Ramp-Up to Algebra

Showing Relationships with Graphs

Unit 6

AMERICA'S CHOICE®

Acknowledgments

Field Test:

America's Choice®, Inc., wishes to acknowledge the following schools for their participation in the field testing of this material during the 2004–2005 school year:

AMERICA'S CHOICE HIGH SCHOOL, Sacramento, CA—Principal: Beate Martinez; Teacher: Nakia Edwards

CHATTOOGA COUNTY HIGH SCHOOL, Summerville, GA—Principal: Roger Hibbs; Math Coaches: Claire Pierce, Terry Haney; Teachers: Renee Martin, David Whitfield

JOHN MARSHALL HIGH SCHOOL, Rochester, NY—Principal: Joseph Munno; Math Coach: Tonette Graham; Teachers: Tonette Graham, Michael Emmerling

WOODLAND HIGH SCHOOL, Cartersville, GA—Principal: Nettie Holt; Math Coach: Connie Smith; Teachers: Leslie Nix, Kris Norris

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America's Choice, Inc., developed and field-tested Ramp-Up to Algebra over a ten-year period. Initial development started with a grant from the Office of Educational Research and Improvement (OERI) of the U.S. Department of Education, and was based on the most recent research in mathematics. During that ten-year period many authors, reviewers, and math consultants—both from the United States and internationally—contributed to this course. The materials underwent three major revisions as we analyzed field-test results and consulted math content experts. During the entire time, Phil Daro guided the development of the entire Ramp-Up to Algebra curriculum. See the *Getting Started* Teacher Resource guide for a complete list of people involved in this effort.

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ISBN 978-1-59896-538-4

First Printing 2007

1 2 3 4 5 6 7 8 9 10   11 10 09 08 07

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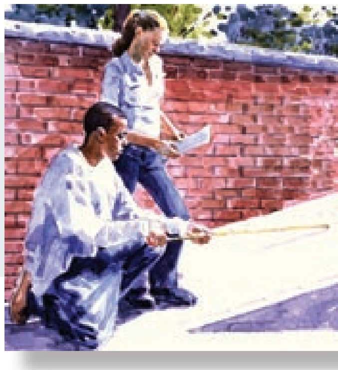

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LESSON

BUILDING THE COORDINATE PLANE

LESSON **1**

CONCEPT BOOK	GOAL
See pages 199–201 in your *Concept Book*.	To define a coordinate plane, and to graph points.

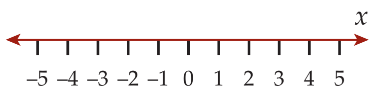
Relationships between quantities can be represented with graphs. One type of relationship is the proportional relationship. The quantities in a proportional relationship have a constant ratio, or *constant of proportionality*, which you studied in Unit 5: *Ratio and Proportionality*. In this unit, you will use graphs to represent proportional relationships.

A *coordinate plane* consists of two number lines that intersect at 0 on each line:

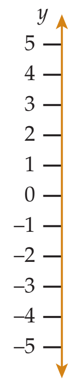
- the x-axis (a horizontal number line)
- the y-axis (a vertical number line)

A number line continues without end in the positive and negative directions, as shown by the arrow tips.

The x-axis is a horizontal number line that continues without end in the positive (right) and negative (left) directions. The x-axis represents x-values.



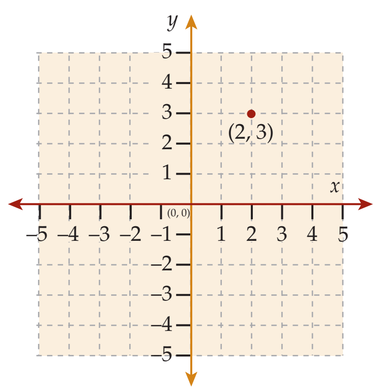
The y-axis is a vertical number line that continues without end in the positive (up) and negative (down) directions. The y-axis represents y-values.

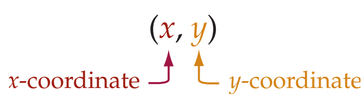


When these two number lines cross each other at zero, they form a coordinate plane. The point at which the lines cross is called the *origin*.

You can identify the position of a point in a coordinate plane by giving its x- and y-coordinates: (x, y).

(x, y) is an ordered pair of numbers that defines or locates a point by giving its horizontal position (along the x-axis) and its vertical position (along the y-axis).





Work Time

1. a. Sketch a horizontal number line. This number line is the x-axis and represents x-values. Mark the points $x = 1$, $x = 7$, $x = –3$, and $x = 0$ on the x-axis.

b. Sketch a vertical number line. This number line is the y-axis and represents y-values. Mark the points $y = 1$, $y = 7$, $y = –3$, and $y = 0$ on the y-axis.

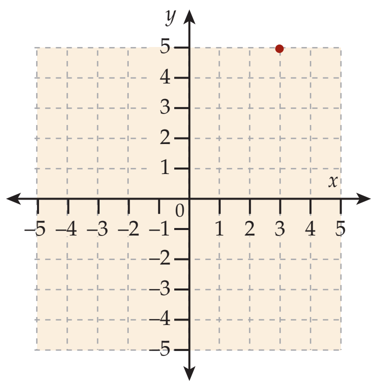
2. a. Sketch a horizontal number line and a vertical number line to form a coordinate plane.

b. What is the x-coordinate of the point at which the number lines cross?

c. What is the y-coordinate of the point at which the number lines cross?

d. Label the point at which the number lines cross with (x, y) coordinates. This point is called the origin.

3. The point shown at right is on neither the x-axis nor the y-axis; but it is has an x-value and a y-value.



a. Describe this point in relation to the x-axis and y-axis.

b. Give the x-coordinate and y-coordinate that tell you its position.

4. Look again at the points you plotted on the x-axis and y-axis in problem 1. The coordinates for $x = 1$ on the x-axis are (1, 0). Give the coordinates for each of the other seven points in problem 1.

5. Sketch a new coordinate plane, and then plot the following points.

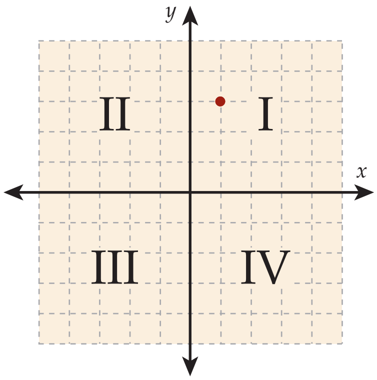
a. (1, 3)　b. (1, –3)　c. (–1, 3)　d. (–1, –3)

e. Connect points (1, 3) to (1, –3) to (–1, –3) to (–1, 3) back to (1, 3) with lines. What simple figure did you graph?

f. What is the length of the vertical line segment between points (1, 3) and (1, –3)?

g. What is the length of the horizontal line segment between points (1, 3) and (–1, 3)?

6. A coordinate plane has four *quadrants*. The four quadrants are labeled using Roman numerals.



Quadrant I: x is positive, y is positive

Quadrant II: x is negative, y is positive

Quadrant III: x is negative, y is negative

Quadrant IV: x is positive, y is negative

The point (1, 3) is in quadrant I.

In which quadrants are each of the other points you plotted in problem 5?

a. (1, –3) b. (–1, 3) c. (–1, –3)

d. What simple figure do the lines connecting these points form?

e. What is the length of the vertical line segment between points (–1, 3) and (–1, –3)?

7. a. Plot a square on a new coordinate plane.

b. Write the coordinates of the vertices.

Preparing for the Closing

8. Think about the following types of values for x and y. For each type, write what you know about how the values would be represented in the coordinate plane.

a. Values for x and y that are not integers.

b. Negative value for x and positive value for y.

c. Positive value for x and negative value for y.

d. Negative values for both x and y.

e. $x = 3$ and y can be any number.

f. $y = 3$ and x can be any number.

g. There is a constant ratio of 0.5 between x and y.

Skills

Solve.

a. Keesha has $7 and Lisa has $9.

Find the ratio of Keesha's money to Lisa's money.

b. There are 31 students in a class, and 17 of them are boys.

Find the ratio of the number of girls to the number of boys in the class.

c. Parcel A weighs 23 kg and parcel B weighs 29 kg.

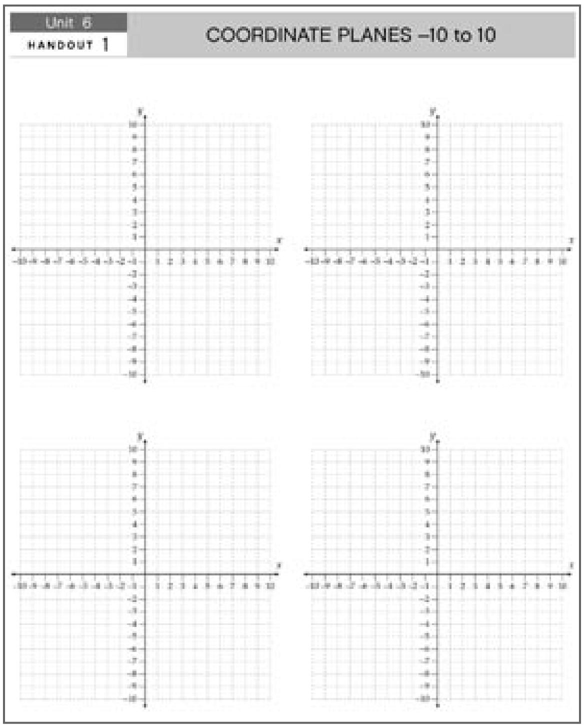
Find the ratio of the weight of parcel B to the weight of parcel A.

d. There are 13 sheep and 11 cows on a farm.

Find the ratio of sheep to cows.

Review and Consolidation

Use Handout 1: *Coordinate Planes –10 to 10* for problems 1–3.



1. Choose four pairs of numbers. Select one point that will lie in each quadrant.

a. Represent each of your pairs as the coordinates of a point in the form (x, y).

b. Plot your four points on a graph.

c. Identify in which quadrant each of your points lies.

d. Work with a partner to confirm that the quadrants you identified in part c are correct.

2. Choose an x-value.

a. Represent three points in the form (x, y) using your x-value with different y-values. Choose at least one y-value that is negative and at least one y-value that is not an integer.

b. Plot your three points on another graph.

3. Choose a *y*-value.

a. Represent three points in the form (*x*, *y*) using your *y*-value with different *x*-values. Choose at least one *x*-value that is negative and at least one *x*-value that is not an integer.

b. Plot your three points on another graph.

4. Write one similarity and one difference between the two sets of points you plotted in part b of problems 2 and 3.

5. Compare your observations in problem 4 with a partner. Did you make the same observations? Can you make any additional observations together?

Homework

Use your second copy of Handout 1: *Coordinate Planes –10 to 10* for problems 1–3. Graph the points with the following *x*- and *y*-coordinates.

Use one graph for each (*x*, *y*) table.

1.

x	*y*
2	1
4	2
8	4
–4	–2

2.

x	*y*
3	6
2	4
4	8
–2	–4

3.

x	*y*
3	–3
2	–2
–1	1
–3	3

LESSON 2

CONSTANT RATIOS AND GRAPHING

GOAL

To identify constant ratios between y-coordinates and x-coordinates for points in a coordinate plane.

CONCEPT BOOK

See pages 201–204 in your *Concept Book.*

Work Time

Use Handout 1: *Coordinate Planes –10 to 10* for problems 1–3 and 6.

1. Every point has an x-coordinate and a y-coordinate. Plot the following points on a coordinate plane. (Remember that on the x-axis, the positive direction is to the right. On the y-axis, the positive direction is upward.)

(1, 3) (–2, –3) (–1, 6)

2. Plot the point (x, y) for each pair of x- and y-coordinates.

a. $x = 5$ and $y = 2$
b. $x = -7.5$ and $y = -3$
c. $x = 10$ and $y = 4$
d. $x = -5$ and $y = -2$
e. $x = 7.5$ and $y = 3$

3. Plot the point (x, y) for each pair of x- and y-values represented in this (x, y) table.

x	3	2	1.5	–1	–2
y	9	6	4.5	–3	–6

4. In problem 2, the constant ratio between the y-coordinates and x-coordinates is $\frac{2}{5}$. Check that this is true for at least three of the (x, y) pairs given in parts a–e of problem 2.

5. Is there a constant ratio between the y-coordinates and x-coordinates given in problem 3? If so, what is it? Use the table in problem 3 to find your answer.

6. Choose any two points from the (x, y) table in problem 3.

a. Mark your two points on one of the coordinate planes.
b. Sketch a straight line that passes through these two points. Extend the line beyond the points in both directions.
c. Find someone in the class who chose points that are different from yours. Do your lines look the same?

Preparing for the Closing

7. Look at the points you plotted in problem 2.

a. Could you sketch a straight line through them?

b. Does this line intersect (cross) the origin, (0, 0)?

c. Is this line steeper or less steep than a line drawn through the points in problem 3?

Skills

Write each ratio in its simplest form.

a. 4 : 10	b. 4 : 24	c. 4 : 36	d. 4 : 5
e. 19 : 38	f. 19 : 95	g. 19 : 19	h. 19 : 190

Review and Consolidation

Use your second copy of Handout 1: *Coordinate Planes –10 to 10* for problems 1 and 3.

1. Plot each point below on the same coordinate plane.

a. (1.5, –1) b. $(\frac{1}{3}, -1)$ c. (–0.75, 1.5) d. (3, –2) e. $(-\frac{3}{4}, \frac{1}{2})$

2. Each point in the coordinate plane has an x-coordinate and a y-coordinate.

a. Which of the points in problem 1 have the same x-coordinate?

b. Do any of the points in problem 1 have the same y-coordinate? If so, which ones?

3. Look at the ratio of y to x in each of the points given in problem 1.

a. Which of the points given in problem 1 have the same ratio $\frac{y}{x}$?

b. Give the coordinates of another point with this same ratio.

c. Plot all four points from parts a and b on a graph and sketch a line through them.

4. Check that the x- and y-coordinates of each of the four points you graphed in problem 3 satisfy the formula $y = kx$ for the constant ratio $k = \frac{y}{x}$.

CONCEPT BOOK

See pages 315–317 for information on constant ratio.

Homework

1. Here are the tables from the homework for Lesson 1.

Find the constant ratio between y and x for each table.

a.

x	y
2	1
4	2
8	4
–4	–2

b.

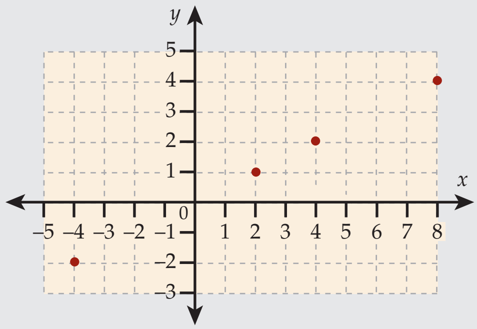
x	y
3	6
2	4
4	8
–2	–4

c.

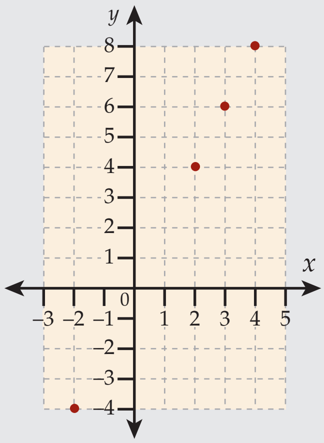
x	y
3	–3
2	–2
–1	1
–3	3

2. Look at the graphs of the points in each table.

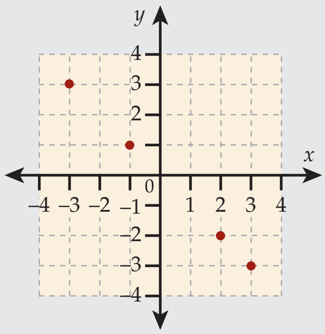
Graph 1



Graph 2



Graph 3



a. Do the points of each graph fall in a straight line?

b. Does each graph pass through the origin (0, 0)?

c. Which of the graphs represent(s) a relationship with a positive constant ratio?

d. Which of the graphs represent(s) a relationship with a negative constant ratio?

HOW STEEP IS THE LINE?

LESSON 3

CONCEPT BOOK	GOAL
See pages 327–328 in your *Concept Book*.	To graph lines and compare the steepness.

The x-axis and y-axis form a coordinate plane. To *graph* a line means to sketch the line in a coordinate plane.

Example

The formula $y = 2x$ defines a line. $y = 2x$ is read as "y equals x multiplied by 2."

To graph this line, identify at least two points on the line, and use a straightedge to draw a line between the points. You can identify points on the line using an (x, y) table.

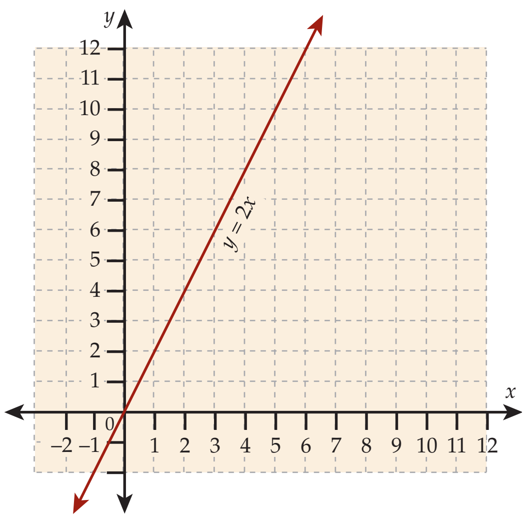
The (x, y) table for the formula $y = 2x$ is a ratio table, and the corresponding y-value for each x-value is equal to x multiplied by 2. The constant of proportionality is 2.

The formula $y = 2x$ describes the relationship between the x- and y-coordinates of every ordered pair for points on the line.

Ratio Table

x	y
1	2
2	4
3	6
6	12
–1	–2

Graph



Formula

$y = 2x$

Work Time

Use Handout 2: *Blank Coordinate Planes* for the Work Time problems.

1. Choose any two points for $y = 2x$ from the (x, y) table on page 9.

a. Plot your two points on one of the coordinate planes.

b. Sketch a straight line that passes through these two points. Extend the line beyond the points in both directions.

c. Find someone in the class who chose points that are different from yours. Do your lines look the same?

2. Make a new (x, y) table to identify at least two (x, y) ordered pairs that satisfy the formula $y = 3x$.

a. Your (x, y) table is a ratio table. Say how you know.

b. Plot your two points on a coordinate plane.

c. Sketch a straight line that passes through these two points. Extend the line beyond the points in both directions. This is the graph of the line $y = 3x$.

d. Find someone in the class who chose points that are different from yours. Do your lines look the same?

3. Make a new (x, y) table to identify at least two points on the line $y = x$. (The corresponding y-value for every x-value will be equal to x.)

a. Your (x, y) table is a ratio table. Say how you know.

b. Plot your two points on a coordinate plane.

c. Sketch a straight line that passes through these two points. Extend the line beyond the points in both directions.

d. Find someone in the class who chose points that are different from yours. Do your lines look the same?

Preparing for the Closing

4. Compare the steepness of the three lines you sketched in problems 1–3.

a. Which line is the steepest?

b. Which line is the least steep?

5. You have sketched graphs of $y = x$, $y = 2x$, and $y = 3x$. Observe that the lines defined by each of these formulas pass through the point (0, 0), the origin.

Every line that passes through the point (0, 0) has the form: $y = kx$.

↑ coefficient of x
constant of proportionality

k is the *coefficient* of *x*.

a. What is the coefficient of x in the formula $y = x$?

b. What is the coefficient of x in the formula $y = 2x$?

c. What is the coefficient of x in the formula $y = 3x$?

6. Consider the formula $y = 4x$ and the formula $y = \frac{1}{2}x$.

a. What is the coefficient of x in each of these formulas?

b. Do you think the line defined by $y = 4x$ is steeper or less steep than the lines you have sketched today? Say why.

c. Do you think the line defined by $y = \frac{1}{2}x$ is steeper or less steep than the lines you have sketched today? Say why.

Skills

Write each ratio as a unit ratio of the form 1 : *n*.

a. 5 : 10 b. 99 : 297 c. 4 : 10 d. 4 : 14

Review and Consolidation

For problems 2 and 3, use Handout 2: *Blank Coordinate Planes*.

1. Each of the following (x, y) tables is a ratio table. Copy the tables into your notebook and fill in the missing values and formulas. Identify the constant ratio $\frac{y}{x}$ in each table.

a. Formula: $y = \frac{1}{2}x$

Constant ratio: ______

x	1	2	3	
y	0.5	1		2

b. Formula: ____________

Constant ratio: ______

x	1	2		112
y	1		15	112

c. Formula: $y = 2x$

Constant ratio: $\frac{2}{1} = 2$

x	1	4		50
y		8	12	

d. Formula: ___________

Constant ratio: $\frac{3}{1} = 3$

x	y
3	9
13	
16	48
	66

e. Formula: $y = 4x$

Constant ratio: ______

x	y
1	
	36
10	40
	100

f. Formula: ___________

Constant ratio: ______

x	y
1	0.25
2	0.5
10	2.5
12	3

2. Choose one of the formulas and (x, y) tables from problem 1.

a. Plot at least two points in the (x, y) table on a coordinate plane.

b. Sketch a straight line that passes through both of the points you plotted.

3. Choose another formula and (x, y) table from problem 1.

a. Plot at least two points in the (x, y) table on another coordinate plane.

b. Sketch a straight line that passes through both of the points you plotted.

4. a. What is the coefficient of x in each of the formulas you chose for problems 2 and 3?

b. Which of your lines is steeper, the one you sketched in problem 2 or the one you sketched in problem 3?

c. Make an observation about how the coefficient, k, and the steepness of the line $y = kx$ are related.

Homework

Use another copy of Handout 2: *Blank Coordinate Planes*.

1. a. Plot the two points (0, 0) and (2, 8) on a coordinate plane.

b. Sketch a straight line that passes through these two points.

c. Use your graph to identify which of the following points lie on this line.

(2, 1) (1, 4) (4, 1) (–2, –8) (8, 2) (3, 12)

d. Make an (x, y) table showing the x- and y-coordinates for any three points on your line.

e. Your (x, y) table in part d is a ratio table. What is the constant ratio $y : x$?

2. a. Plot the two points (0, 0) and (8, 4) on a coordinate plane.

b. Sketch a straight line that passes through these two points.

c. Use your graph to identify which of the following points are on this line.

(1, 2) (2, 1) (4, 1) (–4, –2) (4, 8) (6, 3)

d. Make an (x, y) table showing the x- and y-coordinates for any three points on your line.

e. Your (x, y) table in part d is a ratio table. What is the constant ratio $y : x$?

3. These are the formulas for the lines you graphed: $y = 4x$ and $y = \frac{1}{2}x$.

a. Which line is defined by $y = 4x$, the line from problem 1 or the line from problem 2? How do you know?

b. Which line is defined by $y = \frac{1}{2}x$, the line from problem 1 or the line from problem 2? How do you know?

c. Look at the points you identified on the lines in problem 1c and problem 2c. Check that the x- and y-values for these points satisfy the formula for the line.

For example, for the line $y = \frac{1}{2}x$, you know the point (2, 1) is on the line because the values $x = 2$ and $y = 1$ satisfy the formula $y = \frac{1}{2}x$, $\left(1 = \frac{1}{2} \bullet 2\right)$.

d. Which line is steeper, $y = \frac{1}{2}x$ or $y = 4x$?

LESSON 4 INTRODUCING SLOPE

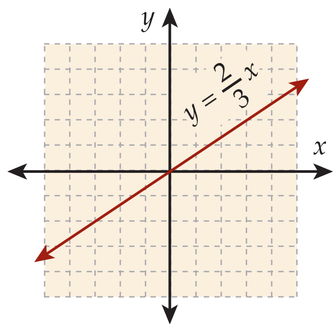
GOAL	CONCEPT BOOK
To introduce the concept of slope as the steepness of a line.	See pages 327–328 in your *Concept Book.*

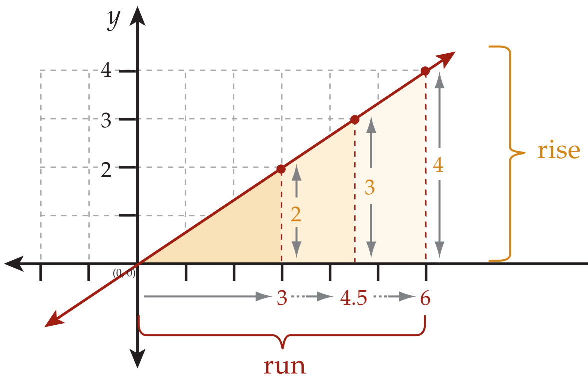
The *slope* of a line is a measure of its steepness.

Example

Consider the line $y = \frac{2}{3}x$.

Between any two points on a line, you can sketch similar right triangles that show the *rise* and *run* of the line between those points.





The *rise* is the vertical distance the line climbs between two points.

The *run* is the horizontal distance the line extends between two points.

In the graph above:

- Between the two points (0, 0) and (3, 2), the rise is 2 and the run is 3.
- Between the two points (0, 0) and (4.5, 3), the rise is 3 and the run is 4.5.

What is the rise and run of the line between (0, 0) and (6, 4)?

The slope of a line is the constant ratio of the corresponding *rise* and *run* between two points on the line:

$$\text{slope} = \frac{\text{rise}}{\text{run}} = \frac{\text{change in } y}{\text{change in } x}$$

Example

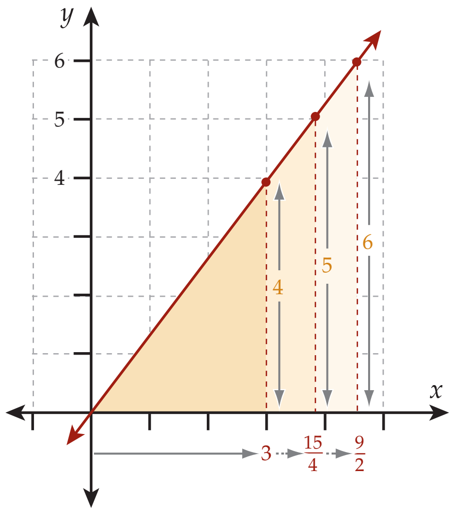
The ratio $\frac{\text{rise}}{\text{run}}$ for $y = \frac{2}{3}x$ is: $\frac{\text{rise}}{\text{run}} = \frac{\text{change in } y}{\text{change in } x} = \frac{2}{3} = \frac{3-2}{4.5-3} = \frac{4-3}{6-4.5}$

Between each pair of points, the slope is the same—the line has a constant slope. Any line with a constant slope will be a straight line.

If you change the ratio of rise to run, the slope of the line will change.

Example

A steeper line has a greater slope:

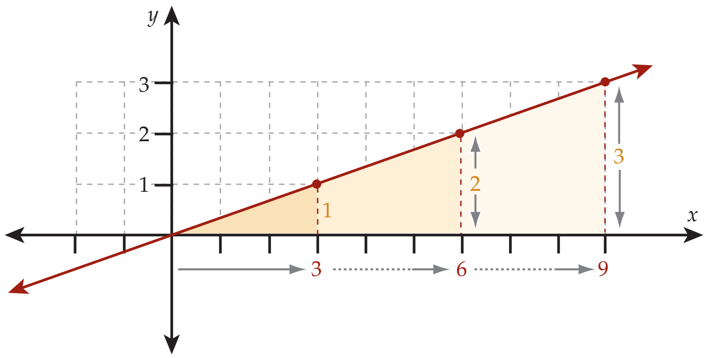


Slope = $\frac{4}{3}$

The constant ratio $\frac{\text{rise}}{\text{run}} = \frac{\text{change in } y}{\text{change in } x} = \frac{4}{3}$ is the slope of the line. $\frac{4}{3} > \frac{2}{3}$, so this line is steeper than the line on the graph on page 14.

Example

A line that is less steep has a smaller slope.



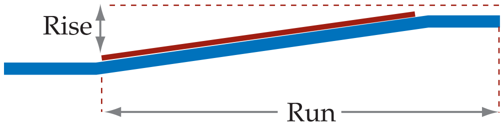
Slope = $\frac{1}{3}$

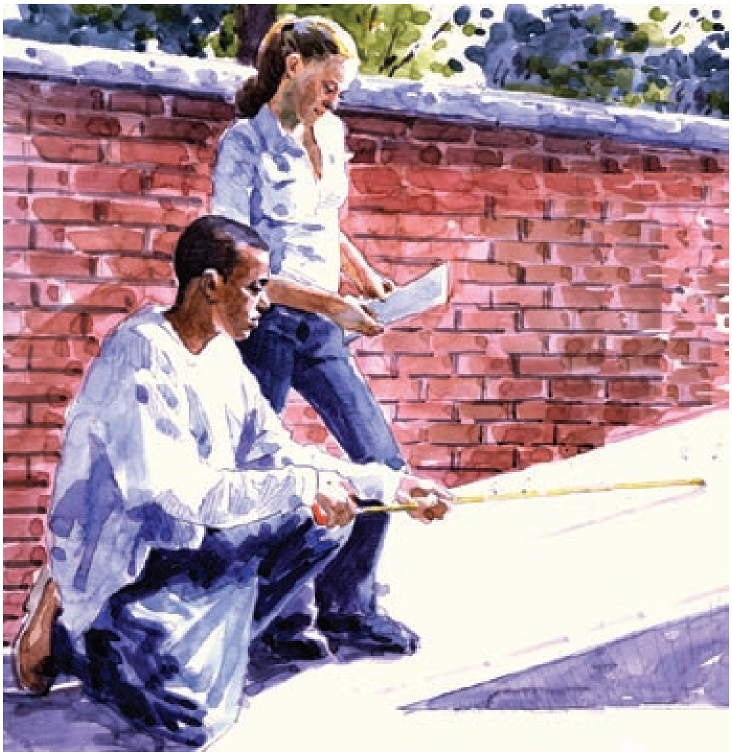
The constant ratio $\frac{\text{rise}}{\text{run}} = \frac{\text{change in } y}{\text{change in } x} = \frac{1}{3}$ is the slope of the line. $\frac{1}{3} < \frac{2}{3}$, so this line is less steep than the line on the graph on page 14.

Work Time

Lisa and Jamal are building a wheelchair ramp for their school store.

This is the diagram they sketched to design the ramp.





1. Suppose the ramp must reach a height of 2 feet. What would the slope of the ramp be if the run was 10 feet?

2. According to the *Americans with Disabilities Act* specifications, the maximum slope allowed for a wheelchair ramp is $\frac{1}{12}$.

   How long does the run need to be for the ramp to reach a height of 2 feet and have a slope of $\frac{1}{12}$?

3. Suppose Lisa and Jamal need to build another ramp that reaches a height of 4 feet. How long does the run have to be for the ramp to have a slope of $\frac{1}{12}$?

4. If the ramp has a run of 6 feet, how high will the ramp have to reach to have a slope of $\frac{1}{12}$?

> **Reminder**
>
> Slope = $\frac{rise}{run}$

Preparing for the Closing

5. In problems 2–4, there were three measures of the wheelchair ramp: the rise, the run, and the constant slope of $\frac{1}{12}$.

   a. Sketch a diagram of each ramp described in problems 2–4.

   b. Sketch a graph to represent the ramps described.

   c. Write a formula that corresponds to each graph.

   d. Does each ramp have a different formula? Why or why not?

6. As the rise of the ramp increases, does the run need to increase or decrease to maintain the same slope?

7. There is another ramp with a rise of 3 feet and a run of 32 feet. Is this ramp steeper or less steep than the ramps that meet *Americans with Disabilities Act* specifications described in problems 2–4?

Skills

Write each ratio as a percent.

a. 4 : 10 b. 57 : 171 c. 6 : 24 d. 8 : 64

Review and Consolidation

For formulas of the form $y = kx$, the constant ratio $k = \frac{y}{x}$ is the slope of the line.

Example

The slope of the line $y = \frac{2}{3}x$ is $\frac{2}{3}$.

You can show these constant ratios by making a ratio table for each formula. Or, you can use right triangles to show the constant ratio between rise and run. You can also use a graph with a constant slope.

1. Make a ratio table showing three ordered pairs of x- and y-values that satisfy the formula $y = \frac{2}{3}x$.

2. Sketch three right triangles that represent three different ramps with a slope of $\frac{2}{3}$. Use the points in your table for problem 1 to define the rises and runs of your ramps.

3. Make a ratio table showing three ordered pairs of x- and y-values that satisfy the formula $y = 2x$.

4. Sketch three right triangles that represent three different ramps with a slope of 2. Use the points in your table for problem 3 to define the rises and runs of your ramps.

5. Make a ratio table showing three ordered pairs of x- and y-values that satisfy the formula $y = \frac{1}{2}x$.

6. Sketch three right triangles that represent three different ramps with a slope of $\frac{1}{2}$. Use the points in your table for problem 5 to define the rises and runs of your ramps.

7. Make a ratio table showing three ordered pairs of x- and y-values that satisfy the formula $y = \frac{1}{12}x$.

8. Sketch three right triangles that represent three different ramps with a slope of $\frac{1}{12}$. Use the points in your table for problem 7 to define the rises and runs of your ramps.

9. Sketch a graph on a coordinate plane that represents your three ramps from problem 6.

10. Sketch a graph on a coordinate plane that represents your three ramps from problem 8.

Homework

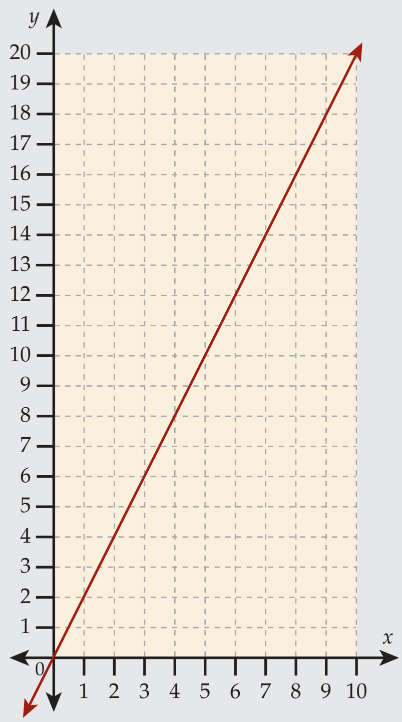
1. Find the rise and run of this line between the points (0, 0) and (5, 10). Sketch a right triangle if it helps you.

2. Find the rise and run of this line between the points (0, 0) and (3.5, 7). Sketch a right triangle if it helps you.

3. Find the rise and run of this line between the points (0, 0) and (0.5, 1). Sketch a right triangle if it helps you.

4. What is the ratio of rise to run in problems 1–3?

5. Write a formula in the form $y = kx$ to represent the line.



GRAPHING NEGATIVE VALUES

LESSON **5**

CONCEPT BOOK	GOAL
See pages 201, 329 in your *Concept Book.*	To work with negative values of two related quantities, and to work with a negative constant of proportionality.

Compare the two (x, y) tables below.

Table 1

x	1.5	2	3.15	5.8	12	25.02
y	4.5	6	9.45	17.4	36	75.06

Table 2

x	–3.15	–2	–1.5	1.5	5.8	25.02
y	–9.45	–6	–4.5	4.5	17.4	75.06

What differences do you notice between the two tables?

What similarities do you notice between the two tables?

Observe that:

- Table 1 has only positive x- and y-values, and Table 2 has both negative and positive values.
- Both tables include non-whole values for x and y.
- Both tables are ratio tables.
- The constant ratio, k, between x- and y-values is 3 in both tables.
- $y = 3x$ represents the relationship between quantities in both tables.

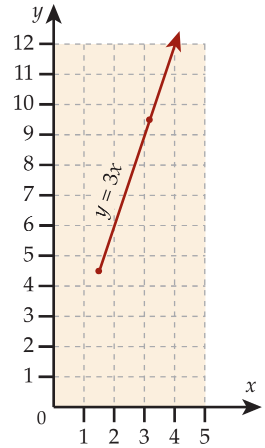
Notice that as x gets larger in the first table, y also gets larger. This is true in the second table as well.

Notice that k is a positive number in this case. But k could also be a negative number. You will see an example of this in Work Time.

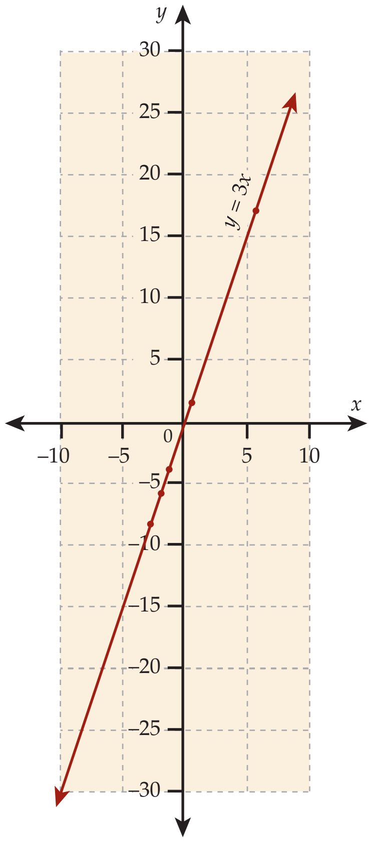
Work Time

1. In both Table 1 and Table 2 on page 19, $k = 3$. Notice that k is a positive number. Here are graphs of the relationship between quantities shown in the two tables:

GRAPH 1



GRAPH 2



a. Notice that there are many more points shown on the graphs than are shown in the tables. Identify at least one more point on each graph that could be included in the corresponding table.

> **Example**
> The point (–4.6, –13.8) could be included in Table 2.

b. Do the graphs climb up to the right or descend down to the right?

c. When x increases by 1 unit, by how much does y increase?

d. Select one point on either graph and find the rise and run of the line between that point and (0, 0).

e. What is the slope of either line?

Use the following table for problems 2–8.

x	–3.15	–2	–1.5	1.5	2	3.15
y	9.45	6	4.5	–4.5	–6	–9.45

2. What is the constant ratio, k, between corresponding values in this table?

3. Is k positive or negative?

4. Compare the two tables at the beginning of the lesson to the table above. In the table above, as x increases does y increase or decrease?

5. Write a formula in the form $y = kx$ to represent the relationship between quantities in the table.

6. Sketch a graph to represent the relationship between the quantities shown in the table.

7. Does your graph slope up to the right (climbing) or slope down to the right (descending)?

8. When x increases by 1 unit, by how much does y increase or decrease? This value is the slope of the graph.

Preparing for the Closing

9. Look again at the graphs in today's lesson.

a. Does each graph intersect the origin, (0, 0)?

b. Do you think that the graph of every relationship with a constant ratio between pairs of values intersects the origin? Say why or why not.

c. Is every graph a straight line?

d. Do you think that the graph of every relationship with a constant ratio between pairs of values is a straight line? Say why or why not.

10. Can a constant of proportionality be a non-whole number? Say how you know.

11. Can a constant of proportionality be a negative number? How would a negative constant of proportionality be represented on a graph?

12. When x increases by 1 unit, the corresponding increase or decrease in y is the slope of the graph. Say why.

Skills

Solve.

a. Jamal and Chen shared 747 mL of juice in the ratio 5 : 4.
How much juice did Jamal get?

b. Keesha cut a piece of string 747 cm long into two pieces in the ratio 1 : 2.
What is the length of the shorter piece of string?

Review and Consolidation

1. Make a table showing at least five ordered pairs of values for a relationship between x and y with each of the following constant ratios.

a. $k = 1$

b. $k = 2$

c. $k = \frac{1}{2}$

d. $k = -1$

e. $k = -2$

f. $k = -\frac{1}{2}$

2. Write a formula to represent each relationship shown in the tables you made in problem 1.

3. Work with a partner. Choose two pairs of values in each of your partner's tables for problem 1. Confirm that each of these pairs of values satisfies the formula that corresponds to the table.

4. Sketch a graph of the relationship with constant ratio $k = \frac{1}{2}$.

5. On the same coordinate axes you used in problem 4, sketch a graph of the relationship with constant ratio $k = -2$.

6. Make at least one observation about how the two graphs in problems 4 and 5 are similar to or different from each other.

Homework

1. Make an (x, y) table in which there is a constant ratio of $-\frac{1}{4}$ between the x- and y-values. Include at least four pairs of corresponding values.
2. Make a prediction: Will the graph of this relationship slope up or down to the right?
3. As x increases by 1 unit, how much does y increase or decrease?
   This value is the slope of the line.
4. Sketch a graph of this relationship.

LESSON 6

RELATIONSHIPS WITHOUT A CONSTANT RATIO

GOAL

To explore graphs of relationships without a constant ratio between corresponding values.

CONCEPT BOOK

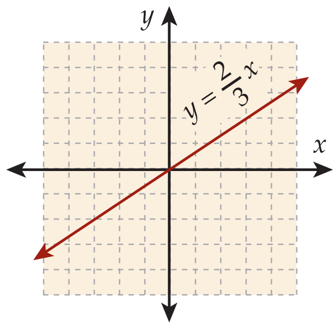
See pages 309–311, 322 in your *Concept Book*.

If there is a constant ratio between pairs of values of two quantities, then this ratio is called a *constant of proportionality*.

The graph of a proportional relationship:

- Is a straight line
- Intersects the origin

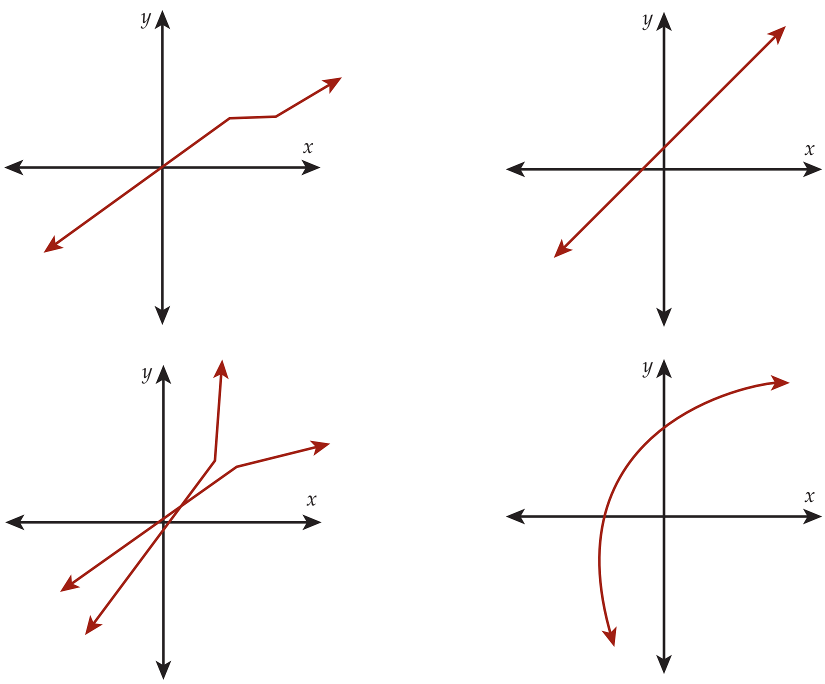
However, not every relationship between quantities has a constant of proportionality; in fact, many do not.



Not Proportional Relationships

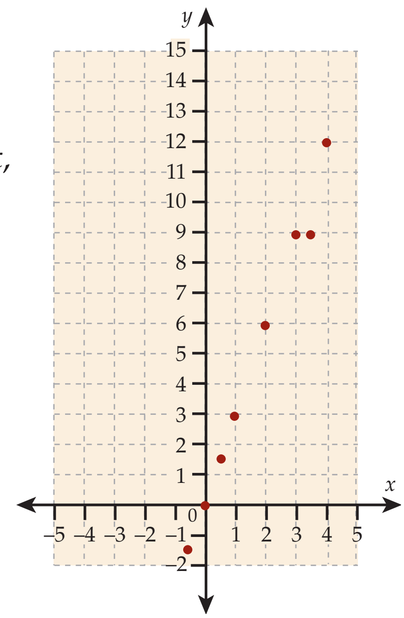
Example

These graphs show relationships between quantities that are *not proportional*. They lack one or both of the characteristics of a proportional relationship.



Work Time

1. a. Identify at least five points shown on the graph and define the coordinates of these points in an (x, y) table. Also, include in your table the coordinates of the point that does not appear to be on the same line as the rest of the points.

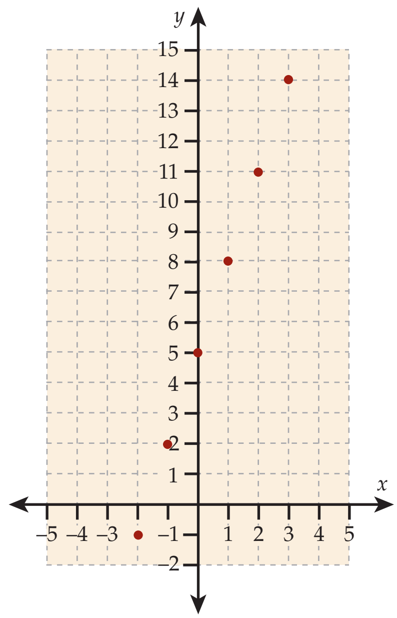


b. Look at your table from part a. Is there a constant ratio, k, between values of x and y? If so, what is it? If not, say how you know.

c. Is this graph a straight line?

d. Does this graph intersect the origin?

2. a. Identify at least five points shown on the graph and define the coordinates of these points in an (x, y) table.



b. Look at your table from part a. Is there a constant ratio, k, between values of x and y? If so, what is it? If not, say how you know.

c. Is this graph a straight line?

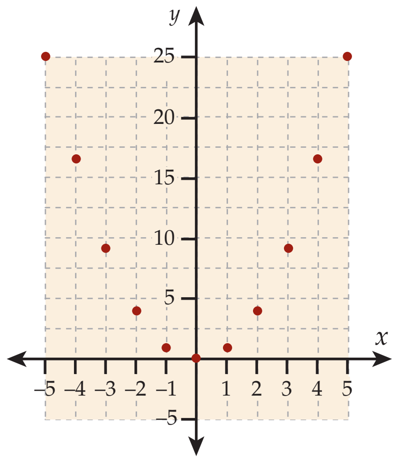
d. Does this graph intersect the origin?

3. a. Identify at least five points shown on the graph and define the coordinates of these points in an (x, y) table.

   b. Look at your table from part a. Is there a constant ratio, k, between values of x and y? If so, what is it? If not, say how you know.

   c. Is this graph a straight line?

   d. Does this graph intersect the origin?



Preparing for the Closing

4. Look again at the graphs in today's problems. None of the graphs represent proportional relationships, but one of the graphs has a constant slope. How can you tell?

5. What features does a proportional relationship display in each representation?

   a. Table b. Graph c. Formula

6. a. Can you think of real-world quantities that might have a relationship represented by the graph for problem 1?

   b. Can you think of real-world quantities that might have a relationship represented by the graph for problem 2?

   c. Can you think of real-world quantities that might have a relationship represented by the graph for problem 3?

7. A formula describes the relationship between every pair of corresponding values for quantities that vary. Do you think it is possible to write a formula for any of the three relationships in today's lesson?

Skills

Solve.

a. A cash prize of $750 was divided between two people in the ratio 7 : 3.

   How much did each person receive?

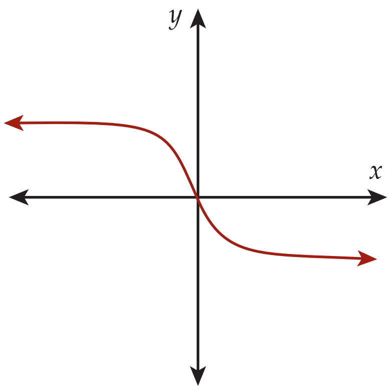
b. There are 748 buckets and spades in a store. The ratio of buckets to spades is 4 : 7.

   How many buckets are in the store?

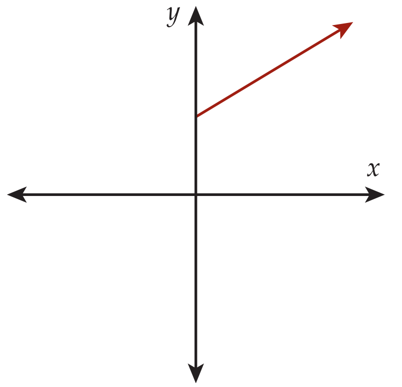
Review and Consolidation

1. Which of the following graphs intersect the origin?

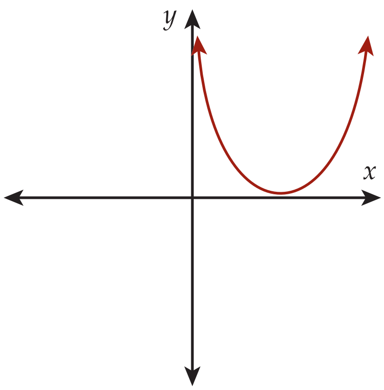
GRAPH 1



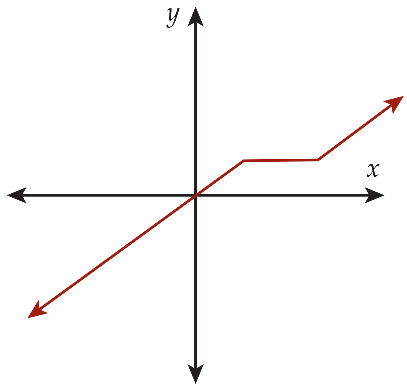
GRAPH 2



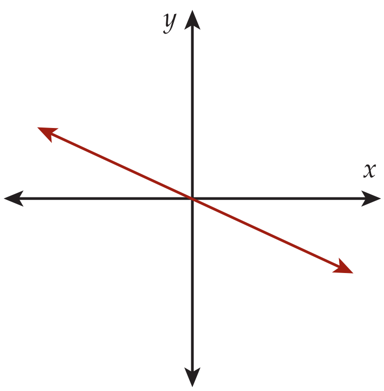
GRAPH 3



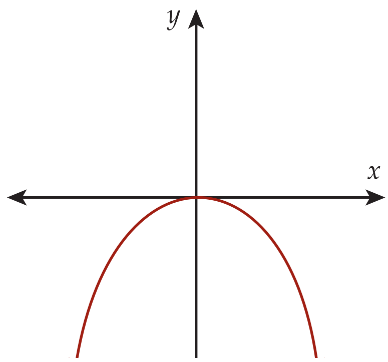
GRAPH 4



GRAPH 5



GRAPH 6



2. Which one of the graphs in problem 1 represents a proportional relationship?

3. Which one of the following tables shows some of the points on the graph that represent the proportional relationship in problem 1?

Ⓐ

x	–10	–2.3	0	1.5	18
y	–40	–9.2	0	6	72

Ⓑ

x	0	2	5	10	24
y	0	1	3	7	17

Ⓒ

x	–1.5	–1	0	1.6	3
y	3	2	0	–3.2	–6

Ⓓ

x	–10.5	–3	0	2.9	12
y	5.25	1.5	0	–1.45	–6

Ⓔ

x	–8	–2.5	0	3	8
y	4	1.25	0	–2	–6

4. Which formula might represent the proportional relationship in problem 1?

Ⓐ $y = -2x$ Ⓑ $y = \frac{1}{2}x$ Ⓒ $y = x + \frac{1}{2}$ Ⓓ $x = \frac{1}{2}y$ Ⓔ $y = -\frac{1}{2}x$

5. How did you determine your answer to problem 4? Be specific.

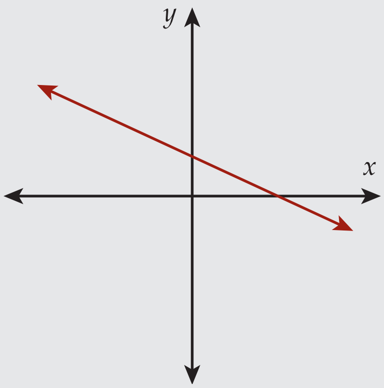
Homework

For each of these graphs:

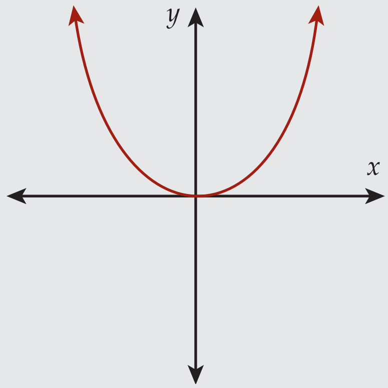
a. State whether or not the graph represents a proportional relationship.

b. Justify your answer to part a by saying how you know.

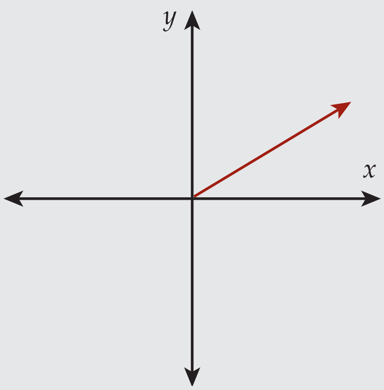
1.



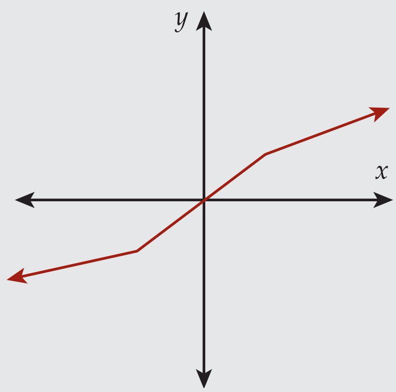
2.



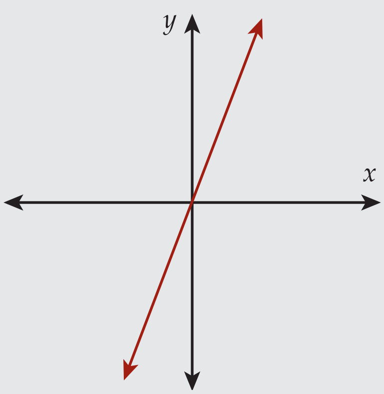
3.



4.



5.



LESSON 7

GRAPHS SHOWING SPEED

GOAL

To graph relationships between distance and time, and to learn that the ratio of distance to time is called speed.

CONCEPT BOOK

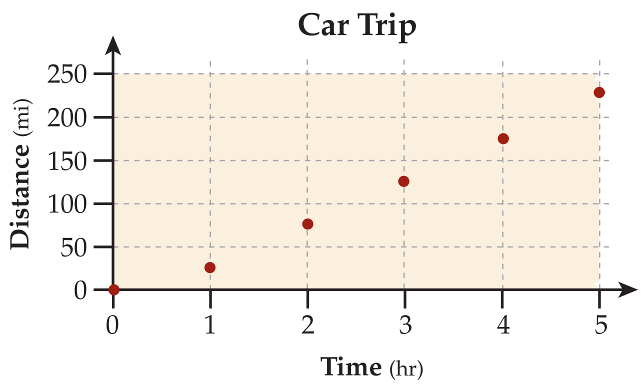
See pages 308–309 in your *Concept Book.*

Dwayne is taking a road trip to the beach with his family. Dwayne is keeping track of the relationship between how much time his family has been driving on the highway (in hours) and the distance they have traveled (in miles). He uses the mile markers along the side of the highway to make notes every hour on how far they have traveled.

The two quantities that vary in relation to each other in this situation are:

- The time spent in the car, in hours
- The distance traveled, in miles

The graph shows the relationship between time and distance during part of Dwayne's family trip.



The ratio $\frac{\textit{distance}}{\textit{time}}$ is a measure of how fast something travels.

The graph shows only part of Dwayne's family trip (the part on the highway). During this part the family traveled at a constant speed. At the beginning and end of the trip (not shown on the graph), the family traveled more slowly.

This ratio is called *speed*. Speed is an example of a *rate of change*. Speed is a rate because the units of the two quantities in the ratio are different.

$$\text{Speed} = \frac{\textit{distance}}{\textit{time}}$$

The speed of Dwayne's family car during the part of the trip represented by the graph above is $\frac{50 \text{ miles}}{1 \text{ hour}}$, or 50 miles per hour (mph).

The slope of the graph is $\frac{\text{rise}}{\text{run}} = \frac{\text{change in } y}{\text{change in } x} = \frac{50}{1} = 50$.

Work Time

1. Sketch a graph that shows Dwayne's family trip from beginning to end, beginning at (0, 0) and ending at (5, 200). Your graph will not be precise because you do not have all of the actual data. Your task is to sketch the general shape of the graph.

2. What do the endpoints of your graph in problem 1 represent?

Use the situation below for problems 3–8.

On her vacation, Rosa used the mile markers along the side of the highway to make notes every hour on how far her family had traveled. This table shows the data that she recorded.

Time (hours)	**Distance** (miles traveled)
0.5	28
1	61
1.5	91
2	118
2.5	152
3	180

3. Plot Rosa's data points on a coordinate plane.

4. Does it make sense to connect the points, or not? Say why.

5. a. During which half-hour interval did Rosa's family travel the farthest distance?

   b. During which half-hour interval did they travel the shortest distance?

6. Remember that *speed* is distance divided by time. *Average speed* during an interval of time is the distance traveled during that interval divided by the amount of time in the interval.

$$Speed = \frac{distance}{time} \qquad average\ speed = \frac{distance\ traveled\ in\ that\ interval}{time\ in\ that\ interval}$$

a. What was the average speed of the car during the first half-hour of the trip?

b. What was the average speed of the car during the second half-hour of the trip?

c. What was the average speed of the car for the whole trip?

7. Is the formula $y = 60x$ (or *distance* = 60 • *time*) a good approximation of Rosa's data? Why or why not?

8. a. An important formula for modeling the mathematical relationship of an object moving at a constant speed is: *distance* = *speed* • *time*. Sketch a graph, with time on the x-axis and distance traveled on the y-axis, to represent a car traveling at 60 miles per hour for 3 hours.

b. How does this graph compare with the graph of Rosa's data?

Preparing for the Closing

9. How is speed represented on a graph?

10. Describe how speed and slope are related to each other.

11. When graphing a proportional relationship described by a formula, you need only two points to give an accurate representation of the relationship between quantities. Say why.

12. When graphing real data, you need more than two data points to give an accurate representation of the relationship between quantities. Say why.

Skills

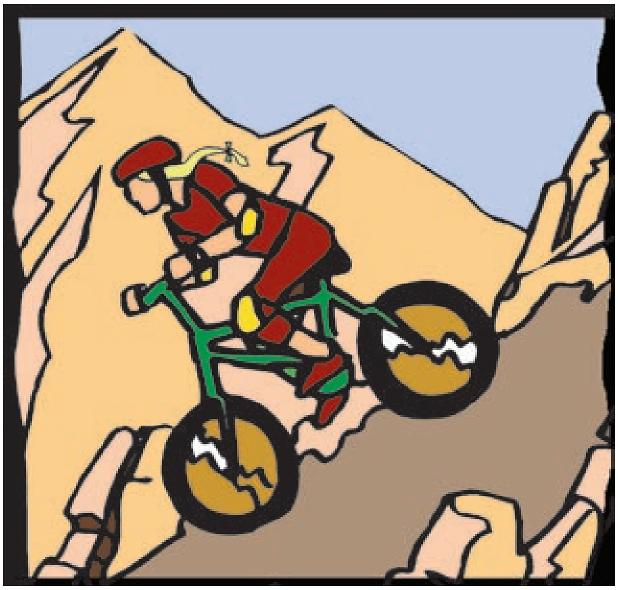
Solve.

a. The ratio of Lisa's weight to her mother's weight is 7 : 8.
If Lisa's weight is 49 kg, find the total weight of Lisa and her mother.

b. The ratio of the number of girls to the number of boys in a class is 5 : 8.
If there are 55 girls in the class, what is the total number of students in the class?

Review and Consolidation

1. A bicycle rider starts at the top of a hill and rides downhill at a constant speed of 20 miles per hour.



a. Make a table showing five ordered pairs for the distance *d* (in miles) and the time *t* (in hours) that the bicycle rider travels when she is riding downhill.

b. Is the ratio $\frac{d}{t}$ the same at every point in the table?
In other words, is the table a ratio table?

c. What is the constant of proportionality, $\frac{d}{t}$, in this situation?

d. Would the graph representing this situation intersect the origin, (0, 0)?

e. Write a formula to represent the relationship between *d* and *t*.

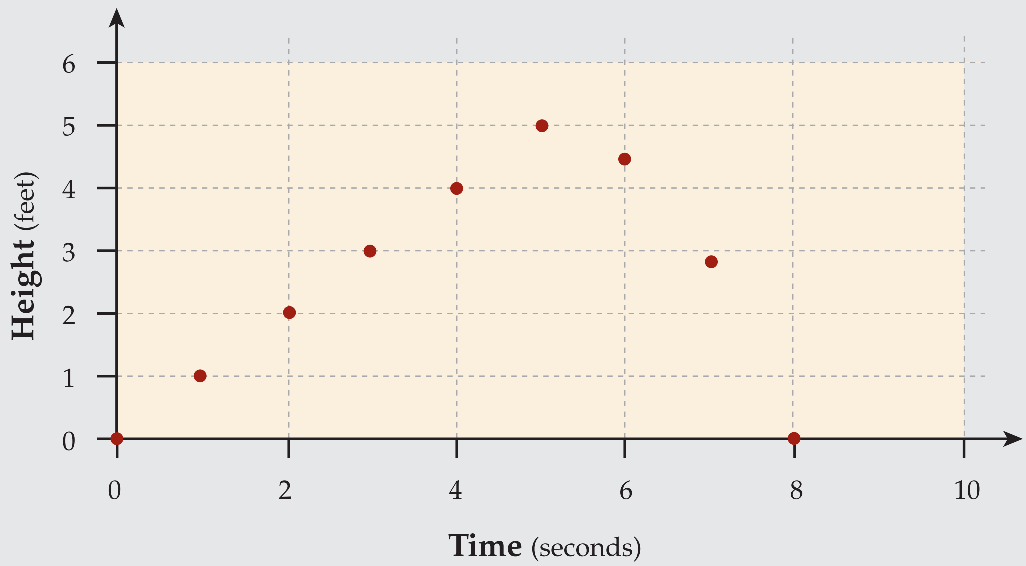
f. When the value of *t* doubles, triples (three times as big), quadruples (four times as big), and so on, does the value of *d* also double, triple, quadruple, and so on?

g. Are *d* and *t* proportional to each other?

Homework

A friend of Jamal is practicing jumping off of a ramp on his skateboard. He skates up the ramp for about 5 seconds, and then he jumps off at the end, and tries to land on his feet. Below is a graph showing his height over time.

1. What part of the graph represents a proportional relationship?
2. What part of the situation does the proportional part of the graph represent?
3. How fast is Jamal's friend moving when he skates up the ramp?
4. How long does it take for the skateboarder to land after jumping off the end of the ramp?



GRAPHING GEOMETRIC RELATIONSHIPS

LESSON 8

CONCEPT BOOK

See page 329 in your *Concept Book*.

GOAL

To use what you know about graphing to represent and interpret geometric relationships.

Here is a formula describing the relationship between a circle's radius and its circumference:

$$C = 2\pi r$$

radius
circumference
constant of proportionality

The two quantities that vary in relation to each other in this formula are:

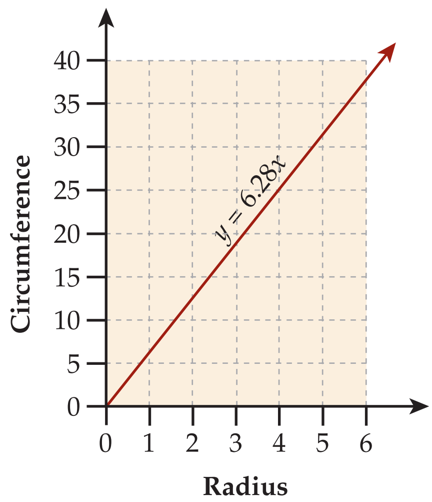
- r, the radius of the circle
- C, the circumference of the circle

The constant of proportionality is 2π.

CONCEPT BOOK

See pages 235–236 and 239–240 for definitions of *radius*, *diameter*, and *circumference* of a circle.

Here is a graph (using 3.14 for π) showing the relationship between the radius, r, and the circumference, C:



$$y = 6.28x$$

radius
circumference
constant of proportionality

Think about why the graph is entirely in the first quadrant.
Negative values would not make sense for either of the quantities in this situation.

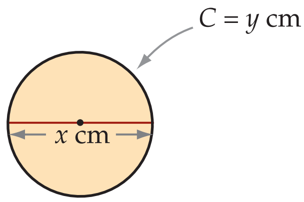
The graph represents a proportional relationship because it is a straight line and it goes through the origin, (0, 0).

Work Time

In today's lesson, you will use 3.14 as an approximation of π. For problems 1–5:

- Make a table of values for x and y.
- Sketch a graph to represent the relationship between x and y.
- Sketch diagrams first if doing so helps you to understand the situation.

1. A circle has a diameter of x cm and a circumference of y cm.



a. Is your graph a straight line?

b. Does your graph intersect the origin?

c. Is the relationship between x and y proportional? Say how you can tell from your graph.

d. What is y when $x = 3$? Label this point on your graph.

e. What is x when $y = 22$? Label this point on your graph.

f. Write a formula to represent the relationship between x and y.

2. A circle has a circumference of x cm and a radius of y cm.

a. Is your graph a straight line?

b. Does your graph intersect the origin?

c. Is the relationship between x and y proportional? Say how you can tell from your graph.

d. What is y when $x = 22$? Label this point on your graph.

e. What is x when $y = 4$? Label this point on your graph.

f. Write a formula to represent the relationship between x and y.

3. A square has side lengths of x cm and a perimeter of y cm.

a. Is your graph a straight line?

b. Does your graph intersect the origin?

c. Is the relationship between x and y proportional? Say how you can tell from your graph.

d. What is y when $x = 4$? Label this point on your graph.

e. What is x when $y = 4$? Label this point on your graph.

f. Write a formula to represent the relationship between x and y.

CONCEPT BOOK

See pages 232–233 and 242 for definitions of the *perimeter* and the *area* of a square.

4. A square has side lengths of x cm and an area of y cm^2.

a. Is your graph a straight line?

b. Does your graph intersect the origin?

c. Is the relationship between x and y proportional? Say how you can tell from your graph.

d. What is y when $x = 4$? Label this point on your graph.

e. What is x when $y = 4$? Label this point on your graph.

f. Write a formula to represent the relationship between x and y.

5. A rectangle has a length of 3 cm, a width of x cm, and a perimeter of y cm.

a. Is your graph a straight line?

b. Does your graph intersect the origin?

c. Is the relationship between x and y proportional? Say how you can tell from your graph.

d. What is y when $x = 1$? Label this point on your graph.

e. What is x when $y = 14$? Label this point on your graph.

f. Write a formula to represent the relationship between x and y.

Preparing for the Closing

6. A square has a side length of s cm. The perimeter of the square is p cm.

For each formula below, sketch a graph showing the relationship between s and p.

$$p = 4s \qquad\qquad s = \frac{1}{4}p$$

Explain why your graphs are two different ways of representing the same relationship.

Skills

Solve.

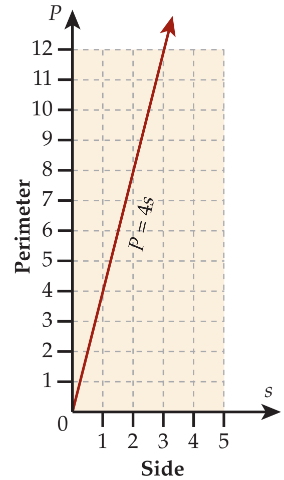
a. The length of a garden is 56 m. The ratio of the garden's width to its length is 3 : 8. What is the width of the garden?

b. The total length of two strings is 136 cm. The ratio of the length of string X to the length of string Y is 9 : 8. What is the length of each piece of string?

Review and Consolidation

Consider the two formulas $P = 4s$ and $s = \frac{1}{4}P$ for problems 1 – 3. Both formulas describe the relationship between the following two quantities:

- s, the side of a square
- P, the perimeter of the square



1. What is the constant of proportionality in each formula?

2. Use the graph at right to approximate these values.

a. What is P when $s = 2$?

b. What is P when $s = 4$?

c. What is s when $P = 4$?

d. What is s when $P = 12$?

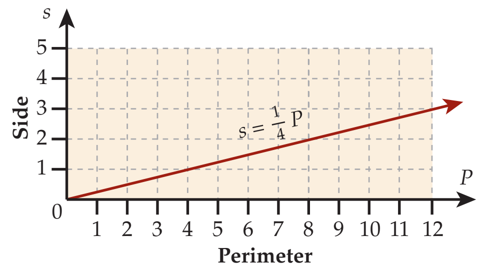
3. Use the graph at right to approximate these values

a. What is s when $P = 4$?

b. What is s when $P = 12$?

c. What is P when $s = 2$?

d. What is P when $s = 4$?



Consider the two formulas $C = \pi d$ and $d = \frac{1}{\pi}C$ for problems 4 – 6. Both formulas describe the relationship between the following two quantities:

- d, the diameter of a circle
- C, the circumference of the circle

4. What is the constant of proportionality in each formula?

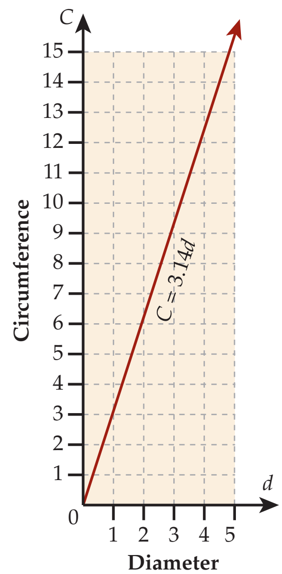
5. Use the graph at right (which uses 3.14 for π) to approximate these values.

a. What is C when $d = 2$?

b. What is C when $d = 4$?

c. What is d when $C = 3.14$?

d. What is d when $C = 9.42$?



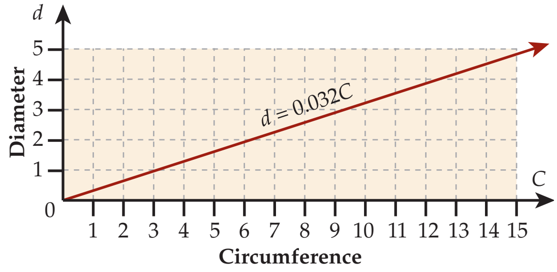
6. Use the graph at right (which uses 3.14 for π) to approximate these values.

a. What is d when $C = 3.14$?

b. What is d when $C = 9.42$?

c. What is C when $d = 2$?

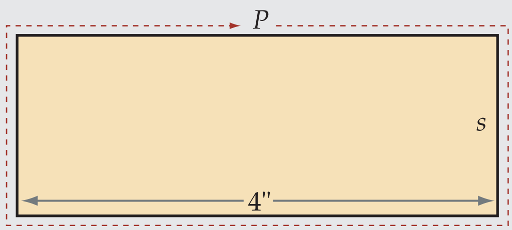
d. What is C when $d = 4$?



Homework

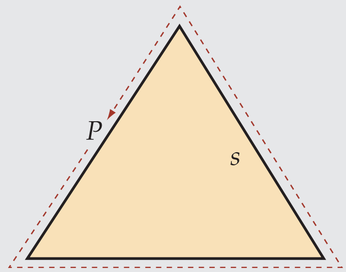
Use diagrams A and B for problems 1 and 2.

Diagram A



The rectangle has a length of 4 inches, a width of s inches, and a perimeter of P inches.

Diagram B



The equilateral triangle has a side length of s and a perimeter of P.

CONCEPT BOOK

See page 220 for the definition of equilateral triangles.

1. Match each formula below to its diagram, A or B.

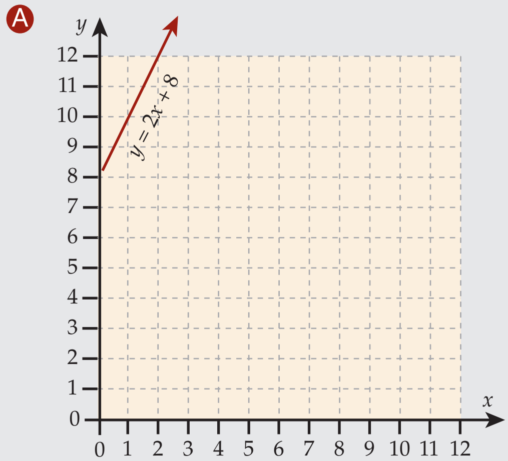
a. $P = 3s$

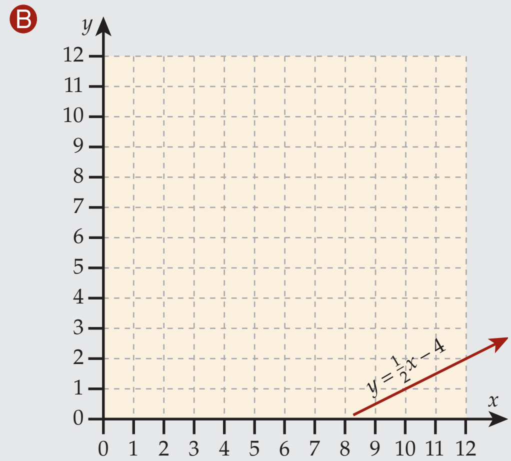
b. $P = 2s + 8$

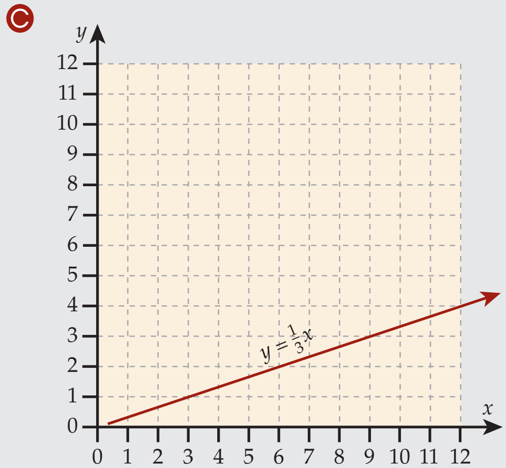
c. $s = \frac{1}{3}P$

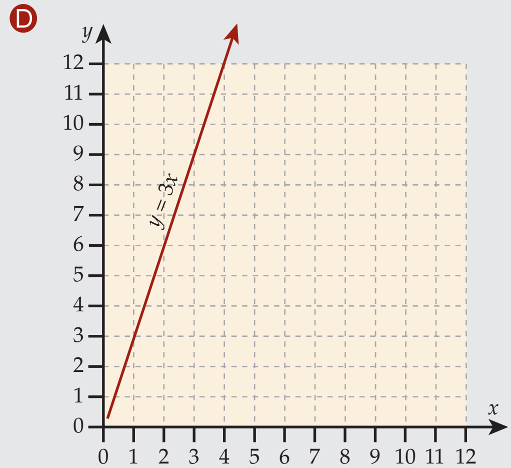
d. $s = \frac{1}{2}P - 4$

2. Match each graph below to its diagram, A or B, shown in problem 1.









3. Which of the graphs in problem 2 have a constant slope?

4. Which of the graphs in problem 2 represent proportional relationships?

GRAPHING DISCRETE AND CONTINUOUS DATA

LESSON **9**

CONCEPT BOOK	GOAL
See pages 205–209 in your *Concept Book.*	To use what you know about graphing proportional relationships to represent and interpret discrete and continuous data.

In this lesson, you will compare data from two different situations, a water tank being filled and emptied, and books being stacked. In both cases, there is a *rate of change*.

In Lesson 7, you learned that speed was an example of a rate of change.

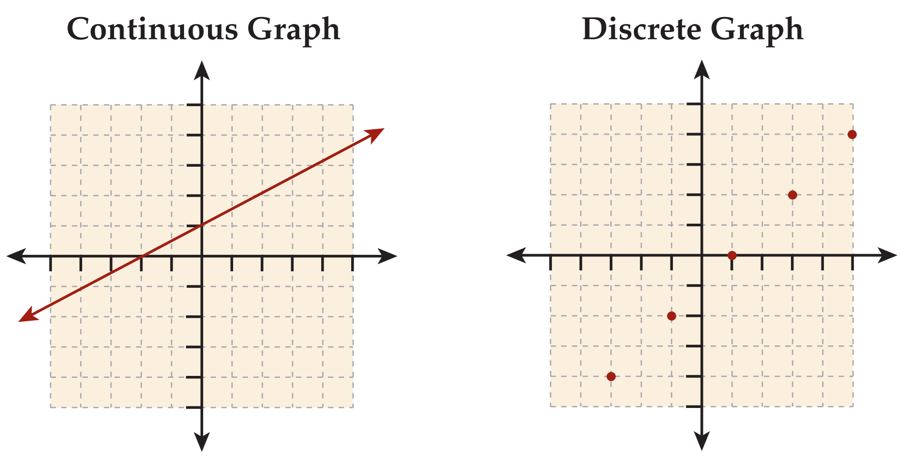
A rate of change can be positive (when both quantities either increase or decrease) or negative (when one quantity decreases as the other quantity increases).

- A graph that shows a *constant increase* is a straight line with a positive slope.
- A graph that shows a *constant decrease* is a straight line with a negative slope.

In some situations, the increase or decrease is *continuous* because the values of both quantities vary without interruption. In other situations, the increase or decrease is *discrete* because one or both of the quantities varies by jumping from one value to the next—for example, number of cans of dog food, or measurements taken at intervals.

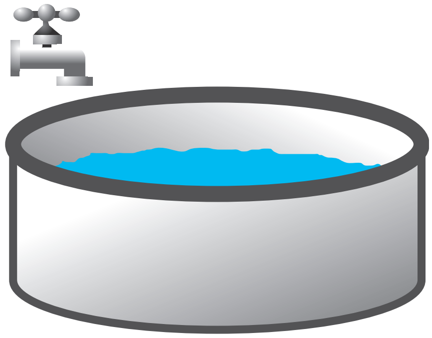
- A graph with data points that are connected is a *continuous* graph.
- A graph with data points that are not connected is a *discrete* graph.

Example



Work Time

Problems 1 and 2: Filling a Water Tank



You are filling an empty water tank.
A faucet sits right above the tank.

When you turn the faucet on, water pours into the tank at a constant rate.

- After one minute, the tank contains 2 liters.
- After two minutes, it contains 4 liters.
- After three minutes, it contains 6 liters.

1. a. Make an (x, y) table to represent this situation.
   - Let x represent the amount of time that has passed (in minutes).
   - Let y represent the amount of water in the tank (in liters).

b. When x increases by 1, by how much does y increase or decrease? This value is the slope.

c. Is there a *constant increase* in y as x increases? In other words, does y increase by the same amount with each increase in x of 1?

d. Are x and y proportional to each other? Say how you know.

2. a. Sketch a graph to represent the relationship between x and y.

b. Does your graph have a positive slope, a negative slope, or a slope of 0 (horizontal line)?

c. Is your graph a straight line?

d. Does your graph intersect the origin?

e. Would negative values for either of the quantities in this situation make sense? Say why or why not.

f. Do the quantities in this situation vary through all values greater than 0, or do discrete data points make sense? Say why or why not.

Check that your graph reflects your answers.

Problems 3 and 4: Stacking Books

You are stacking textbooks. The textbooks are identical in size.

With each book added to the stack, the height of the stack increases.

- A stack with 1 book is 2.4 cm tall.
- A stack with 2 books is 4.8 cm tall.
- A stack with 3 books is 7.2 cm tall.

3. a. Make an (x, y) table to represent this situation.
   - Let x represent the number of books in the stack.
   - Let y represent the height of the stack of books (in cm).

b. When x increases by 1, by how much does y increase or decrease? This value is the slope.

c. Is there a *constant increase* in y as x increases? In other words, does y increase by the same amount with each increase in x of 1?

d. Are x and y proportional to each other? Say how you know.

4. a. Sketch a graph to represent the relationship between x and y.

b. Does your graph have a positive slope, a negative slope, or a slope of 0?

c. Is your graph a straight line?

d. Does your graph intersect the origin?

e. Would negative values for either of the quantities in this situation make sense? Say why or why not.

f. Do the quantities in this situation vary through all values greater than 0, or do discrete data points make sense? Say why or why not.

Check that your graph reflects your answer.

Preparing for the Closing

5. Consider the water tank situation in problems 1–2.

a. How would the graph representing the situation need to be adjusted if the water coming out of the faucet were pouring into the tank faster?

b. How would the graph representing the situation need to be adjusted if the water coming out of the faucet were only dripping into the tank?

c. How would the graph representing the situation need to be adjusted if the faucet were shut off when the tank was filled to 30 liters, and then the tank began to leak?

d. How would the graph representing the situation need to be adjusted if the faucet were turned on and off several times during an hour?

e. How would the graph of the water tank look different if, instead of starting out empty, it started with 5 liters of water in it?

6. Consider the book stacking situation in problems 3–4.

a. How would the graph representing the situation need to be adjusted if the textbooks were thicker than in the example?

b. How would the graph representing the situation need to be adjusted if the textbooks were thinner than in the example?

c. How would the graph representing the situation need to be adjusted if you stopped adding books when the stack reached 240 cm, and then started taking books off the stack to distribute them to students?

d. How would the graph representing the situation need to be adjusted if the books within the stack had different thicknesses?

e. How would the graph of the stack of books look different if instead of being stacked on the floor, the books were stacked on a desk 50 cm tall?

7. Use the situations in today's lesson to explain in your own words the difference between a discrete and a continuous graph.

8. Use the situations in today's lesson to explain in your own words the difference between a positive rate of change and a negative rate of change.

Skills

Write each ratio in its simplest form.

a. 3 : 27 : 15 b. 4 : 36 : 20 c. 5 : 50 : 30 d. 6 : 66 : 42

Review and Consolidation

1. Every week, the town of Lakeside has a farmers' market in which farmers sell the produce they grow. One farmer sells apples for 30 cents each.

a. Sketch a graph to represent the relationship between the number of apples and the total cost of the apples. Think carefully about whether to connect the points on your graph.

b. What is the slope of your graph?

c. Write a formula to represent the relationship between the two quantities in this situation.

d. Is the total cost of the apples proportional to the number of apples purchased? Say how you know.

2. Another farmer is selling apples for $1.50 per pound.

a. Sketch a graph to represent the relationship between the weight of apples and the total cost of the apples. Think carefully about whether to connect the points on your graph.

b. What is the slope of your graph?

c. Write a formula to represent the relationship between the two quantities in this situation.

d. Is the total cost of the apples proportional to the weight of the apples purchased? Say how you know.

3. Which of the situations in problems 1 and 2 has discrete data? Say why.

4. Which of the situations in problems 1 and 2 has continuous data? Say why.

5. Which of the situations in problems 1 and 2 involves a constant increase? Say why.

6. Does either of the situations in problems 1 and 2 involve a constant decrease? Say why.

Homework

1. How thick is each book in the stack?

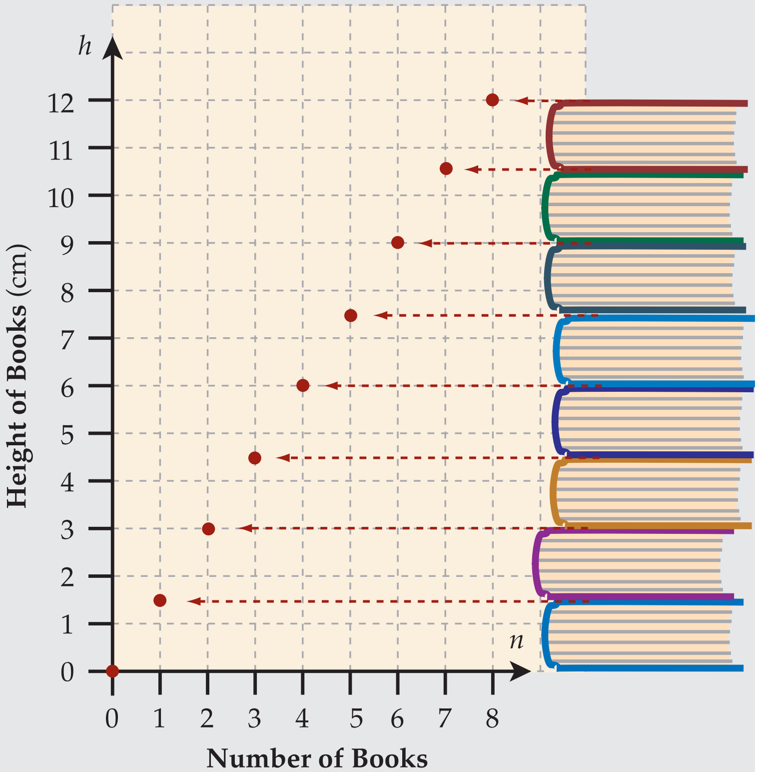
2. How is the thickness of the books shown on the graph?

3. The slope of a graph is the ratio $\frac{\text{rise}}{\text{run}} = \frac{\text{change in } y}{\text{change in } x}$.

   a. What is the change in y on the graph between the points (0, 0) and (2, 3)?

   b. What is the change in x on the graph between the points (0, 0) and (2, 3)?

   c. What is the slope of the graph between the points (0, 0) and (2, 3)?



4. Explain how you can tell that the graph represents each of the following:

   a. A proportional relationship

   b. A relationship of constant increase

   c. Discrete data points

PROGRESS CHECK

LESSON 10

CONCEPT BOOK	GOAL
See pages 205–209, 327–330 in your *Concept Book.*	To review using graphs to represent relationships between quantities that vary, and to identify proportional relationships using graphs, tables, and formulas.

Work Time

For problems 1–4, refer to the following situation:

A car is traveling at a constant speed of 25 miles per hour. The variable y represents distance, in miles; the variable x represents the amount of time that has passed, in hours.

1. Make an (x, y) table showing at least four pairs of corresponding values for x and y. Include 1, 2, and 3 as three of the four x-values.

2. Look at your table from problem 1.

a. When x increases from 1 to 2, by how much does y increase or decrease?

b. When x increases from 2 to 3, by how much does y increase or decrease?

c. Is there a constant increase, constant decrease, or neither in this situation?

3. Sketch a graph representing the relationship between x and y. When sketching your graph, be sure to do the following:

- Decide whether your graph should be continuous or discrete.
- Use scales on each axis that allow you to easily plot the points you need to show.
- Label your axes with the quantities that vary in relation to each other.

4. Is y proportional to x? Say how you can tell from the graph.

For problems 5–8, refer to the following situation:

A water tank contains 10 gallons of water, but starts leaking at a constant rate of 0.5 gallons per minute. The variable y represents the amount of water in the tank, in gallons; the variable x represents the amount of time that has passed, in minutes.

5. Make an (x, y) table showing at least four pairs of corresponding values for x and y. Include 1, 2, and 3 as three of the four x-values.

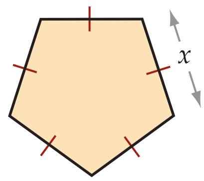
6. Look at your table from problem 5.
   a. When x increases from 1 to 2, by how much does y increase or decrease?
   b. When x increases from 2 to 3, by how much does y increase or decrease?
   c. Is there a constant increase, constant decrease, or neither in this situation?

7. Sketch a graph representing the relationship between x and y.
   When sketching your graph, be sure to do the following:
   - Decide whether your graph should be continuous or discrete.
   - Use scales on each axis that allow you to easily plot the points you need to show.
   - Label your axes with the quantities that vary in relation to each other.

8. Is y proportional to x? Say how you can tell from the graph.

For problems 9–12, refer to the following situation:

A pentagon has sides of equal length (it is a regular pentagon). The variable x represents the length of each side of the pentagon, in cm; the variable y represents the perimeter of the pentagon, in cm.



9. Make an (x, y) table showing at least four pairs of corresponding values for x and y. Include 1, 2, and 3 as three of the four x-values.

10. Look at your table from problem 9.
   a. When x increases from 1 to 2, by how much does y increase or decrease?
   b. When x increases from 2 to 3, by how much does y increase or decrease?
   c. Is there a constant increase, constant decrease, or neither in this situation?

11. Sketch a graph representing the relationship between x and y. When sketching your graph, be sure to do the following:
   - Decide whether your graph should be continuous or discrete.
   - Use scales on each axis that allow you to easily plot the points you need to show.
   - Label your axes with the quantities that vary in relation to each other.

12. Is y proportional to x? Say how you can tell from the graph.

Preparing for the Closing

13. Which of the (x, y) tables in today's lesson are ratio tables? Say how you know.

14. Which of the graphs in today's lesson represent a proportional relationship? Say how you know.

15. Which of the graphs in today's lesson have a constant slope? Say how you know.

16. Which of the graphs in today's lesson have a negative slope? Say how you know.

17. Which of the relationships between x and y in today's lesson involves a rate? Say how you know.

Skills

Solve.

a. Rosa, Lisa, and Lisa's sister Annie share a cash prize in the ratio of 1 : 9 : 5.

If the biggest share is $333, what is the total prize money?

b. The ratio of Rosa's weight to Lisa's sister weight to her friend Sarah's weight is 6 : 5 : 4.

If Sarah weighs 28 kg, what is Rosa's weight?

Review and Consolidation

Look again at the three situations in today's Work Time problems:

Situation 1
A car is traveling at a constant speed of 25 miles per hour. The variable y represents distance, in miles; the variable x represents the amount of time that has passed, in hours.

Situation 2
A water tank contains 10 gallons of water, but starts leaking at a constant rate of 0.5 gallons per minute. The variable y represents the amount of water in the tank, in gallons; the variable x represents the amount of time that has passed, in minutes.

Situation 3
A pentagon has sides of equal length (it is a regular pentagon). The variable x represents the length of each side of the pentagon, in cm; the variable y represents the perimeter of the pentagon, in centimeters.

Use your work during Work Time to help you answer the following questions.

1. Identify the constant of proportionality for each situation that is proportional.

2. Which of the following formulas describes the relationship between quantities in Situation 1?

   (A) $y = x + 25$ (B) $x = 25y$ (C) $y = 25x$ (D) $y = \frac{25}{x}$

3. Which of the following formulas describes the relationship between quantities in Situation 2?

   (A) $y = 10x - \frac{1}{2}$ (B) $y = 10 - \frac{1}{2}x$ (C) $x + y = 10 - \frac{1}{2}$ (D) $y = \frac{1}{2}x + 10$

4. Which of the following formulas describes the relationship between quantities in Situation 3?

   (A) $5y = x$ (B) $y = x + 5$ (C) $x = 5y$ (D) $y = 5x$

5. Translate each of the correct formulas in problems 2–4 into words. Use the descriptions of the situations above to refer to the quantities correctly.

6. Each of the correct formulas in problems 2–4 defines y "in terms of" x. In each case, there is an inverse description of the relationship between x and y. Focus on the inverses in Situations 1 and 3; these formulas define x "in terms of" y:

   Situation 1: $x = \frac{y}{25}$ In words, this formula says, "The number of hours is equal to the number of miles divided by 25."

   Situation 3: $x = \frac{y}{5}$ Translate this formula into words. Use the description of the situation above to refer to the quantities correctly.

Homework

1. a. Review the problems in this lesson and find three that were difficult for you.

   b. Use your *Concept Book* and previous lessons to work through these three problems more carefully until you understand them. If you still have questions about the problems, be prepared to ask them during the next lesson.

2. Write a summary page of the concepts you have learned in Lessons 1 through 9.

LEARNING FROM THE PROGRESS CHECK

LESSON **11**

CONCEPT BOOK	GOAL
See pages 327–330 in your *Concept Book.*	To self-assess errors made on the Progress Check and continue working on similar problems.

Work Time

For each of the following situations:

- Sketch a graph that represents the relationship between x and y. Think carefully about whether each graph should be discrete or continuous, and label the axes with the quantities in the relationship.
- State whether the relationship is proportional.

If you need help, sketch a diagram, make an (x, y) table, and write a formula to represent the relationships between quantities.

1. A rectangle with a length of 5 meters has a width of x meters and a perimeter of y meters.

2. A stack of x identical books that are each 3.6 cm thick has a height of y cm.

3. Chen's younger brother, Winston, is 12 years old and 57 inches tall. Chen is 15 years old and 68 inches tall. His older sister, Amy is 17 years old and 65 inches tall. His older brother, Jong, is 19 years old and 67 inches tall. A person who is x years old has a height of y inches.

4. A square has sides of length x meters and an area of y square meters.

5. A plant that starts out 0 inches tall grows 0.5 inches per week. After x weeks, the plant is y inches tall.

6. The initial temperature of the water in a bathtub is 10°C. The temperature drops at a constant rate of 2 degrees each minute. After x minutes, the temperature is y degrees.

7. A book that weighs 1.2 lb costs \$8.99. Another book that weighs 1.2 lb costs \$15.50. A book that weighs 0.6 lb costs \$5.49. A book that weighs 2.5 lb costs \$26.00. The price of a book with a weight of x pounds is y dollars.

8. A bicycle rider travels at a constant speed of 12 miles per hour. After x hours, he has traveled y miles.

9. The thickness of one coin is 2 mm. A stack of x identical coins is y mm tall.

10. An office supply store is having a sale, and erasers are on sale for \$0.25 each. The total cost of x erasers is y dollars.

Preparing for the Closing

11. Summarize your understanding of the following key concepts from Units 5 and 6. How are these concepts alike? How are they different?
   - Unit ratio
   - Similarity ratio
   - Constant ratio
   - Constant multiple
   - Rate
   - Constant of proportionality
   - Slope

Skills

Solve.

a. A machine fills 55 bottles of ketchup in 11 minutes.
   How many bottles of ketchup does it fill in one minute?

b. Jamal is paid \$66 for working 12 hours.
   What is his rate of pay per hour?

Review and Consolidation

1. a. Make two tables: one that shows a relationship between quantities with a constant ratio between pairs of values, and one that shows a relationship between quantities without a constant ratio between pairs of values.

b. Compare your tables with those of a partner. What do your tables have in common? How do your tables differ? Compare the following features of your tables:

- Positive values versus negative values
- Whole values versus non-whole values
- Constant ratio between values versus no constant ratio
- Whole constant ratio versus non-whole constant ratio

2. a. Sketch two graphs: one that represents a relationship between quantities with a constant ratio between pairs of values, and one that represents a relationship between quantities without a constant ratio between pairs of values.

b. Compare your graphs with those of a partner. What do your graphs have in common? How do your graphs differ? Compare the following features of your graphs:

- Discrete versus continuous
- Straight versus crooked, bent, or curved
- Intersects origin versus crosses y-axis at somewhere other than zero
- Crosses y-axis at point with positive y-value versus crosses y-axis at point with negative y-value

3. a. Write two formulas: one that represents a relationship between quantities with a constant ratio between pairs of values, and one that represents a relationship between quantities without a constant ratio between pairs of values.

b. Compare your formulas with those of a partner. What do your formulas have in common? How do your formulas differ? Compare the following features of your formulas:

- Form $y = kx$ versus another form
- Positive k value versus a negative k value
- Whole k value versus a value of k that is not whole

Homework

1. Think again about the situation in Work Time problem 5: A plant that starts out 0 inches tall grows 0.5 inches per week. After x weeks, the plant is y inches tall.

a. How tall will the plant be after 6 weeks?

- (A) 6 inches
- (B) 0.5 inches
- (C) 1.5 inches
- (D) 1 foot
- (E) 3 inches

b. In how many weeks will the plant have grown to a height of 17.5 inches?

- (A) 17.5 weeks
- (B) 35 weeks
- (C) 8.75 weeks
- (D) 350 weeks
- (E) 8.5 weeks

c. Suppose the seedling started out 3 inches tall and grew at the same rate. How would the graph of this situation be different from the first situation? Choose all that apply:

- (A) The slope would be different—it would be positive.
- (B) The slope would be different—it would be negative.
- (C) Where it crosses the y-axis would be different.
- (D) The graph would be a curve instead of a straight line.
- (E) The graph would be a straight line instead of a curve.
- (F) The graph would represent a proportional relationship instead of a non-proportional relationship.
- (G) The graph would no longer represent a proportional relationship.

2. Look again at the situation in Work Time problem 6: The initial temperature of the water in a bathtub is 10°C. The temperature drops at a constant rate of 2 degrees each minute. After x minutes, the temperature is y degrees.

a. What will be the temperature of the water after 3 minutes?

Ⓐ 16°C

Ⓑ 4°C

Ⓒ 3°C

Ⓓ 6°C

Ⓔ 12°C

b. After how many minutes will the temperature of the water reach 0°C?

Ⓐ 10 minutes

Ⓑ 20 minutes

Ⓒ 50 minutes

Ⓓ 2 minutes

Ⓔ 5 minutes

c. Suppose the water started out at 0°C and was being heated at a constant rate of 2 degrees each minute. How would the graph of this situation be different from the first situation?

Ⓐ The slope would be different—it would be positive.

Ⓑ The slope would be different—it would be negative.

Ⓒ Where it crosses the y-axis would be different.

Ⓓ The graph would be a curve instead of a straight line.

Ⓔ The graph would be a straight line instead of a curve.

Ⓕ The graph would represent a proportional relationship instead of a non-proportional relationship.

Ⓖ The graph would no longer represent a proportional relationship.

LESSON

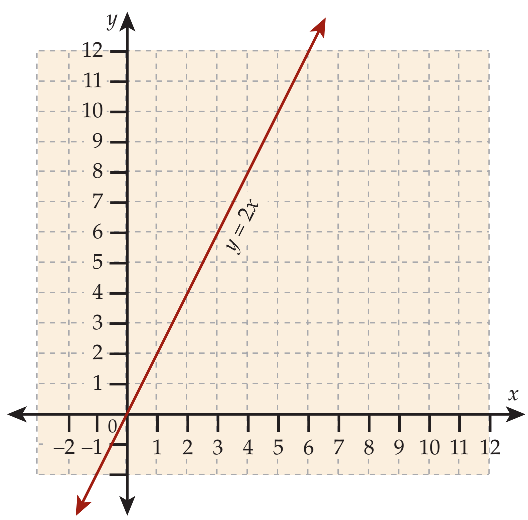
12 LINEAR GRAPHS

GOAL	CONCEPT BOOK
To learn the significance of the *y*-intercept, and to explore linear graphs with a nonzero *y*-intercept.	See pages 311–313, 368 in your *Concept Book*.

You have focused mainly on graphs of proportional relationships. These graphs are straight lines that intersect the origin, (0, 0).

Example

This is a graph of a proportional relationship: $y = 2x$.

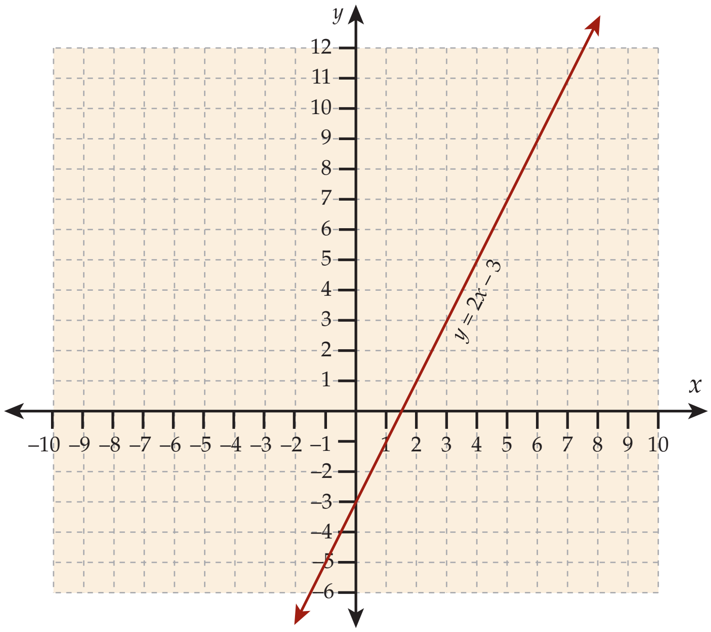


The point at which this line intercepts the vertical axis has a *y*-coordinate of 0. In other words, it has a *y-intercept* of 0.

Many situations can be modeled by graphs that are straight lines that do not intercept the y-axis at 0. Any graph that is a straight line is a *linear* graph, whether or not it has a y-intercept of 0.

Example

The line $y = 2x - 3$ is a linear graph with a y-intercept of –3.



Every linear graph has an equation of the form: $y = kx + b$.

The coefficient k is the slope of the line, and b is the y-intercept.

$y = 2x - 3$ — slope: 2; y-intercept: -3

$y = kx + b$ — slope: k; y-intercept: b

Work Time

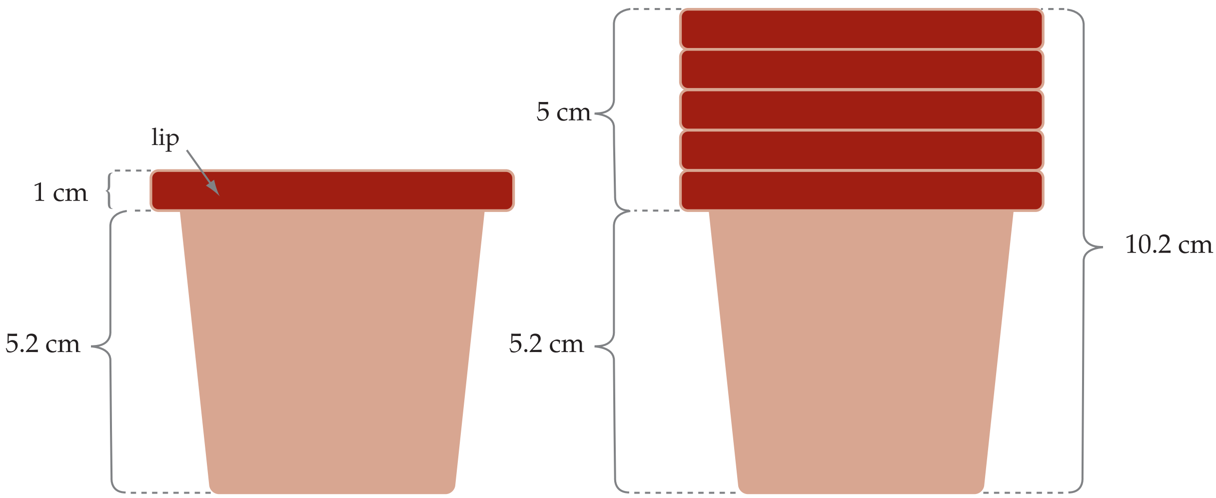
In Lesson 14 of Unit 1: *Foundations of Algebra*, you explored a situation with stacks of cups.

In this lesson you will revisit a similar situation with Keesha and Dwayne in more detail to see why the y-intercept is important.

Keesha was in her backyard with Dwayne, putting plants she had purchased into small clay pots. Keesha stacked 5 empty pots on the table and asked Dwayne how he would describe the height of the stack mathematically.

Dwayne measured the pots and found that they had a lip of 1 cm and a base of 52 cm.

Dwayne then drew this diagram.



Keesha said the two quantities that vary in relation to each other in this situation are:

- n, the number of pots in a stack
- h, the height of the stack of pots, in centimeters

Dwayne described the relationship as, "The height of the stack is equal to the number of pots plus 5.2 centimeters."

Keesha added, "I know a formula that represents this relationship: $h = n + 5.2$."

Use Dwayne's diagram of the pots for problems 1–4.

1. Do negative values make sense for either n or h in this situation?

2. Do number values that are not whole make sense for either n or h in this situation?

3. a. Make a table showing at least five different ordered pairs for n and h to represent the relationship between the number of pots and the height of a stack of pots.

b. Use your table to determine whether there is a constant ratio between corresponding values of n and h.

c. Is the height of the stack of pots proportional to the number of pots in the stack?

d. How much does the height of the stack increase with each pot added to the stack?

e. What does the amount you wrote in part d represent in the situation?

4. a. Sketch a graph of the relationship between n and h.

b. Does your graph extend beyond the first quadrant? Say how you know.

c. Is your graph discrete or continuous? Say how you know.

d. Is your graph a straight line? If so, what is the constant vertical increase or decrease of the graph that corresponds to each horizontal increase of 1 unit?

e. Does the graph intersect the origin? If not, where does it intercept (cross) the y-axis?

f. Why does the graph have the y-intercept you identified for part e? Think about how the graph relates to the meaning of the situation.

5. Dwayne went home and noticed that his flower pots he had different measurements. He said, "The height of a stack of the pots I have can be represented with this formula: $h = n + 4$."

a. How are Dwayne's pots different from Keesha's in the diagram he had sketched? How are they similar?

b. Use the formula for the height of a stack of Dwayne's pots to find out how many of his pots are in a stack that is 50 cm tall.

c. Sketch a graph of the relationship between h and n for Dwayne's pots on the same coordinate plane you used in problem 4.

d. How can your graph be used to check your answer for part b?

e. What is the constant increase or decrease of the graph representing stacks of Dwayne's pots?

f. What is the y-intercept of the graph representing stacks of Dwayne's pots?

Preparing for the Closing

6. The slope of a graph tells you how steep it is. The slope of a graph is equal to the constant vertical increase or decrease of the graph that corresponds to each horizontal increase of 1 unit. Explain what the slope of the graph represents in the situation in today's lesson.

7. The y-intercept of a graph tells you its position in the coordinate plane. Explain what the y-intercept of the graph represents in the situations in today's lesson.

8. a. Sketch a graph of $h = n + 2$ in the same coordinate plane you used for problems 4 and 5. Then compare the three graphs by commenting on these features:

b. The slope of each graph

c. The y-intercept of each graph

d. The shape of each graph

e. Whether any of the graphs are linear

f. Whether any of the graphs represent proportional relationships

Skills

Solve.

a. A lighthouse flashes 6 times per minute. At this rate, how many times does it flash in 8 hours?

b. Dwayne's heart beats at the rate of 380 times in 5 minutes. At this rate, how many times will it beat in 1 hour?

Review and Consolidation

For problems 1–3, match the situation described to one of the tables below.

1. A tank contains 3 liters of water. You add water to the tank at a constant rate of 8 liters per minute.

2. You add water to an empty tank at a constant rate of 8 liters every minute.

3. A water tank contains 8 gallons of water. You pour water out of the tank at a rate of 0.8 gallon per minute.

A

x	0	1	2	3	4	5
y	8	7.2	6.4	5.6	4.8	4

B

x	0	1	2	3	4	5
y	3	11	19	27	35	43

C

x	0	1	2	3	4	5
y	0	8	16	24	32	40

4. a. Which of the situation tables in problems 1–3 involves a proportional relationship?

b. Which of the situation tables in problems 1– 3 involves a linear relationship?

5. a. Sketch a graph to represent each case.

b. Are your graphs discrete or continuous? Say why.

c. What is the slope (constant increase or decrease) of each of your graphs?

d. What is the y-intercept of each of your graphs?

e. Do your graphs show a constant decrease or constant increase? Say why.

Homework

1. For each formula given, state the slope and y-intercept of the graph representing the relationship between x and y, and then sketch the graph for x-values between –5 and 5.

a. $y = \frac{1}{2}x + 2$ b. $y = -\frac{1}{2}x + 2$ c. $y = 2x + \frac{1}{2}$ d. $y = 2x - \frac{1}{2}$

LESSON 13

FOCUS ON SLOPE

GOAL	CONCEPT BOOK
To explore the slope of linear graphs in more detail.	See pages 327–330 in your *Concept Book.*

The slope-intercept equation of a line always has this form: $y = kx + b$.

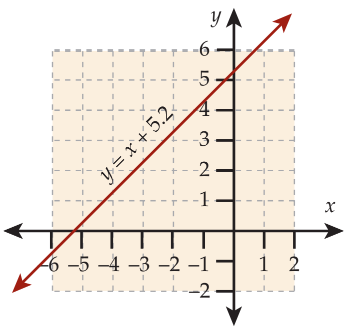
You can easily identify the slope and y-intercept if the equation of the line is in this form.

Example

- $y = 2x - 2$ — The slope of the line is 2. The y-intercept is –2.
- $y = -4x + 3$ — The slope of the line is –4. The y-intercept is 3.
- $y = 5x$ — The slope of the line is 5. The y-intercept is 0.

In the previous lesson, you graphed three lines:

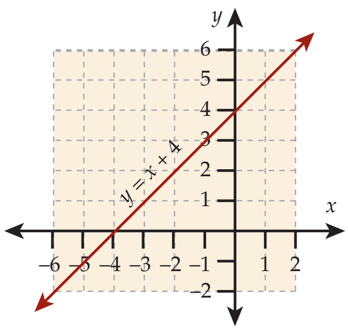
$y = x + 5.2$



slope = 1

y-intercept = 5.2

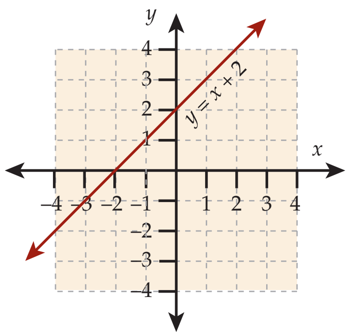
$y = x + 4$



slope = 1

y-intercept = 4

$y = x + 2$



slope = 1

y-intercept = 2

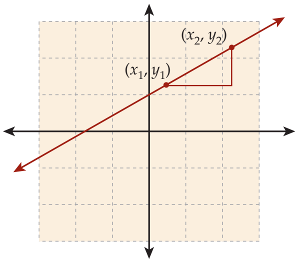
These three lines have the same slope but different y-intercepts. The slope of each line is 1 (the coefficient of x), and the y-intercepts are 5.2, 4, and 2, respectively.

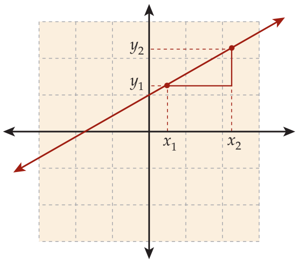
Notice that none of these lines represents a proportional relationship. This means that there is not a constant ratio between y and x. As you have seen, when there is a constant ratio between y and x, the slope of the graph is equal to the constant ratio $\frac{y}{x}$.

Without a constant ratio, you can find the slope of any linear graph by finding the change in y-values over the change in x-values between any two points on the graph.

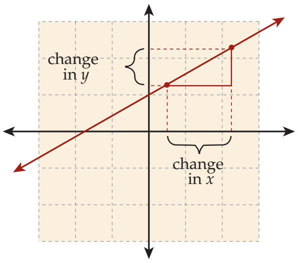
Finding Positive Slopes

Example

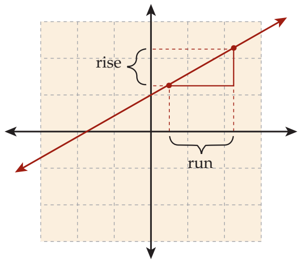




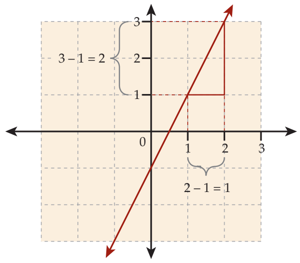
$$\text{slope} = \frac{y_2 - y_1}{x_2 - x_1}$$



$$\frac{\text{change in } y\text{-values}}{\text{change in } x\text{-values}}$$



$$\text{slope} = \frac{\text{rise}}{\text{run}}$$



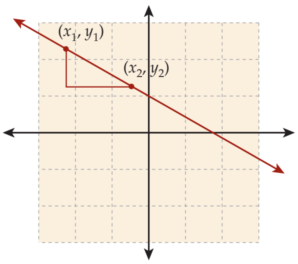
When run = 1 unit,

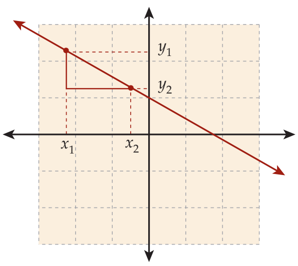
rise = slope

because $\frac{\text{rise}}{\text{run}} = \frac{\text{rise}}{1}$

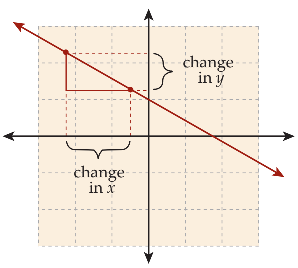
Finding Negative Slopes

Example

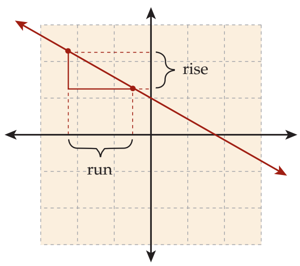




$$\text{slope} = \frac{y_2 - y_1}{x_2 - x_1}$$



$$\frac{\text{change in } y\text{-values}}{\text{change in } x\text{-values}}$$



$$\text{slope} = \frac{\text{rise}}{\text{run}}$$

You can find the slope of a line using any two points on the line.

Example

Between the points (1, 3) and (5, 11), the change in y-values is $11 - 3 = 8$, and the change in x-values is $5 - 1 = 4$.

So, the slope is $\frac{8}{4} = 2$.

Look at each pair of x- and y-values in the (x, y) table below.

x	1	5	6	–5	–0.5
y	3	11	13	–9	0

The formula $y = 2x$ does not describe the relationship between the x- and y-values in each pair. The relationship between y and x is not proportional, and the y-intercept is not 0.

You know the formula is $y = 2x + b$ because you know the slope is 2. But what is the value of b, the y-intercept?

Work Time

Example

Write a slope-intercept equation for the line that passes through the following points:

x	1	5	6	–0.5
y	3	11	13	0

A slope-intercept equation has the form $y = kx + b$.
To write the equation, you must find the value of k and the value of b.

Step 1: Find k.

k is the slope. You can use any two points in the table to find the slope, for example the points (5, 11) and (–0.5, 0):

change in y-value $= 0 - 11 = -11$

change in x-value $= -0.5 - 5 = -5.5$

$$\text{slope} = \frac{\text{change in } y}{\text{change in } x} = \frac{-11}{-5.5} = 2$$

You know $k = 2$, so the equation is $y = 2x + b$. But what is the value of b, the y-intercept?

Comment

If you already know the slope of a line, you can skip Step 1.

Step 2: Choose a point and substitute its x- and y-coordinates into the equation.

You can choose any point from the table; for example, the point (1, 3). (This point tells you that when x is 1, y must be 3.)

$$y = 2x + b$$
$$3 = 2(1) + b$$
$$3 = 2 + b$$

Step 3: Find b.

To find the value of b, solve the equation from Step 2.
b has to be 1 for this equation to be true.

Step 4: Write the equation of the line.

Use the values you found for k and b to write the equation.
The equation is $y = 2x + 1$.

1. Find the value of b in the equation for the same line as in the example above using a point other than (1, 3). Use the following steps.

a. Choose a point other than (1, 3) from the table in the example.

b. Substitute the x- and y-coordinates of your chosen point into the equation $y = 2x + b$. (Step 2)

c. Find the value of b by solving your equation for b. (Step 3)

d. Find someone in the class who chose a different point than you. Did you both find the same value for b? If not, review and correct your work together.

e. Sketch a graph of this line by plotting two points from the table in the example. Confirm that the y-intercept matches the value you found for b.

2. a. Write a slope-intercept equation for the line that passes through the following points:

x	–1	0	2.25	7
y	–4	–2	2.5	12

b. Graph the line. Confirm that the y-intercept of your graph matches the value you found for b.

3. a. Write a slope-intercept equation for the line that passes through these points. Use the steps outlined in the example.

(–8, 5)   (1, 0.5)   (4, –1)   (9, –3.5)   (10, –4)

b. Sketch a graph of the line.

Preparing for the Closing

4. If a graph is linear, you can find the slope of the graph, k, using any two points on the line. Say why.

5. If a graph is linear and you know its slope and a point on the line, you can find the y-intercept, b, using any point on the line. Explain how.

6. What does "substitute values into an equation" mean?

7. What does "solve an equation" mean?

8. Explain what the following sentences mean in your own words:

a. Every pair of x- and y-values that is a solution to the equation of a line defines a point on that line.

b. Every pair of x- and y-values that defines a point on a line is a solution to the equation of that line.

Skills

Solve.

a. Mr. Jackson's secretary types 45 words per minute.

At this rate, how long will she take to type 135 words?

b. Water is flowing from a tap at the rate of 45 liters per minute.

At this rate, how long will it take to fill a 135-liter container?

Review and Consolidation

1. What is k, the coefficient of x, in each of these equations?

$$y = \frac{1}{2}x - 1 \qquad\qquad y = -2x - 1$$

2. a. Identify two points on the line $y = \frac{1}{2}x - 1$ by finding two pairs of x- and y-values that satisfy the equation.

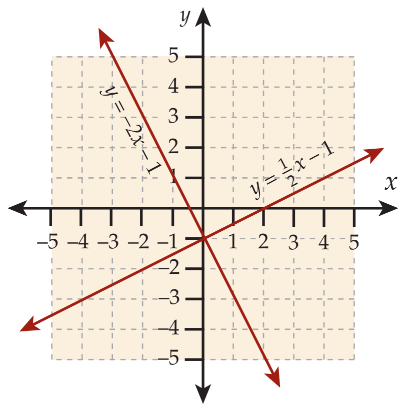
b. Confirm that $k = \frac{\text{change in } y}{\text{change in } x}$ for the x- and y-coordinates of your two points.

3. a. Identify two points on the line $y = -2x - 1$ by finding two pairs of x- and y-values that satisfy the equation.

b. Confirm that $k = \frac{\text{change in } y}{\text{change in } x}$ for the x- and y-coordinates of your two points.

4. Look at the two lines at right.

Confirm that the four points you identified on each line in problems 2 and 3 are, in fact, on the lines by pointing to their locations with your finger.



You know that the slope of a line defined by a slope-intercept equation of the form $y = kx + b$ is k, the coefficient of x.

You also know that in the equation $y = kx + b$, the y-intercept of the line is b.

In proportional relationships, b is 0 and k is equal to the constant of proportionality, $\frac{y}{x}$.

5. Write a slope-intercept equation and sketch a graph of a proportional relationship with a constant of proportionality of –1.

6. Write a slope-intercept equation and sketch a graph of a linear relationship with a slope of –1 and a y-intercept of –1.

Homework

1. Use the table below for parts a–e. Refer to the example from Work Time if necessary.

x	–5	–1	2.5	4
y	4	0	–3.5	–5

a. Is there a constant of proportionality in this table? If so, what is it?

b. Find the slope, using slope = $\frac{\text{change in } y}{\text{change in } x}$, between any two points from the table.

c. Use the slope from part b to write as much as you can of the slope-intercept equation for this line.

d. Find the y-intercept by substituting the x- and y-coordinates of any point from the table into the equation you wrote in part c and then solving for b.

e. Write the slope-intercept equation for this line.

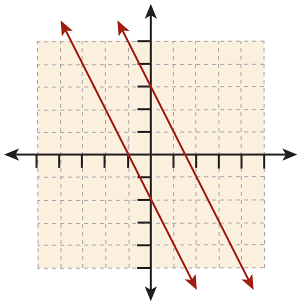
PARALLEL AND PERPENDICULAR LINES

LESSON **14**

CONCEPT BOOK	GOAL
See page 334, 368–372 in your *Concept Book*.	To identify and graph parallel and perpendicular lines.

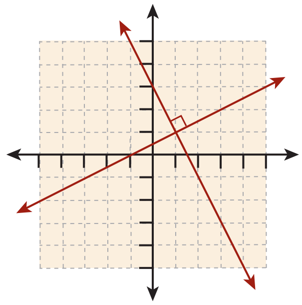
Parallel lines never intersect each other.

They have the same slope, but different *y*-intercepts.



Perpendicular lines cross each other at a right angle.

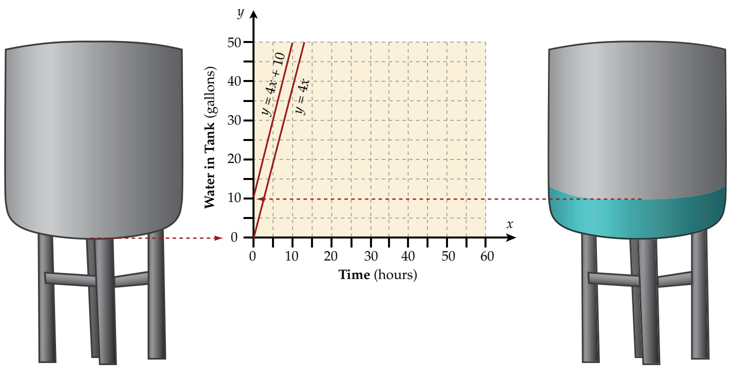
Their slopes are *negative inverses* of each other.



Parallel Lines

Example

In this water tank situation, the two lines represent the relationship between time and the amount of water in the tank. One shows the tank starting out empty, and the other shows the tank starting out with 10 gallons of water.



The two lines are parallel:

$y = 4x$	$y = 4x + 10$
slope = 4	slope = 4
y-intercept = 0	*y*-intercept = 10

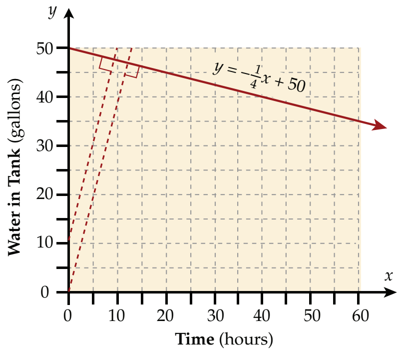
The lines in the water tank example on the previous page are parallel.

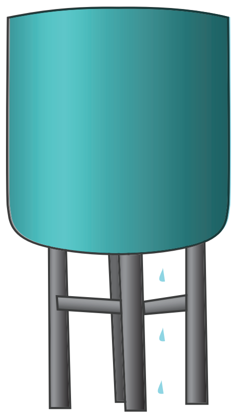
- The rate of change is the same (equal to the amount of water added per hour) in both cases, so the graphs have the same slope.
- The y-intercepts are different, because the initial amounts of water are different.

Perpendicular Lines

Example

This graph shows a 50-gallon water tank that begins full, and leaks at a rate of $\frac{1}{4}$ gallon per hour.





The line representing this tank is *perpendicular* to those on the previous page.

The slope of the graph, $-\frac{1}{4}$, is the *negative inverse* of the slope of the first two lines, 4.

Parallel		Perpendicular
$y = 4x$	$y = 4x + 10$	$y = -\frac{1}{4}x + 50$
slope = 4	slope = 4	slope = $-\frac{1}{4}$

Two numbers, or two expressions with letters and numbers, are multiplicative inverses if their product is 1.

Example

$\frac{3}{4}$ and $\frac{4}{3}$ are multiplicative inverses.

The *negative* multiplicative inverse of $\frac{3}{4}$ is $-\frac{4}{3}$.

$4a$ and $\frac{1}{4a}$ are multiplicative inverses.

The negative multiplicative inverse of $4a$ is $-\frac{1}{4a}$.

$\frac{1}{x+2}$ and $x + 2$ are multiplicative inverses.

The negative multiplicative inverse of $\frac{1}{x+2}$ is $-(x + 2)$, and so on.

Work Time

Use Handout 2: *Blank Coordinate Planes* for the Work Time problems.

1. Use the same coordinate plane on your handout to sketch each of these graphs.

   a. $y = x + 2$ b. $y = 2x + 2$ c. $y = 2x - 2$ d. $y = 2x$

2. Which of the graphs in problem 1 represent a proportional relationship, and what is the constant of proportionality for the graph(s)?

3. Which of the graphs in problems 1 are parallel to each other?

4. Which of the graphs in problems 1 have the same y-intercept?

5. a. Sketch a graph of $y = -\frac{1}{2}x$ on the same coordinate axes that you used for problem 1.

   b. To which lines from problem 1 is this line perpendicular?

6. a. On a second coordinate plane, sketch graphs of the following lines.

$y = 1.25x + 1$ $\quad$ $y = \frac{5}{4}x - 1$ $\quad$ $y = \frac{4}{5}x + 1$ $\quad$ $y = 0.8x$ $\quad$ $y = -\frac{4}{5}x - 1$

b. Identify lines that are parallel to each other.

c. Identify lines that are perpendicular to each other.

Preparing for the Closing

7. Consider the following lines:

$y = x$ $\qquad$ $y = -x$

These lines are perpendicular to each other. Say how you can tell from the equations.

8. Consider the following lines:

$y = 2x$ $\qquad$ $y = -2x$

These lines are not perpendicular to each other. Say how you can tell from the equations.

9. Parallel lines cannot have the same y-intercept. Say why.

Skills

Solve.

a. A photocopier can print 45 copies in 3 minutes.
At this rate, how long will it take to print 135 copies?

b. A car can travel 45 miles on 4 gallons of gas.
At this rate, how far can the car travel on 135 gallons of gas?

Review and Consolidation

1. a. Write a slope-intercept equation of a proportional relationship with a constant of proportionality of $\frac{2}{5}$.

b. Sketch the graph.

2. a. Write a slope-intercept equation of a linear relationship with a slope of –2.5 and a y-intercept of –3.

b. Sketch the graph.

3. a. Identify two points on the line $y = \frac{2}{5}x$ by finding two pairs of x- and y-values that satisfy the equation.

b. Confirm that $k = \frac{\text{change in } y}{\text{change in } x}$ for the x- and y-coordinates of your two points.

4. a. Identify two points on the line $y = (-2.5)x - 3$ by finding two pairs of x- and y-values that satisfy the equation.

b. Confirm that $k = \frac{\text{change in } y}{\text{change in } x}$ for the x- and y-coordinates of your two points.

5. Look at the graphs you sketched in problems 1 and 2. These lines *look* perpendicular. Say how you can be *sure* that they are perpendicular.

Homework

1. Use your handout from Work Time. Sketch graphs of the following lines on the same coordinate plane.

$y = 3x + 1$ $\quad$ $y = -3x - 3$ $\quad$ $y = -\frac{1}{3}x + 1$ $\quad$ $y = -3x + 1$ $\quad$ $y = -\frac{1}{3}x - 3$

2. Identify the slope and y-intercept of each of the lines in problem 1.

3. Identify lines in problem 1 that are parallel to each other.

4. Identify lines in problem 1 that are perpendicular to each other.

LESSON 15

INTERPRETING GRAPHS

GOAL

To explore different ways of representing and interpreting the same data using graphs.

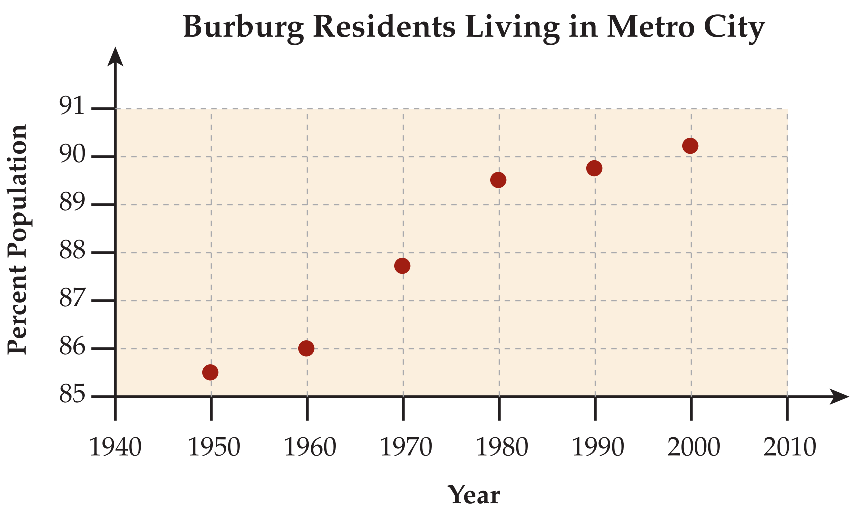
CONCEPT BOOK

See pages 327–330 in your *Concept Book.*

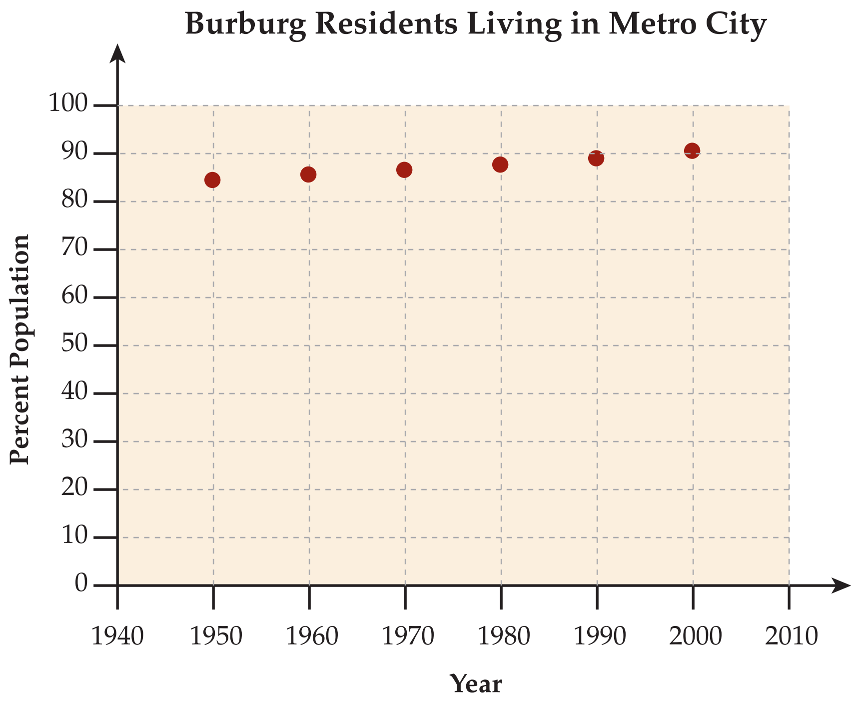
Work Time

There are often different ways to represent the same real-world data using graphs. Graphs tell a story and the scale determines what parts of the story are being examined. Decisions about how to set up a graph determine how the data will be interpreted, and how a story is told.

1. The following two graphs represent the same data: the percent of the population of Burburg County that lived in Metro City, from 1950 to 2000. The graphs have different scales, and give different ways of interpreting the same data. Notice the values at the origin of each graph.



The first graph shows that the percentage of people living in the city grew every year. There was a much faster increase in the percent of the population in the city between 1960 and 1980. By 2000, the increase was still taking place but much more slowly.



The second graph more clearly shows that the large majority (over 85%) of the population of Burburg County has lived in Metro City since 1950.

a. What are the values where the *x*-axis and the *y*-axis cross?

b. Why does the horizontal axis not start with 0 on either graph?

c. Why does one of the graphs look linear while the other does not?

d. What claims could you make more easily using the first graph?

e. What claims could you make more easily using the second graph?

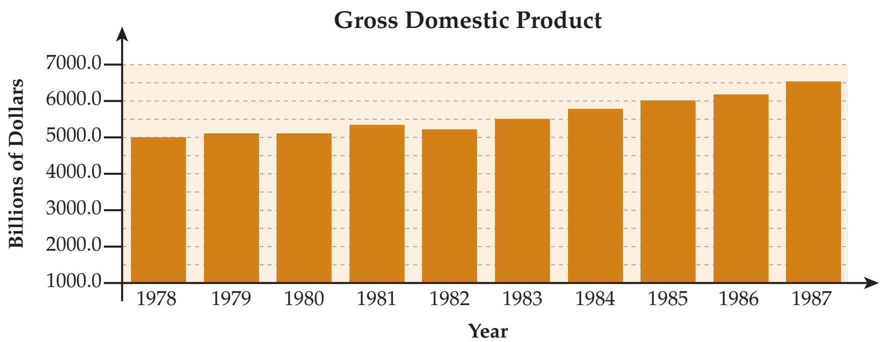
Use the following situation for problems 2–4.

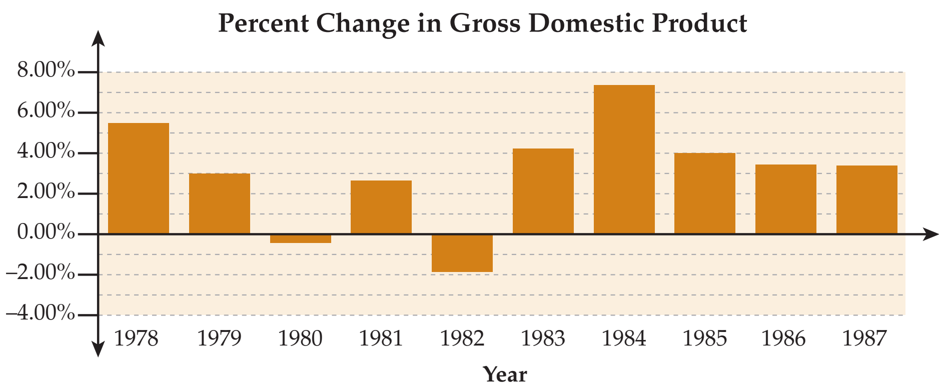
Jamal needs to write a report for his U.S. history class about how the economy has changed in recent decades. He has learned that a conventional measure of the total economic activity in the United States is the Gross Domestic Product.

He finds the table below, from the U.S. Department of Commerce, which shows the Gross Domestic Product (GDP) for each year from 1978 to 1987:

Year	1978	1979	1980	1981	1982	1983	1984	1985	1986	1987
GDP ($ billions)	5015.0	5173.4	5161.7	5291.7	5189.3	5423.8	5813.6	6053.7	6263.6	6475.1

With further research, Jamal finds two different bar charts of the GDP from 1978–1987. The first one shows the total GDP, and the second one shows the percent change from year-to-year:





2. Suppose Jamal wants to decide in which of the years the economy was doing the best.

a. Looking at the first chart, which year do you think was the best? Say why.

b. Looking at the second chart, which year do you think was the best? Say why.

3. Sketch two graphs, one to represent each of the bar charts. Each data point on the bar charts should be a point on one of your graphs. Be sure to label your axes appropriately.

a. Which way of representing the data can be more closely approximated with a linear graph?

b. Which way of representing the data shows more unevenness of growth from year to year?

4. a. Why might Jamal want to choose the first chart or graph for his report?

b. Why might he want to choose the second chart or graph?

5. Garrison City, with a population of 5000 people, has a very popular city park. For two years, city officials counted the number of people using the park. They compared the average number of people in the park to the temperature outside, rounded to the nearest ten degrees. This table shows the results of their study.

Temperature (°F)	0	10	20	30	40	50	60	70	80	90	100	110
People in Park	20	21	23	27	55	146	517	1002	1342	415	105	24

a. Sketch a graph that shows how the number of people in the park changed as the temperature changed.

b. Say why the scales you chose for your axes in part a are appropriate for the data in this situation.

c. Did you connect the data points on your graph? Say why or why not.

d. Sketch a graph that shows what *percent of the total population* was in the park at different temperatures.

e. Say why the scales you chose for your axes in part d are appropriate for the data in this situation.

f. Compare the graphs you sketched in parts a and d. How might these two graphs lead to different interpretations of the same data?

Preparing for the Closing

6. Think about the different ways of representing data using graphs, and the different interpretations that different graphs can lead to. What kinds of decisions might you need to make when setting up a graph to represent data?

7. What impact could the decisions you listed in problem 6 have on how the graph appears?

8. If data can be approximated by a linear graph, what kinds of claims can be made about it?

Skills

Solve.

The workers in a factory are paid the following rates.

Weekdays	\$28 per day
Week Nights	\$38 per night
Saturdays and Sundays—Weekend Day	\$48 per day
Saturdays and Sundays—Weekend Night	\$58 per night

a. Jamal's uncle worked 4 weekdays and 1 weekend night.

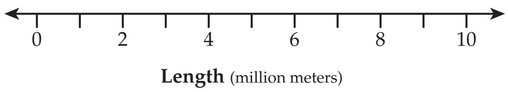
How much was he paid?

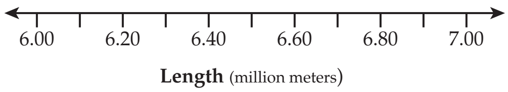
b. Rosa's aunt worked 4 week nights and on Sunday during the day.

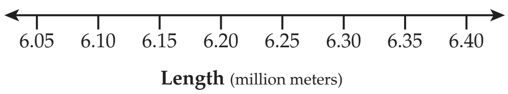
How much was she paid?

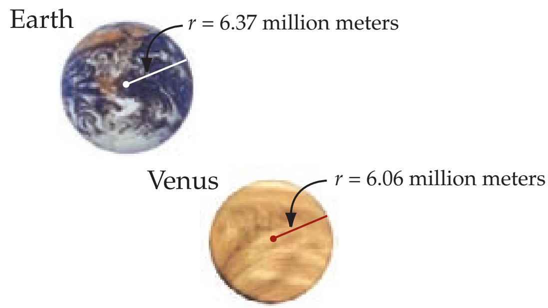
Review and Consolidation

1. Planet Earth has a radius of about 6.37 million meters. Venus has a radius of about 6.06 million meters. Which one of these number lines best emphasizes the difference between the two planets' radii? Say why.

A


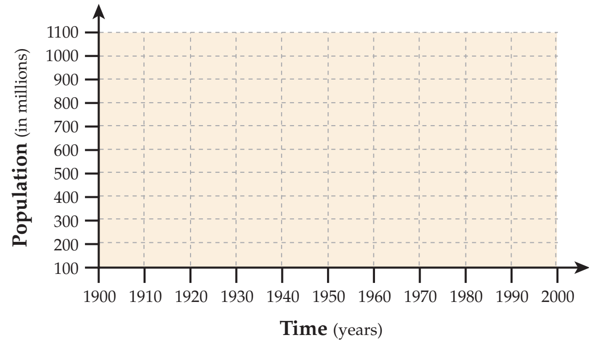
B


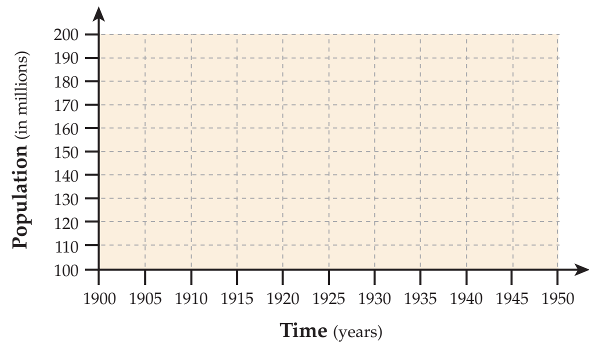
C




2. In 1910, the population of the United States was about 92 million. In 1990, the United States population was about 250 million.

Which one of these coordinate planes shows this population growth over time most precisely?

A


B


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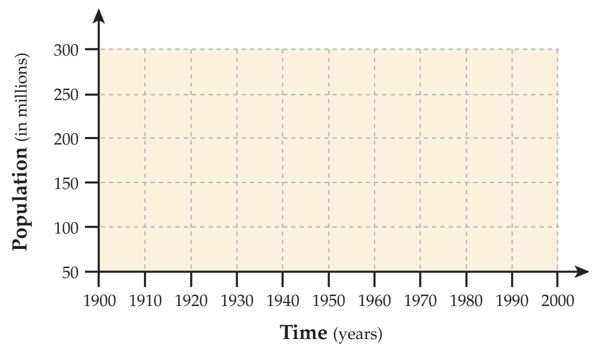
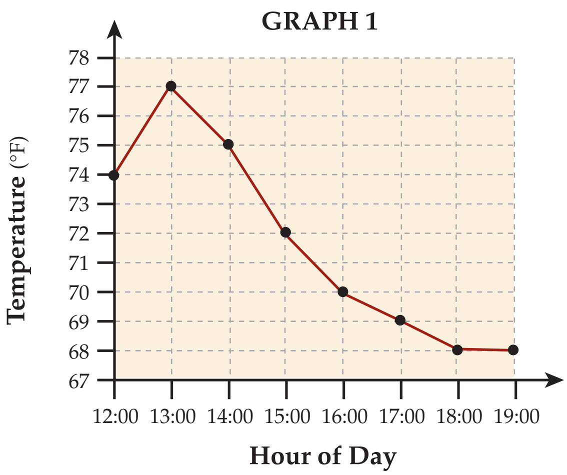


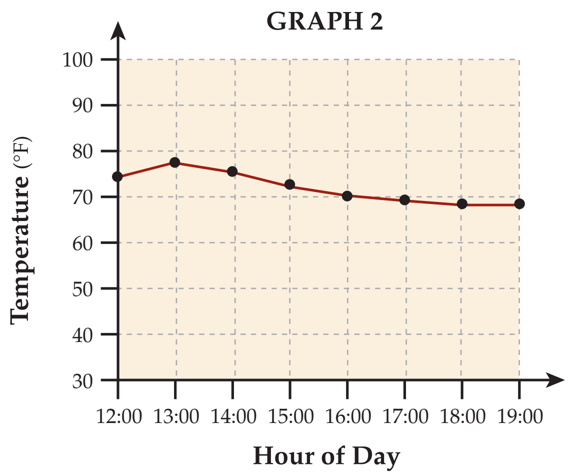
3. This table shows a room's temperature at different times of day.

> **Comment**
> 13:00 hours is the same as 1:00 PM.
> 14:00 hours is the same as 2:00 PM.

Time (24-Hour Time)	12:00	13:00	14:00	15:00	16:00	17:00	18:00	19:00
Temperature (°F)	74	77	75	72	70	69	68	68

Here are two graphs showing this data in different ways.





Although both graphs represent the same data, they look very different. The choice of scale determines how the data looks on the graph.

The change in temperature looks very large in Graph 1, while the data line looks flat in Graph 2. Although they convey different messages, both representations are accurate.

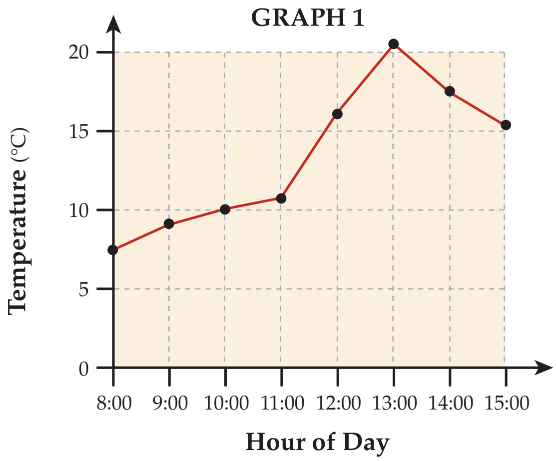
a. How are the coordinate axes of the two graphs similar and different?

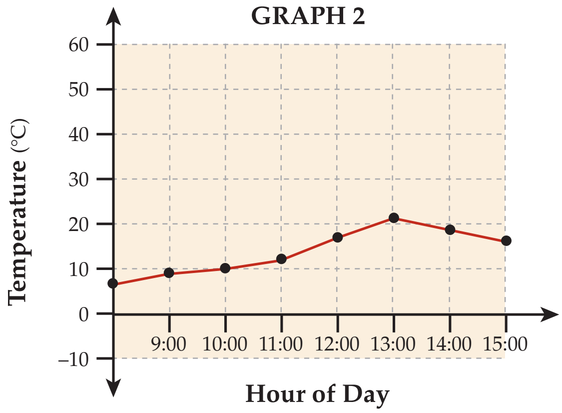
b. Which graph would be appropriate if you wanted to emphasize the room's maximum temperature during this time period? Say why.

c. Which graph would be appropriate if you wanted to compare the room's temperature during this day to a very hot or cold day? Say why.

4. Here is another table and two graphs showing time and temperature.

Time (24-Hour Time)	**Temperature** (°C)
8:00	7
9:00	9
10:00	10
11:00	12
12:00	17
13:00	21
14:00	18
15:00	16





a. The quantities in this situation are the same as in problem 3, but one of the quantities is measured differently. Which quantity is measured differently?

b. Could you come up with an even more precise scale than the one used for the top graph? Describe the scale.

5. These two graphs show the height of a growing plant. Graph 1 uses straight line segments to join points. Graph 2 uses a single, smooth curve.

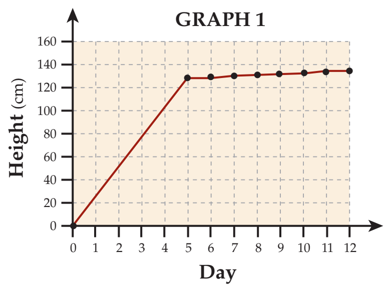
a. Use Graph 1 to estimate the height of the plant on day 3.

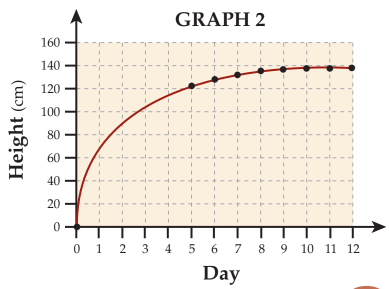
b. Use Graph 2 to estimate the height of the plant on day 3.

c. Use Graph 1 to estimate the age of the plant when it reached a height of 60 cm.

d. Use Graph 2 to estimate the age of the plant when it reached a height of 60 cm.

e. Discuss with a partner which graph you think gives a better approximation of the plant's growth, and why.





This table shows the total cost per month of leaving lights on per day.
Use this table for problems 6–10.

Hours per Day	5	7	9
Cost per Month	$5.10	$7.14	$9.18

6. a. Sketch a graph with scales that are appropriate for estimating the cost of leaving the lights on for 3.5 hours per day.

b. Explain to a partner how you chose the scale for each axis.

c. Use your graph to estimate the cost of leaving the lights on for 3.5 hours per day. Confirm that you and your partner found the same value.

7. a. Sketch a graph with scales that are appropriate for estimating the cost of leaving the lights on for 20 hours per day.

b. Explain to a partner how you chose the scale for each axis.

c. Use your graph to estimate the cost of leaving the lights on for 20 hours per day. Confirm that you and your partner found the same value.

8. Are either of your graphs linear? If so, which one(s)?

9. Do either of your graphs represent proportional relationships? If so which one(s)?

10. a. Identify at least one other way that either of the quantities in this situation, monthly cost or time, could be measured. For example, think about how percent might be useful.

b. Describe how measuring one of the quantities in a different way would change the appearance of the graph.

Homework

1. Sketch a graph to represent each of the following:

a. A situation that involves a proportional relationship between quantities.

b. A relationship between quantities that can be represented by a linear graph.

c. A situation with data that can be approximated by a linear graph.

d. A relationship between quantities that can be represented by a graph that is not linear.

THE UNIT IN REVIEW

LESSON 16

CONCEPT BOOK	GOAL
See pages 199–209, 308–322, 327–333 in your *Concept Book*.	To review the important concepts in this unit in preparation for the End-of-Unit Assessment.

Work Time

1. Consider the following situations and then answer the questions on the following page.

Situation 1
A 14-inch candle is lit and begins burning down at a constant rate of 2 inches per hour.

Situation 2
A row of CDs that are each 0.8 cm thick is on a shelf. The length of the row increases with each CD added to the shelf.

Situation 3
The table below shows how much time it takes a runner to reach different distances during a race.

Distance (miles)	0	0.25	0.5	0.75	1	1.25
Time (minutes)	0	1.63	3.19	4.64	5.86	7.47

Situation 4
Lisa is building a square garden bed. She must decide where to cut the boards for the edges of the bed so that she has enough area in which to plant her garden. The area of the square bed increases as the length of the boards increases.

Situation 5
A circle has a diameter of d cm and a circumference of C cm.

d (cm)	1	1.5	3	6	7.5
C (cm) (rounded to the nearest hundredth)	3.14	4.71	9.42	18.84	23.55

a. Which of the situations involve a proportional relationship between quantities? Say how you know.

b. Which of the situations could be represented by a linear graph? Say how you know.

c. Which of the situations could be approximated by a linear graph? Say how you know.

d. You will need two copies of Handout 2: *Blank Coordinate Graphs.* Sketch a graph of each of the situations 1–5, and confirm your answers to parts a–c.

2. Look at the (x, y) table below.

x	–10	–4	–1	1	4	10
y	–6	–3.6	–2.4	–1.6	–0.4	2

a. Write the slope-intercept equation for the line through the points in the table by finding the slope between any two points and then finding the y-intercept.

b. Use your second copy of the handout to sketch a graph of the line and check that the slope and y-intercept match the equation you wrote in part a.

3. a. Which of the following lines are parallel to the line $y = 0.4x - 2$.

b. Which of the following lines are perpendicular to the line $y = 0.4x - 2$?

Ⓐ $y = -0.4x + 5$ Ⓑ $y = \frac{2}{5}x + 2$ Ⓒ $y = -2.5x$

Ⓓ $y = -\frac{1}{4}x - 2$ Ⓔ $y = -\frac{5}{2}x + 5$ Ⓕ $y = 2.5x$

Ⓖ $y = 0.4x + 5$ Ⓗ $y = \frac{1}{4}x - \frac{1}{2}$ Ⓘ $y = \frac{5}{2}x - 2$

4. The school theater group is performing a play to raise money to renovate their auditorium. The following table shows the total amount of money raised with each additional night of the performance.

Number of Nights	1	2	3	4	5
Total Amount Raised ($)	486	718	942	1228	1582

Sketch two different graphs to represent this data. For one graph, choose scales for the coordinate axes that suggest that the data is approximately linear. For the other graph, choose scales for the coordinate axes that show that the data is not approximated by a linear graph.

Preparing for the Closing

5. Look at the graphs that you sketched and the graphs that were given for this lesson.

a. Which of the graphs have a constant positive slope?

b. Which of the graphs have a constant negative slope?

c. If a graph has a constant slope, does it mean that the graph represents a proportional relationship?

d. If a graph has a constant slope, identify at least two things you know about this graph.

6. Look back at your work for problem 2.

a. Say why you can use any two points on a line to find the slope of the line.

b. If you know the slope of a line, say why you can use any point on a line to find the y-intercept of the line.

Skills

Solve.

In a city, a taxi charges the following rates.

For the First Mile	$3.50
For Every Additional Mile	$0.70

a. Calculate the taxi fare for a journey of 6 miles.

b. If a taxi fare was $9.80, about how many miles did the passenger travel?

Review and Consolidation

Assessing Your Work

1. Review your work in this unit, and select a piece that you believe is the best example of your understanding of the mathematics presented in the unit. The work that you select could show a graph, table, and formula for a linear equation. You might show how to determine the slope and y-intercept. The piece of work could show more than one concept.

   When choosing your piece of work, show that you have done the following:

   - Used the concept accurately to solve the problem
   - Represented the concept in multiple ways (numbers, graphs, symbols, diagrams, or words)
   - Explained your solution and the concept well to the reader

2. Identify the piece in the manner described by your teacher.

3. Write a brief explanation of why you chose this piece and how it demonstrates your knowledge and ability.

Homework

1. a. Review the problems in this lesson and find three that were difficult for you.

   b. Use your *Concept Book* and previous lessons to work through these three problems more carefully until you understand them. If you still have questions about the problems, be prepared to ask them during the next lesson.

2. Write a summary page of the concepts you have learned in the unit.

COMPREHENSIVE REVIEW

UNITS

1–6

1. Complete this table.

a	4	5	6	12	15	120
6*a*	24					

2. Explain why $3 + (-7) = 3 - (+7)$.

3. Write each fraction as an equivalent fraction in simplest form.

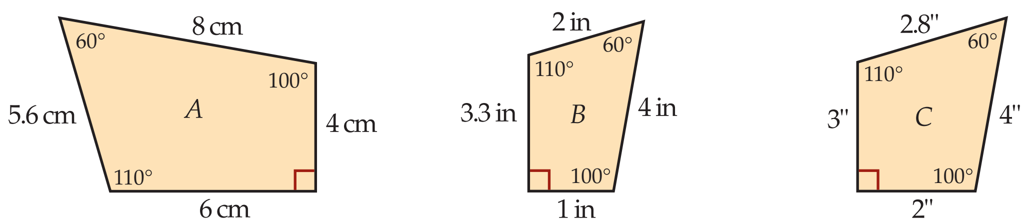
a. $\frac{10}{15}$ b. $\frac{9}{27}$ c. $\frac{16}{64}$

4. Here are four numbers: 830 3800 38 338,000

a. In which number does the digit 8 have the greatest value? Say how you know.

b. What is the value of the digit 8 in each number?

5. Look at this set of three figures below. Decide which two figures are similar. Say how you know.



6. Calculate.

a. $-7 \bullet [4 - (-6)]$

b. $(-7 \bullet 4) - (-6)$

c. $-7 \bullet 4 - (-6)$

d. $(-7 \bullet 4) - [-7 \bullet (-6)]$

e. Which of these equations give the same answers?

7. Calculate each product. Use either mental strategies or the standard method.

a. 153 • 7 = b. 153 • 8 = c. 153 • 80 = d. 153 • 70 =

8. Using parentheses and a fraction bar where needed, rewrite these expressions.

a. $a \bullet b \div 4 + 8$ b. $14 - a \bullet b \div 4$ c. $7 - b \div a \bullet 2$

d. Substitute the values $a = 4$ and $b = 6$ into each expression you wrote. Calculate the value of each expression.

9. a. List all of the factors of 28.

b. List the first 5 multiples of 28.

10. An artist wants to reduce a picture that is 40 cm wide by 52 cm high. He wants the reduced picture to be 39 cm high.

What will be the width of the picture? Explain your reasoning.

11. Set up a coordinate plane from –10 to 10. Graph the data below.

x	6	4	2	0	–2	–4	–6
y	3	2	1	0	–1	–2	–3

12. Graph $y = \frac{1}{3}x$ on another coordinate plane.

Page numbers in red are found in the *Concept Book*.

Algebra and Functions

Gr. 4 AF: 1.5	Understand that an equation such as $y = 3x + 5$ is a prescription for determining a second number when a first number is given. 9–29, 41–68; 201, 205–209, 309–313, 322, 327–330, 363–364
Gr. 5 AF: 1.4	Identify and graph ordered pairs in the four quadrants of the coordinate plane. 1–5; 201–204
Gr. 5 AF: 1.5	Solve problems involving linear functions with integer values; write the equation; and graph the resulting ordered pairs of integers on a grid. 9–23, 47–55; 201, 205–209, 327–330
Gr. 6 AF: 2.3	Solve problems involving rates, average speed, distance, and time. 30–34; 308–309
Gr. 7 AF: 3.3	Graph linear functions, noting that the vertical change (change in y-value) per unit of horizontal change (change in x-value) is always the same and know that the ratio ("rise over run") is called the slope of a graph. 9–23, 47–68, 83–86; 199–209, 308–322, 327–333, 363–364
Gr. 7 AF: 3.4	Plot the values of quantities whose ratios are always the same (e.g., cost to the number of an item, feet to inches, circumference to diameter of a circle). Fit a line to the plot and understand that the slope of the line equals the quantities. 6–13, 35–40, 83–86; 199–209, 308–322, 327–333
Gr. 7 AF: 4.2	Solve multistep problems involving rate, average speed, distance, and time or a direct variation. 30–34, 41–46; 205–209, 308–309

Measurement and Geometry

Gr. 4 MG: 2.0	Students use two-dimensional coordinate grids to represent points and graph lines and simple figures: 1–5; 199–204
Gr. 4 MG: 2.1	Draw the points corresponding to linear relationships on graph paper (e.g., draw 10 points on the graph of the equation $y = 3x$ and connect them by using a straight line). 6–8; 201–204
Gr. 4 MG: 2.2	Understand that the length of a horizontal line segment equals the difference of the x-coordinates. 1–5; 199–204
Gr. 4 MG: 2.3	Understand that the length of a vertical line segment equals the difference of the y-coordinates. 1–5; 199–204
Gr. 4 MG: 3.1	Identify lines that are parallel and perpendicular. 69–73, 83–86; 199–209, 308–322, 327–334, 368–372
Gr. 7 MG: 1.3	Use measures expressed as rates (e.g., speed, density) and measures expressed as products (e.g., person-days) to solve problems. 41–46; 205–209

Statistics, Data Analysis, and Probability

Gr. 5 SDP: 1.4	Identify ordered pairs of data from a graph and interpret the meaning of the data in terms of the situation depicted by the graph. 19–29, 74–82; 201, 309–311, 322, 327–330
Gr. 5 SDP: 1.5	Know how to write ordered pairs correctly; for example, (x, y). 1–5; 201–204

Mathematical Reasoning

Gr. 5 MR: 2.5 Gr. 5 SDP: 1.4	Indicate the relative advantages of exact and approximate solutions to problems and give answers to a specified degree of accuracy. 74–82; 327–330
Gr. 7 MR: 2.4 Gr. 7 AF: 3.3	Make and test conjectures by using inductive reasoning. 56–61; 311–313, 363–364
Gr. 7 MR: 2.5 Gr. 7 AF: 3.3	Use a variety of methods, such as words, numbers, symbols, charts, graphs, tables, diagrams, and models, to explain mathematical reasoning. 51–55; 327–330
Gr. 7 MR: 2.6 Gr. 7 AF: 4.2	Express the solution clearly and logically by using the appropriate mathematical notation and terms and clear language; support solutions with evidence in both verbal and symbolic work. 30–34; 308–309

Page numbers in red are found in the *Concept Book*.