and Functions

Gr. 3 AF: 2.1	Solve simple problems involving a functional relationship between two quantities (e.g., find the total cost of multiple items given the cost per unit). 75–80; 307
Gr. 4 AF: 1.5	Understand that an equation such as $y = 3x + 5$ is a prescription for determining a second number when a first number is given. 111–116; 201, 205–209, 309–313, 322, 327–330, 363–364
Gr. 5 AF: 1.5	Solve problems involving linear functions with integer values; write the equation; and graph the resulting ordered pairs of integers on a grid. 111–116; 201, 205–209, 327–330
Gr. 6 AF: 2.0	Students analyze and use tables, graphs, and rules to solve problems involving rates and proportions. 21–28, 34–62, 68–74, 91–110; 308–311, 315–321
Gr. 6 AF: 2.1	Convert one unit of measurement to another (e.g., from feet to miles, from centimeters to inches). 81–84; 307–308
Gr. 6 AF: 2.2	Demonstrate an understanding that rate is a measure of one quantity per unit value of another quantity. 29–37, 72–74; 305–313
Gr. 7 AF: 3.4 Topic 8	Plot the values of quantities whose ratios are always the same (e.g., cost to the number of an item, feet to inches, circumference to diameter of a circle). Fit a line to the plot and understand that the slope of the line equals the quantities. 75–80, 91–110, 130–132; 315–319, 327–329
Gr. 7 AF: 4.2 Topic 6	Solve multistep problems involving rate, average speed, distance, and time or a direct variation. 75–84, 122–125, 130–132; 271–289, 305–313

Page numbers in red are found in the *Concept Book*.

Measurement and Geometry

Gr. 7 MG: 1.3 Topic 8	Use measures expressed as rates (e.g., speed, density) and measures expressed as products (e.g., person-days) to solve problems; check the units of the solutions; and use dimensional analysis to check the reasonableness of the answer. 29–33, 85–90; 271–289

Mathematical Reasoning

Gr. 6 MR: 1.0 Gr. 6 AF: 2.0	Students make decisions about how to approach problems: 72–74; 23–44
Gr. 6 MR: 1.2 Gr. 6 AF: 2.0	Formulate and justify mathematical conjectures based on a general description of the mathematical question or problem posed. 21–24; 23–44
Gr. 6 MR: 1.3 Gr. 6 AF: 2.0	Determine when and how to break a problem into simpler parts. 21–24; 23–44
Gr. 6 MR: 2.2 Gr. 6 AF: 1.3 AF: 2.0	Apply strategies and results from simpler problems to more complex problems. 25–28, 63–67, 126–129; 23–44
Gr. 6 MR: 2.4 Gr. 6 NS: 1.2 NS: 1.3 AF: 2.0	Use a variety of methods, such as words, numbers, symbols, charts, graphs, tables, diagrams, and models, to explain mathematical reasoning. 16–20, 25–28, 63–67, 126–129; 23–44
Gr. 6 MR: 2.5 Gr. 6 NS: 1.3 AF: 2.0	Express the solution clearly and logically by using the appropriate mathematical notation and terms and clear language; support solutions with evidence in both verbal and symbolic work. 25–28, 63–67, 126–129; 23–44
Gr. 7 MR: 2.0 Gr. 7 AF: 4.2	Students use strategies, skills, and concepts in finding solutions: 122–125, 130–132; 23–44
Gr. 7 MR: 2.5 Gr. 7 AF: 4.2	Use a variety of methods, such as words, numbers, symbols, charts, graphs, tables, diagrams, and models, to explain mathematical reasoning. 122–125, 130–132; 23–44
Gr. 7 MR: 2.6 Gr. 7 AF: 4.2	Express the solution clearly and logically by using the appropriate mathematical notation and terms and clear language; support solutions with evidence in both verbal and symbol work. 122–125, 130–132; 23–44
Gr. 7 MR: 2.8 Gr. 7 MG: 1.3	Make precise calculations and check the validity of the results from the context of the problem. 85–90; 23–44
Gr. 7 MR: 3.1 Gr. 7 MG: 1.3	Evaluate the reasonableness of the solution in the contest of the original situation. 85–90; 23–44

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