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# Table of Contents

**LESSON**

1. **Representing Quantities with Expressions**
   - Work Time ................................................................. 3
   - Skills ........................................................................ 4
   - Review and Consolidation ....................................... 4
   - Homework .................................................................. 5

2. **Evaluating Expressions**
   - Work Time ................................................................. 7
   - Skills ........................................................................ 9
   - Review and Consolidation ....................................... 9
   - Homework .................................................................. 10

3. **Exponents**
   - Work Time ................................................................. 13
   - Skills ........................................................................ 14
   - Review and Consolidation ....................................... 14
   - Homework .................................................................. 15

4. **Operations with Exponents**
   - Work Time ................................................................. 18
   - Skills ........................................................................ 19
   - Review and Consolidation ....................................... 20
   - Homework .................................................................. 20

5. **Expressions and Area Models**
   - Work Time ................................................................. 22
   - Skills ........................................................................ 24
   - Review and Consolidation ....................................... 25
   - Homework .................................................................. 26

6. **Combining Like Terms**
   - Work Time ................................................................. 28
   - Skills ........................................................................ 29
   - Review and Consolidation ....................................... 29
   - Homework .................................................................. 30

7. **Combining Quantities**
   - Work Time ................................................................. 32
   - Skills ........................................................................ 35
   - Review and Consolidation ....................................... 35
   - Homework .................................................................. 36
### Table of Contents

**8. Adding and Subtracting Expressions** 37–42
- Work Time ................................................................. 38
- Skills .......................................................................... 39
- Review and Consolidation ........................................... 39
- Homework ................................................................. 42

**9. Parentheses and Exponents** 43–47
- Work Time ................................................................. 45
- Skills .......................................................................... 46
- Review and Consolidation ........................................... 47
- Homework ................................................................. 47

**10. Negative Exponents** 48–52
- Work Time ................................................................. 50
- Skills .......................................................................... 51
- Review and Consolidation ........................................... 51
- Homework ................................................................. 52

**11. Rational Exponents** 53–57
- Work Time ................................................................. 54
- Skills .......................................................................... 56
- Review and Consolidation ........................................... 56
- Homework ................................................................. 57

- Work Time ................................................................. 59
- Skills .......................................................................... 61
- Review and Consolidation ........................................... 61
- Homework ................................................................. 61

**13. The Pythagorean Theorem** 62–68
- Work Time ................................................................. 64
- Skills .......................................................................... 67
- Review and Consolidation ........................................... 68
- Homework ................................................................. 68

**14. Putting the Pythagorean Theorem to Work** 69–72
- Work Time ................................................................. 69
- Skills .......................................................................... 71
- Review and Consolidation ........................................... 71
- Homework ................................................................. 72
## Table of Contents

### LESSON

#### 15. Progress Check  **73–75**
- Work Time ............................................................ 73
- Skills ......................................................................... 75
- Review and Consolidation ....................................... 75
- Homework ............................................................... 75

#### 16. Putting the Math to Work  **76–78**
- Work Time ............................................................ 76
- Skills ......................................................................... 78
- Review and Consolidation ....................................... 78
- Homework ............................................................... 78

#### 17. Writing Equivalent Expressions  **79–82**
- Work Time ............................................................ 80
- Skills ......................................................................... 81
- Review and Consolidation ....................................... 81
- Homework ............................................................... 82

#### 18. Using Expressions in Geometry  **83–87**
- Work Time ............................................................ 84
- Skills ......................................................................... 86
- Review and Consolidation ....................................... 86
- Homework ............................................................... 87

#### 19. Writing Equations  **88–94**
- Work Time ............................................................ 90
- Skills ......................................................................... 92
- Review and Consolidation ....................................... 92
- Homework ............................................................... 94

#### 20. The Addition Property of Equality  **95–100**
- Work Time ............................................................ 97
- Skills ......................................................................... 98
- Review and Consolidation ....................................... 99
- Homework ............................................................... 100

- Work Time ............................................................ 103
- Skills ......................................................................... 105
- Review and Consolidation ....................................... 105
- Homework ............................................................... 106
# Table of Contents

## LESSON

<table>
<thead>
<tr>
<th>Lesson</th>
<th>Title</th>
<th>Page Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>22.</td>
<td>Inequalities</td>
<td>107–111</td>
</tr>
<tr>
<td></td>
<td>Work Time</td>
<td>108</td>
</tr>
<tr>
<td></td>
<td>Skills</td>
<td>110</td>
</tr>
<tr>
<td></td>
<td>Review and Consolidation</td>
<td>110</td>
</tr>
<tr>
<td></td>
<td>Homework</td>
<td>111</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Lesson</th>
<th>Title</th>
<th>Page Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>23.</td>
<td>The Unit in Review</td>
<td>112–117</td>
</tr>
<tr>
<td></td>
<td>Work Time</td>
<td>112</td>
</tr>
<tr>
<td></td>
<td>Skills</td>
<td>114</td>
</tr>
<tr>
<td></td>
<td>Review and Consolidation</td>
<td>115</td>
</tr>
<tr>
<td></td>
<td>Homework</td>
<td>117</td>
</tr>
</tbody>
</table>

**Comprehensive Review (Units 1–7)** 118–119

**California Mathematics Content Standards** 120–121

**Index** 122
An expression is a combination of variables, numerical values, and operators that does not contain an equals sign.

**Example**

\[5 + 2, \ -5t, \ 2x + 1, \ -4a + 3b, \text{ and } x^2\] are all expressions.

**Numerical expressions** contain only numbers and operations; they do not contain variables.

**Example**

\[5 + 2\] is a numerical expression.

**Variable expressions** contain one or more variables; they may also contain numbers.

**Example**

\[\ -5t, \ 2x + 1, \ -4a + 3b, \text{ and } x^2\] are all variable expressions.

**Expressions Are Used to Represent Quantities**

**Example**

The numerical expression \[4 \cdot 6 \text{ cm}\] can represent the height of a stack of 4 books that are each 6 cm thick.

The variable expression \[6n\] can represent the height of a stack of \(n\) books that are each 6 cm thick.
Lesson 1
Expressions, Equations, and Exponents

Example

Suppose you buy a gift that costs $p$ dollars, and $p$ is less than $10.

$p$ stands for the price of the gift. The variable expression $10 - p$ represents the amount of change you will receive when you pay for the gift with a ten-dollar bill.

Expressions Are Composed of Terms

A term is a number, a variable, or a product of numbers and variables, that is part of an expression.

The terms of an expression are separated by addition or subtraction signs.

Example

The expression $2x + 1$ has two terms.

$2x$ is a variable term. The coefficient of $x$ is 2.

1 is a numerical term.

Example

The expression $4 \cdot 6$ has one term.

$4 \cdot 6$ is a numerical term, equal to 24.

Example

The expression $6n$ has one term.

$6n$ is a variable term. The coefficient of $n$ is 6.

Example

The expression $10 - p$ can be written as $10 + (-p)$. It has two terms.

10 is a numerical term.

$-p$ is a variable term. The coefficient of $p$ is $-1$. 
1. Imagine a sheet of stamps. For each situation, write an expression that represents the cost of the stamps. Sketch diagrams if they help you write the expressions.
   a. Each stamp costs 35 cents, and there are 13 stamps.
   b. Each stamp costs 35 cents, and there are $n$ stamps.
   c. Each stamp costs $d$ cents, and there are 10 stamps.
   d. Each stamp costs $d$ cents, and there are $n$ stamps.

2. An exam takes $a$ minutes, and you have been working on it for 25 minutes.
   a. Sketch a diagram to represent this situation.
   b. Write an expression to represent the amount of time, in minutes, you have left to finish the exam.

3. Rosa is $x$ years old. She is 3 years younger than her brother Carlos.
   a. Sketch a diagram to represent this situation.
   b. Write an expression to represent Carlos’s age in terms of Rosa’s age.

4. Suppose a rabbit hops 20 meters and then stops. Starting at the same point, a turtle crawls $b$ meters in the same direction, but has not yet reached the rabbit.
   a. Sketch a diagram to represent this situation.
   b. Write an expression to represent the distance, in meters, that the turtle still needs to crawl in order to reach the rabbit.

Preparing for the Closing

5. Look at the variables in the Work Time problems: $n, d, a, x,$ and $b$.
   a. Which of the variables $n, d, a, x,$ and $b$ could have negative values?
   b. Which of the variables $n, d, a, x,$ and $b$ could have values that are not integers?

6. Look at the expressions that you wrote in today’s lesson, and determine how many terms are in each.
Lesson 1

7. Which of your expressions are variable expressions? Which is a numerical expression?

8. What is the coefficient of each variable term in the variable expressions that you wrote today?

Skills

Solve.

a. \( \frac{1}{2} \) of 5664 = 2832
b. \( \frac{1}{3} \) of 5664 = 1888

c. \( \frac{1}{5} \) of $5665 = $1133
d. When you divide 56,650 by 5 you get 11,330

e. \( \frac{1}{5} \) of 56,550 = 11,310
f. \( \frac{1}{5} \) of \( k \) = \( \frac{k}{5} \)

g. \( \frac{1}{4} \) of 22,600 = 5650
h. 5664 ÷ 4 = 1416

Review and Consolidation

In problems 1–5, sketch a diagram to represent the description. Label your diagrams with the numbers and letters in the description.

1. There are \( n \) booklets in a stack. Each booklet is 12 mm thick.

2. There are 12 booklets in a stack. Each booklet is \( x \) mm thick.

3. There are 5 more girls than boys in the class. There are \( b \) boys in the class.

4. There are 5 fewer boys than girls in the class. There are \( g \) girls in the class.

5. There are 5 boys in the class. There are \( m \) more girls than boys in the class.

6. Compare your diagrams in problems 1–5 with a partner. Are your diagrams labeled with all of the information given in the descriptions? Are your diagrams clear and easy to understand?
7. Use your diagrams from problems 1–5 to write variable expressions that represent the quantities identified below.
   a. Problem 1: The height of the stack of booklets.
   b. Problem 2: The height of the stack of booklets.
   c. Problem 3: The total number of students in the class.
   d. Problem 4: The total number of students in the class.
   e. Problem 5: The total number of students in the class.

8. Compare the expressions you wrote in problem 7 with a partner. Are the expressions you and your partner wrote the same? Are the expressions you and your partner wrote equivalent?

Homework

Write a variable expression to answer each problem. Sketch a diagram if it helps you write the expression.

1. Keesha has $m$ dollars in the bank. She deposits $100$ more dollars. How much money does Keesha now have in the bank?

2. Suppose that in the morning, the temperature in degrees Fahrenheit is equal to $d$. During the afternoon, the temperature goes up 4 degrees. What is the temperature in the afternoon?

3. A bookshelf holds a row of 10 books of equal width. The width of each book is $w$ centimeters. How long is the row of books?

4. One day, 3 students are absent from a class with a total number of students equal to $s$. How many students are in class?

5. Dwayne is $x$ years old. He is 5 years older than his brother. How old is his brother?

6. Look at the variables in the expressions you wrote in problems 1–5.
   a. Which of these variables could have negative values?
   b. Which of these variables could have values that are not integers?
Dwayne wants to buy some pencils and pens.

The price of pencils is 20 cents each.  
The price of pens is 30 cents each.

Let \( m \) stand for the number of pencils Dwayne wants to buy.

Let \( n \) stand for the number of pens Dwayne wants to buy.

The total cost of \( m \) pencils and \( n \) pens can be represented by this expression.

\[
20m + 30n \text{ cents}
\]

How much does it cost to buy 6 pencils and 4 pens?

\[
(20 \cdot m) + (30 \cdot n) \\
\downarrow \quad \downarrow \quad \downarrow \quad \downarrow \\
(20 \cdot 6) + (30 \cdot 4) = 240 \text{ cents}
\]

Here, the specific values 6 and 4 are substituted for the variables \( m \) and \( n \).

To \textit{evaluate} an expression means to substitute values in for the variables of the expression, and then to calculate the numerical value of the quantity represented by the expression.

The numerical value of the quantity represented by \( 20m + 30n \) is 240 when evaluated for \( m = 6 \) and \( n = 4 \).
1. Refer to the situation described on the previous page, in which pencils cost 20 cents each and pens cost 30 cents each.
   a. What is the total cost for 5 pencils and 5 pens?
   b. Evaluate the expression $20m + 30n$ for $m = 9$ and $n = 12$. This is the total cost for 9 pencils and 12 pens.
   c. Evaluate the expression $20m + 30n$ for $m = 12$ and $n = 9$. This is the total cost for 12 pencils and 9 pens.

2. Some CDs in cases are stacked sideways on a shelf as shown in this diagram. Each CD case has a width of 9 millimeters.
   a. Write an expression to represent the width of a stack of $x$ CD cases.
   b. What is the width of the stack if there are 32 CD cases in the stack? Show how you found the answer.
   c. What is the width of the stack if $x = 48$? Show how you found the answer.

3. Suppose there are also some CDs in boxed sets on the shelf and that each boxed set is 27 millimeters thick.
   a. Write an expression for the width of a stack of $x$ CDs in cases and $y$ boxed sets.
   b. What is the width of a stack of 10 CD cases and 5 boxed sets? Show how you found the answer.
   c. Evaluate the expression you wrote in part a for $x = 32$ and $y = 15$. 
4. Create three multiple-choice answers for each of these multiple-choice questions. Use these rules when you create your answer choices.

• One and only one answer choice must be correct.
• The remaining two answer choices must be incorrect.
• One of the incorrect choices must result from a common mistake.
• Be prepared to explain the mistake or misconception leading to each incorrect answer choice.

a. Which of the following represents the value of the expression $3a + 4b$ when $a = 2$ and $b = -3$? (Create three answer choices, using the rules above.)

b. Which of the following represents the value of the expression $2a + 3b$ when $a = 5$ and $b = -2$? (Create three answer choices, using the rules above.)

c. Which of the following represents the value of the expression $2a + (-5b)$ when $a = 2$ and $b = -3$? (Create three answer choices, using the rules above.)

d. Which of the following represents the value of the expression $a + -\frac{1}{3}b$ when $a = 2$ and $b = -3$? (Create three answer choices, using the rules above.)

Preparing for the Closing

5. Compare the answer choices you wrote in Work Time problem 4 with other students’ answer choices.

a. Identify the mistake or misconception leading to each incorrect answer choice in another student’s answers.

b. What advice would you give to a student taking one of the exams you and your classmates have written?

6. In your own words, say what “evaluate an expression” means.
Skills

Fill in the missing numerator or denominator, or solve for $x$.

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<table>
<thead>
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</thead>
<tbody>
<tr>
<td>a.</td>
<td>$\frac{4}{5} = \frac{8}{\Box}$</td>
<td>b.</td>
<td>$\frac{4}{\Box} = \frac{12}{15}$</td>
<td>c.</td>
</tr>
<tr>
<td>d.</td>
<td>$\frac{16}{\Box} = \frac{8}{10}$</td>
<td>e.</td>
<td>$\frac{5}{6} = \frac{15}{\Box}$</td>
<td>f.</td>
</tr>
<tr>
<td>g.</td>
<td>$\frac{\Box}{18} = \frac{45}{54}$</td>
<td>h.</td>
<td>$\frac{20}{24} = \frac{\Box}{72}$</td>
<td>i.</td>
</tr>
<tr>
<td>j.</td>
<td>$\frac{10}{x} = \frac{30}{36}$</td>
<td>k.</td>
<td>$\frac{x}{18} = \frac{45}{54}$</td>
<td>l.</td>
</tr>
</tbody>
</table>

Review and Consolidation

1. In Work Time problem 2, you wrote an expression for the width of a stack of $x$ CDs in cases that are each 9 millimeters thick.
   a. What are the terms of the expression you wrote?
   b. What do the terms represent in the situation?
   c. What are the coefficients of the terms?
   d. What do the coefficients represent in the situation?

2. In Work Time problem 3, you wrote an expression for the width of a stack of $x$ CDs in cases that are each 9 millimeters thick, and for $y$ boxed sets that are each 27 millimeters thick.
   a. What are the terms of the expression you wrote?
   b. What do the terms represent in the situation?
   c. What are the coefficients of the terms?
   d. What do the coefficients represent in the situation?

3. Suppose Dwayne wants to buy $x$ regular pencils for 20 cents each and $y$ mechanical pencils for 85 cents each.
   a. What is the total cost for 5 regular pencils and 1 mechanical pencil?
   b. What is the total cost for 1 regular pencil and 5 mechanical pencils?
c. What do $x$ and $y$ represent in the problem?

d. What are the values of $x$ and $y$ in parts a and b?

e. Write a variable expression using $x$ and $y$ for the total cost of $x$ regular pencils and $y$ mechanical pencils.

f. What do the coefficients in the variable expression you wrote for part e represent?

**Homework**

1. Evaluate the variable expression $4 - \frac{1}{3}z$ when $z = 12$.

2. Evaluate the variable expression $4 + \frac{k}{9}$ when $k = 27$.

3. What is the coefficient of each of the variables in the expressions in problems 1 and 2?

4. Evaluate the expression $4m$ for each given value of $m$. Find the value either by multiplying $m$ by 4 or by evaluating the equivalent expression $m + m + m + m$.
   
a. $m = 20$  
b. $m = 12$  
c. $m = 6$  
d. $m = 2$

5. Evaluate the expression $\frac{k}{-4}$ for each given value of $k$.

   Find the value by dividing $k$ by $-4$ or by evaluating the equivalent expression $-\frac{1}{4}k$.
   
a. $k = 20$  
b. $k = 12$  
c. $k = 6$  
d. $k = 2$

6. Find the value of $-p + 10$ for each given value of $p$.

   a. $p = 1$  
b. $p = 2.55$  
c. $p = 10$  
d. $p = 11$

7. Find the value of $10 - p$ for each given value of $p$.

   a. $p = 2.55$  
b. $p = 1.55$  
c. $p = 0.55$  
d. $p = 7.89$

8. Notice that the expressions in problems 6 and 7 are equivalent. Which number property or properties explain this equivalence?
**Exponential Form**

The expression $5 \cdot 5 \cdot 5$ has three repeated factors of 5. You have already learned that you can represent this as $5^3$ using exponential form.

The expression $5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5$ has twelve factors of 5. You can see that it is much easier to express this number as $5^{12}$.

Here are some more examples to show how numbers can be expressed in exponential form.

<table>
<thead>
<tr>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0.7 \cdot 0.7 \cdot 0.7 \cdot 0.7 \cdot 0.7 = 0.7^6$</td>
</tr>
<tr>
<td>$89,000 \cdot 89,000 \cdot 89,000 \cdot 89,000 = 89,000^4$</td>
</tr>
<tr>
<td>$(–6)(–6)(–6)(–6)(–6)(–6)(–6)(–6) = (–6)^8$</td>
</tr>
<tr>
<td>$\frac{2}{3} \cdot \frac{2}{3} \cdot \frac{2}{3} \cdot \frac{2}{3} \cdot \frac{2}{3} = \left(\frac{2}{3}\right)^5$</td>
</tr>
<tr>
<td>$2 \cdot 6 \cdot 6 \cdot 8 \cdot 6 \cdot 2 \cdot 6 \cdot x = 2^2 \cdot 6^4 \cdot 8x$</td>
</tr>
</tbody>
</table>

Notice that since $a = a^1$ the exponent 1 need not be written.

A base may be a variable or an exponent may be a variable, or both may be variables.

<table>
<thead>
<tr>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y \cdot y \cdot y \cdot y \cdot y \cdot y = y^8$ This expression represents 8 factors of $y$.</td>
</tr>
<tr>
<td>$3^x$ This expression represents $x$ factors of 3.</td>
</tr>
<tr>
<td>$p^q$ This expression represents $q$ factors of $p$.</td>
</tr>
</tbody>
</table>
Using a Calculator

It is often useful to use a calculator to evaluate numbers expressed in exponential form. Note that different types of calculators operate in different ways.

Some calculators use the caret button $\wedge$ to raise numbers to a given power.

Other calculators have a power key like $y^x$.

Your teacher will help you with this.

Parentheses and Exponents

Think about the number $-4^6$.

Without parentheses you read this number as $-(4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4) = -4096$.

If you want to represent “$-4$ raised to the power of $6$” you must use parentheses.

\[
\text{Example} \\
(-4)^6 = (-4)(-4)(-4)(-4)(-4)(-4) = +4096
\]

Parentheses are essential when you express a negative number in exponential form.

(See your calculator to see how it handles this situation.)

Think about the expression $(2 + 3)^4$.

This is equivalent to $5^4$ but it is not equivalent to $2^4 + 3^4$.

It is very important to remember that the exponent 4 does not distribute over the operations of addition or subtraction within parentheses.

Comment

$5^4 = 625$ and $2^4 + 3^4 = 97$

Be sure that you understand why $(2 + 3)^4$ is not equivalent to $2^4 + 3^4$.

Applications of Exponential Form

Place value notation uses exponents where the base number is always 10.

\[
\text{Example} \\
349 \text{ can be represented as } (3 \cdot 10^2) + (4 \cdot 10^1) + (9 \cdot 10^0)
\]

You also know already that prime factorizations can be expressed using exponents.

\[
\text{Example} \\
7^5 \cdot 11^4 = 7 \cdot 7 \cdot 7 \cdot 7 \cdot 11 \cdot 11 \cdot 11 \cdot 11 = 246,071,287
\]
1. Write each expression in simplest exponential form.
   a. \(7 \cdot 7 \cdot 7 \cdot 7\)
   b. \((-83)(-83)(-83)(-83)(-83)(-83)(-83)(-83)(-83)(-83)(-83)(-83)(-83)(-83)\)
   c. \(0.06 \cdot 0.06 \cdot 0.06 \cdot 0.06 \cdot 0.06 \cdot 0.06 \cdot 0.06 \cdot 0.06\)
   d. \(2 \cdot p \cdot 6 \cdot p \cdot 6\)
   e. \(y \cdot y \cdot (-y) \cdot y \cdot (-y)\)

2. Write each expression as repeated factors.
   a. \(12^9\)
   b. \(3^2 \cdot 3^5\)
   c. \(17^6 \div 17^5\)
   d. \((9 + 2)^5\)
   e. \((9^4)^2\)
   f. \(t^8 \cdot t^2\)
   g. \((2x)^4\)

3. What base would you need to get 32 when the exponent is 5? Explain how you know.

4. Use a calculator to evaluate. Write each answer with all of the decimal places that your calculator shows.
   a. \(17^2\)
   b. \(-8.5^4\)
   c. \((-8.5)^4\)
   d. \(\left(\frac{3}{5}\right)^3\)
   e. \(0.02^3\)
   f. \(11.6^5\)
   g. \((8 - 3.2)^5\)

5. Write in standard form: \((5 \cdot 10^4) + (4 \cdot 10^3) + (1 \cdot 10^2) + (7 \cdot 10^1)\).
Preparing for the Closing

8. Keesha told Rosa that she thought the expression \(a^5\) meant “\(a\) multiplied by itself 5 times.” Rosa disagreed with Keesha. What is wrong with Keesha’s definition?

9. The inverse operation of squaring a number is called finding the square root. The inverse of cubing a number is called finding the cube (or third) root.
   a. What operation is represented by the symbol \(\sqrt[3]{x}\)?
   b. Explain how this operation differs from \(x^6\).
   c. Give at least two numerical examples to support your reasoning.

10. Explain the difference between the expressions \((y + y)^2\) and \(y \cdot y\).

Skills

Express each of these fractions in its simplest form.

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>a. (\frac{17}{34})</td>
<td>b. (\frac{9}{12})</td>
<td>c. (\frac{24}{64})</td>
<td>d. (\frac{5}{225})</td>
</tr>
<tr>
<td>e. (\frac{18}{108})</td>
<td>f. (\frac{63}{84})</td>
<td>g. (\frac{16}{56})</td>
<td>h. (\frac{31}{61})</td>
</tr>
</tbody>
</table>

Review and Consolidation

1. Complete the table:

<table>
<thead>
<tr>
<th>Expression</th>
<th>Base</th>
<th>Exponent</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>a. (6^3)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>b. (0.2)</td>
<td>2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>c. (3^3)</td>
<td></td>
<td>2</td>
<td>27</td>
</tr>
<tr>
<td>d. (\text{–}8)</td>
<td></td>
<td></td>
<td>4096</td>
</tr>
</tbody>
</table>

2. How many factors of 7 are there in each of these expressions?
   a. \(7 \cdot 7 \cdot 7 \cdot 7 \cdot 7 \cdot 7 \cdot 7\)
   b. \(7^4\)
   c. \((7^6)^2\)
   d. \(7^x\)
   e. \(-(7^3)\)
3. Explain how you can evaluate $\left(\sqrt{12,345}\right)^2$ without using a calculator.

4. Decide if each of the following is true or false. Write a brief justification for each choice.
   a. In the number $58^2$, 58 is the exponent.
   b. $15^3 = 45$
   c. $72 = 2^3 \cdot 3^2$

---

**Homework**

1. Write in simplest exponential form.
   a. $8 \cdot 8 \cdot 8$
   b. $10.6 \cdot 10.6 \cdot 10.6 \cdot 10.6 \cdot 10.6 \cdot 10.6$
   c. $m \cdot m \cdot m \cdot m$
   d. $5 \cdot p \cdot 5 \cdot p \cdot 5$

2. Simplify without using a calculator.
   a. $18^2$
   b. $6.5^3$
   c. $\frac{11}{12}^2$

3. Simplify using a calculator. Write each answer to two decimal places.
   a. $11.7^7$
   b. $\left(\frac{11}{12}\right)^6$

4. The square of $58^2$ is equivalent to $58 \cdot 58 \cdot 58 \cdot 58$. Say why.

5. The cube of $67^2$ is equivalent to $67^6$. True or false? Justify your choice.

6. Simplify $(8 \cdot 10^5) + (6 \cdot 10^3) + (4 \cdot 10^2) + (7 \cdot 10)$.

7. Write the prime factorization of 16,000 in exponential form.

8. Write each of these as repeated factors.
   a. $(-7)^9$
   b. $8^2 \cdot 3^5$
   c. $(x^4)^2$
   d. $t^5 \cdot t^2$
   e. $10^8 \div 10^5$
Multiplication

Look carefully at these multiplication problems and their products.

Example

\[ 2^4 \cdot 2^5 = (2 \cdot 2 \cdot 2 \cdot 2)(2 \cdot 2 \cdot 2 \cdot 2 \cdot 2) = 2^9 \]

four factors + five factors = nine factors

\[ 6 \cdot 6^8 \cdot 6^3 = (6)(6 \cdot 6 \cdot 6 \cdot 6 \cdot 6 \cdot 6 \cdot 6 \cdot 6)(6 \cdot 6 \cdot 6) = 6^{12} \]

one factor + eight factors + three factors = twelve factors

\[ y^7 \cdot y = (y \cdot y \cdot y \cdot y \cdot y \cdot y \cdot y)(y) = y^8 \]

seven factors + one factor = eight factors

The examples above show you that if you need to multiply two exponential numbers with the same nonzero base you can just add the exponents.

Here is an example to show how this also works with exponents represented by letters.

Example

\[ 10^m \cdot 10^n = (10 \cdot 10 \cdot 10 \cdot 10 \ldots)(10 \cdot 10 \cdot 10 \cdot 10 \ldots) = 10^{m+n} \]

\[ m \text{ factors } + n \text{ factors } = m + n \text{ factors} \]

The general rule is \( a^m \cdot a^n = a^{m+n} \), where \( a \neq 0 \).
Division
Now think about these quotients.

\[
2^5 \div 2^2 = \frac{2 \cdot 2 \cdot 2 \cdot 2 \cdot 2}{2 \cdot 2} = 2^3
\]

\[
p^8 \div p = \frac{p \cdot p \cdot p \cdot p \cdot p \cdot p \cdot p \cdot p}{p} = p^7
\]

\[
5^m \div 5^n = \frac{5 \cdot 5 \cdot 5 \cdots}{5 \cdot 5 \cdot 5 \cdots} = 5^{m-n}
\]

You can see that in each case the common factors have been cancelled.

The examples above show that you can divide two exponential numbers with the same nonzero base by subtracting the exponents.

The general rule is \(a^m \div a^n = a^{m-n}\), where \(a \neq 0\).

The Zero Exponent
If a division results in the complete cancellation of all factors then the answer is 1.

\[
\frac{2 \cdot 2 \cdot 2 \cdot 2 \cdot 2}{2 \cdot 2 \cdot 2 \cdot 2 \cdot 2} = 1
\]

Since the equivalent quotient in exponential form is \(\frac{2^5}{2^5} = 2^0\), it follows that \(2^0 = 1\).

This result is true for all real bases, except zero (\(0^0\) is not defined): \(a^0 = 1\)
1. Write each of these products in expanded factor form. Use the method of counting the factors to write each product in simplest exponential form.
   a. \(2^4 \cdot 2^3\)
   b. \((-7)^2(-7)^7\)
   c. \(x^8 \cdot x^2 \cdot x^5\)
   d. \(2 \cdot p^4 \cdot 2^3 \cdot p^2\)
   e. \(a^2 \cdot b^3 \cdot a^2 \cdot b\)

2. You can obtain the answers to each part of problem 1 more quickly using another method. Explain.

3. Write each of these quotients in expanded factor form. Use the method of canceling and counting the factors to write each quotient in simplest exponential form.
   a. \(\frac{5^5}{5^3}\)
   b. \(2^7 \div 2^5\)
   c. \((-10)^8 \div (-10)^4\)
   d. \(x^5 \div x\)

4. You can obtain the answers to each part of problem 3 more quickly using another method. Explain.

5. Simplify each of these expressions by adding or subtracting the exponents where appropriate.
   a. \(5^8 \cdot 5^3 \div 5^2\)
   b. \(12^{200} \div 12^{200}\)
   c. \(7^6 \cdot 7^5 \cdot 5^8\)
   d. \(\frac{x^6 \cdot x^5 \cdot x^6}{x^3 \cdot x^{22}}\)

6. Simplify each of these expressions by adding or subtracting the exponents where appropriate.
   a. \(\frac{p^6 q^5}{p^2}\)
   b. \(\frac{h^7 k^7}{h^4 k}\)
   c. \(\frac{2^6 m^8 n^4}{2^3 m^3}\)
   d. \(\frac{10a^6 b^5}{b^2} \cdot \frac{10^3 a^4 b}{a^2 b^2}\)
Preparing for the Closing ————————————————————

7. Lisa and Dwayne were discussing the product $2^6 \cdot 3^4$.

Lisa said that the product could be simplified to $6^{10}$.

Dwayne said that the product could not be simplified using exponents.

Do you agree with either Lisa or Dwayne? Explain your response and point out any errors that you think Lisa or Dwayne made.

8. Rosa and Chen were discussing the quotient $2^{10} \div 2^{10}$.

Chen said the quotient cannot be found because if you subtract the exponents you get zero and it is impossible to raise a number to the power of zero.

Rosa said that you must be able to find the quotient because both the numerator and the denominator are equal to 20, and 20 divided by 20 is 1.

Do you agree with either of these students? Explain your response and point out any errors that you think Chen or Rosa made.

Skills

Express each of these as a mixed number or a whole number.

a. $\frac{17}{4}$  b. $\frac{18}{4}$  c. $\frac{19}{4}$  d. $\frac{16}{4}$  e. $\frac{21}{4}$

Change these mixed numbers to improper fractions.

f. $\frac{1}{2}$  g. $\frac{3}{4}$  h. $\frac{2}{3}$  i. $15\frac{1}{2}$  j. $15\frac{3}{4}$
Review and Consolidation

1. Use scientific notation to express the answer to the problem “one thousand times nine million.”

2. Copy and complete this table.

<table>
<thead>
<tr>
<th>Exponent Form</th>
<th>Expanded Form</th>
</tr>
</thead>
<tbody>
<tr>
<td>a. (2x^5)</td>
<td></td>
</tr>
<tr>
<td>b. (5 \cdot y \cdot y \cdot p)</td>
<td></td>
</tr>
<tr>
<td>c. (d \cdot 5 \cdot p \cdot p \cdot 2)</td>
<td></td>
</tr>
<tr>
<td>d. (7x^3y^2)</td>
<td></td>
</tr>
<tr>
<td>e. ((7x^3)(8y)^2)</td>
<td></td>
</tr>
<tr>
<td>f. ((-4)^2m^3)</td>
<td></td>
</tr>
</tbody>
</table>

3. Evaluate each of these expressions using the values \(a = -2\) and \(b = 3\).
   a. \(2a^5\)  
   b. \(ab^2\)  
   c. \(a^b\)  
   d. \((-ab)^2\)

4. Simplify these expressions by adding or subtracting exponents where appropriate.
   a. \(x^4 \cdot x^3\)  
   b. \(m^8 \div m^6\)  
   c. \(\frac{n^5p^2}{n}\)  
   d. \(\frac{k^7b^8}{b^5} \cdot \frac{k^2}{b}\)

Homework

1. Write each of these products in expanded factor form and then write it in simplest exponential form.
   a. \(2^5 \cdot 2^9\)  
   b. \((-6)^2(-6)^9\)  
   c. \(x^9 \cdot x^2 \cdot 3^5\)

2. Write each of these quotients in expanded factor form and then write it in simplest exponential form.
   a. \(\frac{8^5}{8^3}\)  
   b. \(12^8 \div 12^5\)  
   c. \((-2)^9 \div (-2)^4\)  
   d. \(x^{17} \div x^{12}\)

3. Simplify each of these expressions by adding or subtracting the exponents where appropriate.
   a. \(5^8 \div 5^3 \cdot 5^2\)  
   b. \((8^6 \cdot 8^5) \div (8^8 \cdot 8)\)  
   c. \(\frac{c^9q^4}{q^7}\)
   d. \(\frac{3^6m^8n^6}{3^3m^7}\)  
   e. \(\frac{8a^8b^5}{b} \cdot \frac{8^3a^2b^6}{a^7b}\)
Think about the expression $2n^2 + 1$.

How would you give your friend instructions for what to do to $n$? If you said, “Add one to $n$, then multiply $n$ by two, and then square the answer,” would your friend write the correct expression? What is wrong with these instructions?

Here are the correct instructions. “Square $n$, multiply the answer by two, and then add one.”

Here is an area model showing $2n^2 + 1$.

In this area model,

- $n$ represents $n^2$,
- $n$ represents $2n^2$, and
- 1 represents 1.
1. For each of the area models shown below in parts a–g, identify the numerical expression or expressions from the choices below that represent the same quantity.

A. \(3^2 + 4^2\)

B. \(2(3 + 4)\)

C. \((3 + 4)^2\)

D. \(3 \cdot 4^2\)

E. \((3 \cdot 4)^2\)

F. \(\frac{3}{2} + \frac{4}{2}\)

G. \((2 \cdot 3) + 4\)

H. \(4 + (3 \cdot 2)\)

I. \(\frac{3}{2} + 4\)

J. \((2 \cdot 3) + (2 \cdot 4)\)

K. \(\frac{1}{2} (3 + 4)\)

L. \(3^2 + 4^2 + (2 \cdot 3 \cdot 4)\)

M. \(\frac{3 + 4}{2}\)

N. \(\frac{3}{2} + 2\)

Example:

- a.\[\text{Area} = 3 \times 4 + 2 \times 1 = \frac{3 + 4}{2}\]
- b.\[\text{Area} = 4 \times 4 \times 4 = 4 \times 3^2\]
- c.\[\text{Area} = 4 \times 4 \times 4 = 4 \times 3^2\]
EXPRESSions and aREa ModELs

2. For each of the area models below, identify the corresponding verbal instructions from the choices below and then write a variable expression or expressions to represent the quantity. You will discover one area model that has no corresponding verbal instructions. Write your own instructions and expression for this area model.

A Multiply \(n\) by two, then add six.  
B Multiply \(n\) by three, then square the answer.  
C Add six to \(n\), then multiply by two.  
D Add six to \(n\), then divide by two.  
E Add three to \(n\), then multiply by two.  
F Add six to \(n\), then square the answer.  
G Multiply \(n\) by two, then add twelve.  
H Divide \(n\) by two, then add six.  
I Square \(n\), then add six.  
J Square \(n\), then multiply by nine.

Example

a. \[
\begin{array}{|c|c|}
\hline
n & 3 \\
\hline
\end{array}
\]

b. \[
\begin{array}{|c|}
\hline
\frac{1}{2}n & 12 \\
\hline
\end{array}
\]

- C Add six to \(n\), then multiply by two: \(2(n + 6)\).  
- E Multiply \(n\) by two, then add twelve: \(2n + 12\).  

Comment

Some area models will have more than one corresponding set of verbal instructions, and more than one variable expression.
Expressions, Equations, and Exponents

Lesson 5

c. $n \quad n \quad n$
   $n$
   $n$
   $n$

d. $\frac{1}{2} \quad n \quad 6$

Preparation for the Closing

3. For the area models in problems 1–2 that have more than one corresponding expression, explain why these expressions are equivalent. In some cases, a number property will be the only explanation you need.

Skills

Match each of these ordered pairs with a point on the coordinate grid.

(5, 0)  (0, –8)
(–8, –8) (–4, 0)
(0, 0)  (0, 3)
(4, –4)  (–6, 3)
(3, 6)
1. a. Calculate $2^2 + 5^2$.
   
   b. Sketch a geometric figure with an area that can be represented by $2^2 + 5^2$.

2. a. Calculate $(2 + 5)^2$.
   
   b. Sketch a geometric figure with an area that can be represented by $(2 + 5)^2$.

3. a. Calculate $(3 \cdot 2) + 7$.
   
   b. Sketch a geometric figure with an area that can be represented by $(3 \cdot 2) + 7$.

4. a. Calculate $3(2 + 7)$.
   
   b. Sketch a geometric figure with an area that can be represented by $3(2 + 7)$.

5. Evaluate these expressions for $d = -3$.
   a. $d^2$
   b. $-d^2$
   c. $(-d)^2$
In problems 1–7, identify the corresponding verbal instructions from the choices listed below (A–G).

A. Add 1 to \(x\), then square the result, then multiply the result by 4.

B. Add 7 to 3, then cube the result.

C. Square 2, then square 5, then add the two results.

D. Square \(x\), then multiply the result by 4, then add 1 to the result.

E. Multiply \(x\) by 4, then add 1 to the result.

F. Add 7 to 3, then multiply the result by three.

G. Add 2 to 5, then square the result.

1. \(2^2 + 5^2\)  
2. \((2 + 5)^2\)  
3. \(3(3 + 7)\)  
4. \((3 + 7)^3\)

5. \(4x + 1\)  
6. \(4(x + 1)^2\)  
7. \(4x^2 + 1\)

8. Write two variable expressions for the area of each of these figures.

a. \(x + 1\)  

    4

b. \(n\)  

    \(\begin{array}{c|c|c|c}
    \hline
    & 2 & 6 & 1 \\
    \hline
    4 & & & \\
    \hline
    \end{array}\)

9. Which number properties explain why the two expressions you wrote in each part of problem 8 are equivalent?
Think about this expression: \(5x + 9y + 2x - 5y\).
The expression has four terms: \(5x\), \(9y\), \(2x\), and \(-5y\).

The terms \(5x\) and \(2x\) are called *like terms* because the variable, \(x\), is the same in each term. The terms \(9y\) and \(-5y\) are also *like terms* because the variable, \(y\), is the same in each term.

You can simplify expressions by combining like terms using the number properties.

### Example

<table>
<thead>
<tr>
<th>Action</th>
<th>Purpose</th>
<th>Justification</th>
</tr>
</thead>
<tbody>
<tr>
<td>(5x + 9y + 2x - 5y = 5x + 2x + 9y - 5y)</td>
<td>Rearrange terms</td>
<td>To get like terms next to each other</td>
</tr>
<tr>
<td>(= x(5 + 2) + y(9 - 5))</td>
<td>Group coefficients of like terms</td>
<td>To combine like terms</td>
</tr>
<tr>
<td>(= x7 + y4)</td>
<td>Add coefficients</td>
<td>To simplify each term</td>
</tr>
<tr>
<td>(= 7x + 4y)</td>
<td>Rewrite each term</td>
<td>To use conventions correctly</td>
</tr>
</tbody>
</table>

The expression \(5x + 9y + 2x - 5y\) is equivalent to the expression \(7x + 4y\).

### Note on Variable Terms

*Like terms* not only have the same variable, but the variable is raised to the same power. For example, even though the terms in the expression \(x + x^2\) have the same variable, they cannot be written as one term.

On the other hand, \(x^2\) and \(10x^2\) are like terms. The variable is the same and it is raised to the same power in both terms. They can be combined using the distributive property, just like other like terms.

### Example

\[3x^2 + 10x^2 = (3 + 10)x^2 = 13x^2\]
Note on Numerical Terms
Numerical terms are like terms. They are just numbers.
For example, the expression \(-2x + 3 + 4y - 1\) can be simplified to \(-2x + 4y + 2\).
In this case, the 3 and the \(-1\) are like terms and were simply added.

Work Time

1. Simplify these expressions by using the distributive property to combine like terms. In expressions with subtraction, practice adding the inverse of a term rather than subtracting.

Example

Think about the expression \(6x^2 - (-2x^2)\).
You can combine like terms in this expression because the variable is the same in both terms; but what about the subtraction and the negative sign?
Remember the inverse property of addition: \(-a\) and \(a\) are “additive inverses” of each other.

The additive inverse of \(-2\) is 2.
The additive inverse of \(-2x^2\) is \(2x^2\).
Instead of subtracting the term \((-2x^2)\) from \(6x^2\), you can add its inverse, \(2x^2\).
\(6x^2 - (-2x^2)\) is equivalent to \(6x^2 + 2x^2\).
Now, combining like terms is easy using the distributive property.
\(6x^2 + 2x^2 = (6 + 2)x^2 = 8x^2\)

a. \(3x + 7x\)
b. \(4x + (-3x)\)
c. \(-2m + (-6m)\)
d. \(7x + 2y + (-8x) - 3y\)
e. \(6a + 4b + 4a + 5a\)
f. \(a + 5 + b + 7a + (-2) - b\)
g. \(3x^2 - 7x^2\)
h. \(4x^3 - (-3x^3)\)
i. \(6a^3 + 4b^3 - 4a^2 - (-5a^2)\)
j. \(a^3 + 5 + b^3 - 7a^2 - 2 - b^2 + 3a - b + 4a^3 - b^2\)
2. Simplify these expressions by combining like terms. Use the distributive property to rewrite the expressions first so that it is easier to combine like terms.
   a. $5b + 3(b - 3)$
   b. $-3x(3x - 5) - 15x^2$
   c. $5t - (t^2 + 4t)$

Preparing for the Closing

3. The expression $ax + bx$ has like terms that can be combined using the distributive property, like this: $ax + bx = (a + b)x$.
   a. Sketch a geometric figure with an area that can be represented by $ax + bx$.
   b. Identify the parts of the figure that correspond to $ax$, $bx$, and $(a + b)x$.
   c. Can you think of a nongeometric quantity that can be represented by $(a + b)x$? Describe the quantity in words.

4. Imagine that you are a teacher scoring the following student work. Explain what the student did wrong, and then correct the work.
   a. $4m + 8 = 12m$
   b. $(5n)^2 - n = 4n^2$

Skills

Plot the given points on a coordinate grid, and tell what simple figure they make when line segments are drawn connecting the points in order.
   a. $A (8, 8); B (9, 12); C (7, 12)$
   b. $A (5, 7); B (7, 7); C (7, 10); D (5, 10)$
   c. $A (3, 12); B (1, 12); C (1, 8); D (3, 8)$
   d. $A (7, 5); B (9, 5); C (8, 0); D (6, 0)$
   e. $A (1, 0); B (2, 1); C (1, 3); D (0, 2)$

Review and Consolidation

In problems 1–8, write the additive inverse of each expression.

1. $x$
2. $-x$
3. $-(-x)$
4. $10x^4$
5. $-10x^4$
6. $-(-10x^4)$
7. $8y^4$
8. $-5z^5$
In problems 9–12, use the distributive property and the commutative property of multiplication to rewrite each of these expressions, as shown in the examples. Be sure to show both steps illustrated in the examples for each problem.

Example

\[4a + 8a = a(4 + 8) = (4 + 8)a\]

\[2m - m = m(2 - 1) = (2 - 1)m\]

\[x^4 - 10x^4 = x^4(1 - 10) = (1 - 10)x^4\]

9. \(9y - 8y\)  
10. \(z - 5z\)  
11. \(9y^4 - 8y^4\)  
12. \(z^5 - 5z^5\)

13. Simplify these expressions by combining like terms. Use the distributive property and the commutative property, as you did in problems 9–12, but simplify as far as possible.

a. \(7x + 2x\)  
b. \(7x^2 - 2x^2\)  
c. \(7x + (-2x)\)  
d. \(7x^2 - (-2x^2)\)

Homework

1. Simplify these expressions using the distributive property to combine like terms. Remember to add the inverse of a term instead of subtracting the term in expressions with subtraction, and be sure to combine only terms with the same variable, raised to the same power.

a. \(6y + (-4y)\)  
b. \(6y - (-4y)\)

c. \(x - 3y + y\)  
d. \(x - 3y - y\)

e. \(7x^3 + 2y^3 + 11x^2 - 1 + 3x^3 + y^3\)  
f. \(7x^3 + 2y^3 - 11x^2 - 1 + 3x^3 - y^3\)

g. \(3x + 2 + x\)  
h. \(3x - 2 + x\)

2. Use the distributive property and the inverse property to simplify these expressions by combining like terms.

a. \(3x - 2(1 + x)\)  
b. \(3x - x(2 + x)\)
When an expression represents a quantity, you need to use the correct units for the expression. Think back to the pencils and pens Dwayne was buying in Lesson 2.

The price of pencils is 20 cents each. The price of pens is 30 cents each.

In this case, the prices of both pens and pencils are given in cents. The total cost of \( m \) pencils and \( n \) pens can be represented by the expression

\[
20m + 30n \text{ cents}
\]

where \( m \) = the number of pencils and \( n \) = the number of pens.

Sometimes a problem involves expressing one unit of measurement in terms of another unit of measurement.

Rosa is also shopping for pencils and pens, but the pens she wants to buy are fancier.

The pencils she buys are the same: 20 cents each. The pens she buys are more expensive: 3 dollars each.

In this case, the prices of pens and pencils are given in different units. To write an expression to represent the total cost of \( m \) pencils and \( n \) pens, each term must have the same units.

If you express the cost in cents, the expression is \( 20m + 300n \) cents.

If you express the cost in dollars, the expression is \( 0.2m + 3n \) dollars.
1. What is the sum of 5 yards and 7 feet?
   a. Give your answer in yards.
   b. Give your answer in feet.

2. This diagram shows the sum of $a$ yards and $b$ feet.

![Diagram of a yard and feet]

Write an expression for the sum of $a$ yards and $b$ feet.

a. Give your answer in yards.
b. Give your answer in feet.

Example

This diagram shows the sum of $a$ feet and $b$ inches.

![Diagram of feet and inches]

If you use feet as the unit, then the sum is $a + \frac{b}{12}$ feet.

If you use inches as the unit, then the sum is $12a + b$ inches.

3. Write an expression for the sum of $a$ dollars and $b$ cents.
   a. Give the answer in dollars.
   b. Give the answer in cents.
Working with a partner, represent each of the quantities described in problems 4–6 using an expression with the units indicated.

4. The cost of $m$ pencils that cost 25 cents each and $n$ pens that cost 2 dollars each.
   a. Give the answer in cents.
   b. Give the answer in dollars.

5. The cost of 10 pencils that cost $x$ cents each and 6 pens that cost $y$ dollars each.
   a. Give the answer in cents.
   b. Give the answer in dollars.

6. Keesha is serving juice at a party. She has 20 glasses. 12 of her glasses hold 2 cups of liquid and 8 of her glasses hold 12 ounces of liquid. If she wants to fill all of her glasses, how much juice does she need?
   a. Give your answer in cups.
   b. Give your answer in ounces.
7. Jamal needs two types of ribbon for an art project. One type of ribbon is sold by the yard for $1.60 per yard. The other type of ribbon is sold by the foot for $0.65 per foot.

Example

This diagram shows the total cost for $b$ units of ribbon that costs $a$ dollars per unit.

1 unit would cost $a$ dollars.

$b$ units would cost $ab$ dollars.

a. Write an expression for the total cost of $x$ yards of the first type of ribbon and $y$ feet of the second type of ribbon.

b. What is the total cost for 3 yards of the first type of ribbon and 5 feet of the second type?

c. What is the total cost for 4 yards of the first type of ribbon and 2.5 feet of the second type?

d. If Jamal buys 2 yards and 5 feet of ribbon, how much ribbon does he have in total? First write your answer in feet, and then write your answer in yards.

e. If Jamal buys 2.5 yards and 4 feet of ribbon, how much ribbon does he have in total? First write your answer in feet, and then write your answer in yards.
Preparing for the Closing

8. Suppose a friend missed class today. Explain to her how to combine two quantities that are measured in different units.

9. Using your explanation from problem 8, express the sum of \( x \) yards and \( y \) inches:
   a. in yards
   b. in inches

10. Using your explanation from problem 8, express the sum of \( x \) days and \( y \) seconds:
    a. in days
    b. in seconds

Skills

Plot the given points on a coordinate grid, and tell what simple figure they make when line segments are drawn connecting the points in order.

| a. \( A \) (5, 4); \( B \) (3, 4); \( C \) (3, 7) |
| b. \( A \) (4, 10); \( B \) (5, 12); \( C \) (4, 13); \( D \) (3, 11) |
| c. \( A \) (12, 1); \( B \) (12, 3); \( C \) (14, 0); \( D \) (14, 1) |
| d. \( A \) (12, 12); \( B \) (13, 9); \( C \) (12, 7); \( D \) (11, 9) |
| e. \( A \) (11, 7); \( B \) (10, 6); \( C \) (11, 3); \( D \) (12, 4) |

Review and Consolidation

1. State whether the inequalities are *true* or *false*.

   For each inequality that is false, rewrite it so that it is true using one of these methods:
   - Switch the inequality symbol
   - Change the quantity with larger units
   - Change the quantity with smaller units

   Use each of the methods listed above in at least one of these inequalities.

   a. \( 5 \text{ yd} < 17 \text{ ft} \)  
   b. \( $13.22 < 1299 \text{ cents} \)  
   c. \( 28 \text{ ounces} > 4 \text{ cups} \)  
   d. \( 72 \text{ m} > 7199 \text{ cm} \)  
   e. \( 3.5 \text{ hr} < 220 \text{ min} \)  
   f. \( 72 \text{ km} < 7201 \text{ m} \)
Lesson 7

The inequalities in problems 2 and 3 are written with variable expressions on either side of the inequality sign. Evaluate the expressions for the given values and state whether the inequalities are true or false for those values.

2. $6a < 12b$
   a. $a = 10, b = 5$
   b. $a = 100, b = 51$
   c. $a = 9, b = 4$

3. $2x + 1 > x^2$
   a. $x = 1$
   b. $x = 2$
   c. $x = 3$

Homework

Pay close attention to the units of each variable in the situations as you write your expressions.

1. Write an expression to represent the time you still have left to finish an exam if the whole exam takes $t$ hours and you have been working on the exam for 25 minutes.
   a. Give the answer in minutes.
   b. Give the answer in hours.

2. Rosa is training for a track meet. She can run a mile in 6 minutes and 30 seconds. She wants to cut her time down by $n$ seconds. Write an expression for the mile time Rosa is aiming for.
   a. Give the answer in seconds.
   b. Give the answer in minutes.

3. A rabbit has walked 20 meters and a turtle has walked $b$ centimeters from the same starting point in the same direction. Write an expression to represent the distance that the turtle needs to walk to catch up to the rabbit.
   a. Give the answer in meters.
   b. Give the answer in centimeters.

4. Write an expression for the sum of $x$ inches and $y$ feet.
   a. Give the answer in feet.
   b. Give the answer in inches.
You have seen how like terms can be combined by adding and subtracting. Whole expressions can also be added and subtracted.

**Example**

Suppose that you want to add these two expressions.

\[2(3a + 2) \text{ and } (4a - 5)\]

Addition is commutative, so it does not matter in what order you add them.

\[2(3a + 2) + (4a - 5) \text{ or } (4a - 5) + 2(3a + 2)\]

You can add them in either order.

Using the distributive property, you can rewrite the expression \(2(3a + 2)\) as \((6a + 4)\).

Now, the problem is to add \((6a + 4)\) and \((4a - 5)\).

\[
\begin{array}{c}
6a + 4 \\
4a - 5 \\
10a - 1
\end{array}
\]

You can line up \(6a\) and \(4a\) because they are like terms.
You can line up \(4\) and \(-5\) because they are like terms as well.
First add like terms \(6a\) and \(4a\) to get \(10a\). Then add like terms \(4\) and \(-5\) to get \(-1\).
The final expression is \(10a - 1\).
Example

Suppose that you want to subtract \((4a - 5)\) from \(2(3a + 2)\).

Subtraction is not commutative, so the order in which you set up the expressions in your problem \textit{does} matter. The expressions must be in the correct order.

\[
2(3a + 2) - (4a - 5)
\]

As you know, instead of subtracting a number or a term, you can add the inverse of the number or the term. You can do the same with expressions.

Instead of subtracting an expression, you can add the inverse of the expression.

Instead of subtracting \((4a - 5)\) from \(2(3a + 2)\), you can add the inverse of \((4a - 5)\) to \(2(3a + 2)\). The additive inverse of \((4a - 5)\) is \(-\frac{1}{2}(4a - 5)\) or \((-4a + 5)\).

Observe that \(-\frac{1}{2}(4a - 5) = -\frac{1}{2} \cdot 4a + 5\) because of the distributive property.

\[
-\frac{1}{2}(4a - 5) = -1(4a - 5) = \left[\frac{1}{2} \cdot (4a)\right] + \left[\frac{1}{2} \cdot (-5)\right] = -4a + 5
\]

Thus the original problem can be rewritten as: \(2(3a + 2) + (-4a + 5)\).

Using the distributive property again, you can rewrite \(2(3a + 2)\) as \(6a + 4\).

Now, the problem is to add \(6a + 4\) and \(-4a + 5\).

\[
\begin{align*}
6a + 4 & \quad \text{Line up 6a and -4a because they are like terms.} \\
-4a + 5 & \quad \text{Line up 4 and 5 because they are like terms as well.} \\
\hline
2a + 9 & \quad \text{Be careful to add 6a and -4a to get 2a.} \\
& \quad \text{Then add like terms 4 and 5 to get 9.} \\
& \quad \text{The final expression is 2a + 9.}
\end{align*}
\]

Work Time

1. Perform these calculations vertically, as shown in the example above.
   a. Add \((2x + 5)\) and \((3x - 7)\).
   b. Add \((5x - 4y)\) and \((4x - 7y)\).
   c. Add \(-3(x - 4)\) and \(2(x + 5)\).
   d. Add \(5m(3m - 2)\) and \(2m(m - 3)\).
2. Use the distributive property to show that the additive inverse of $2x - 7$ is $-2x + 7$.

3. Use the distributive property to show that the additive inverse of $4x - 5$ is $-4x + 5$.

4. Perform these calculations vertically. Practice rewriting subtraction problems by adding the inverses of expressions instead of subtracting them.
   
   a. Subtract $(3x - 2)$ from $(4x + 7)$.
   
   b. Subtract $2(7n - 5)$ from $(3 - 2n)$.
   
   c. Subtract $(m - 3n)$ from $5(3m - 2n)$.

Preparing for the Closing

5. In today’s problems, you added and subtracted problems vertically, as shown at the beginning of the lesson. You can also perform these calculations horizontally.

   Redo your work in problem 1 part a and in problem 4 part c by adding or subtracting the expressions horizontally, instead of vertically.

Skills

Plot the given points on a coordinate grid, and tell what simple figure they make when line segments are drawn connecting the points in order.

   a. $A(-4, -3)$; $B(4, -3)$; $C(0, 5)$
   
   b. $A(0, 5)$; $B(1, 2)$; $C(4, 2)$; $D(2, 0)$; $E(3, -3)$; $F(0, -1)$; $G(-3, -3)$; $H(-2, 0)$; $I(-4, 2)$; $J(-1, 2)$
   
   c. $A(-6, 0)$; $B(-3, -3)$; $C(5, 0)$; $D(-3, 3)$

Review and Consolidation

1. Use the distributive property to simplify these expressions.
   
   a. $-5(x - y)$
   
   b. $-5(x - 3)$
   
   c. $-5(20 - 3)$

2. Work with a partner. Explain to your partner how you handled the subtraction and negative signs in your work for problem 1.
3. When Lisa and Jamal completed Work Time problem 4 from this lesson, in each case, only one of them worked the part correctly. Your task is to:

- Identify the mistakes they made.
- Explain the misconception that led to each mistake.
- Correct their work.

**Lisa’s Work Time**

**a.** \((4x + 7) - (3x - 2)\)

- \((4x + 7) + (-1)(3x - 2)\)
- \(4x + 7 - 3x + 2\)
- \(4x - 3x + 7 + 2\)
- \((4 - 3)x + 7 + 2\)
- \(x + 9\)

**b.** \((3 - 2n) - 2(7n - 5)\)

- \(3 - 2n + -2 \cdot 7n - -2 \cdot -5\)
- \(3 - 2n + -14n + -10\)
- \(3 + -10 - 2n + -14n\)
- \((-2 + -14)n + 3 - 10\)
- \(-16n - 7\)

**c.** \(5(3m - 2n) - (m - 3n)\)

- \(5[3m + (-1)2n] + (-1)[m + (-1)3n]\)
- \(15m - 2n - m + 3n\)
- \(15m - m - 2n + 3n\)
- \((15 - 1)m - (2 + 3)n\)
- \(14m + 5n\)
Jamal’s Work Time

a. \((4x + 7) - (3x - 2)\)
   \[= 4x + 7 - 3x - 2\]
   \[= 4x - 3x + 7 - 2\]
   \[= (4 - 3)x + 5\]
   \[= x + 5\]

b. \((3 - 2n) - 2(7n - 5)\)
   \[= 3 + -2n + (-2)[7n + (-1)5]\]
   \[= 3 + -2n + -14n + 10\]
   \[= (-2 + -14)n + 3 + 10\]
   \[= -16n + 13\]

c. \(5(3m - 2n) - (m - 3n)\)
   \[= 5(3m - 2n) + (-1)(m - 3n)\]
   \[= 5(3m - 2n) + (-1)m - (-1)3n\]
   \[= 5(3m - 2n) - m + 3n\]
   \[= (5)3m - (5)2n - m + 3n\]
   \[= 15m + (-10)n - m + 3n\]
   \[= (15 - 1)m + (-10 + 3)n\]
   \[= 14m - 7n\]

4. Look at Lisa’s and Jamal’s work shown in problem 3.

   a. Identify whose calculation was correct for each part.

   b. Describe what they did in each step of their work, including any number properties they used.

Example

b. Jamal’s work was correct.

   \((3 - 2n) - 2(7n - 5)\)
   \[= 3 + -2n + (-2)[7n + (-1)5]\]
   \[= 3 + -2n + -14n + 10\]
   \[= (-2 + -14)n + 3 + 10\]
   \[= -16n + 13\]
   
   - Dropped unnecessary parentheses; added inverse instead of subtracting
   - Distributed \(-2\)
   - Applied distributive property to combine like terms
   - Added numbers
1. Add these expressions. Calculate horizontally as shown, or rewrite the calculation vertically, as in today’s lesson.
   
   a. \((4x + 1) + (9x - 1)\)  
   b. \(-2k(k + 3) + k(3k + 5)\)  
   c. \((5x - 2y) + 5(x + 2y)\)

   Remember that adding the inverse of an expression is equivalent to subtracting that expression.

   **Example**

   To subtract \(2x - 7\) from \(4x + 5\), you can add the inverse of \(2x - 7\) to \(4x + 5\).

   \[
   4x + 5 - (2x - 7) = 4x + 5 + (-2x + 7)
   \]
   
   Add the inverse instead of subtracting.

   \[
   = 4x + (-2x) + 5 + 7
   \]
   
   Reorganize terms using commutative and associative properties of addition.

   \[
   = 2x + 12
   \]
   
   Combine like terms.

2. Subtract \(4x - 5\) from \(2x - 7\) using the procedure shown in the example. Arrange the expressions horizontally.

3. Subtract these expressions. Calculate horizontally as shown, or rewrite the calculation vertically, as in today’s lesson.
   
   a. \((4x + 1) - (9x - 1)\)  
   b. \(-2k(k + 3) - k(3k + 5)\)  
   c. \((5x - 2y) - 5(x + 2y)\)

4. Correct these calculations by changing either side of each equation.
   
   a. \((x + 1) - (2x + 1) = -x + 2\)
   
   b. \((4p^2 - 3q) - 4(p^2 - q) = -7q\)
   
   c. \(5k(k + 1) - (k^2 + 5) = 4k^2\)
Parentheses and Negative Bases

In Lesson 3 you learned that it is very important to use parentheses when the base number is negative.

Example

\((-10)^4\) means \((-10)(-10)(-10)(-10) = +10,000.

Without parentheses, \(-10^4\) means \(-(10 \cdot 10 \cdot 10 \cdot 10) = -10,000.\)

Parentheses and Numbers in Exponential Form

Parentheses are necessary for representing situations in which a number in exponential form is raised to another power.

Example

\((2^4)^2\) = \(2^4 \cdot 2^4 = 2^{4+4} = 2^8\)

\((p^6)^3\) = \(p^6 \cdot p^6 \cdot p^6 = p^{6+6+6} = p^{18}\)

The same results can be obtained using multiplication instead of repeated addition.

Example

\((2^4)^2\) = \(2^4 \cdot 2^2 = 2^8\)

\((p^6)^3\) = \(p^6 \cdot 3 = p^{18}\)

These examples show that you can simplify by calculating the product of the exponents.

The general rule is: \((a^m)^n = a^{mn} \cdot n = a^{mn},\) where \(a \neq 0.\)
Parentheses with Products

Think about this example.

Example
\[(2^4 \cdot 5^2)^3 = (2^4 \cdot 5^2)(2^4 \cdot 5^2)(2^4 \cdot 5^2) = 2^{4+4+4} \cdot 5^{2+2+2} = 2^{12} \cdot 5^6\]

A quicker way to obtain the exponents is to use multiplication instead of repeated addition.

Example
\[(2^4 \cdot 5^2)^3 = 2^4 \cdot 3 \cdot 5^2 \cdot 3 = 2^{12} \cdot 5^6\]

Here is another example.

Example
\[(6h)^5 = (6h)(6h)(6h)(6h)(6h) = (6 \cdot 6 \cdot 6 \cdot 6 \cdot 6)(h \cdot h \cdot h \cdot h \cdot h) = 6^{1+1+1+1+1} \cdot h^{1+1+1+1+1} = 6^5 \cdot h^5 = 6^5h^5\]

Using the method of multiplying the exponents gives:

Example
\[(6h)^5 = 6^1 \cdot 5 \cdot h^1 \cdot 5 = 6^5h^5\]

Parentheses with Quotients

Think about this example.

Example
\[(2^4 \div 5^2)^3 = \left(\frac{2^4}{5^2}\right)\left(\frac{2^4}{5^2}\right)\left(\frac{2^4}{5^2}\right) = \frac{2^{4+4+4}}{5^{2+2+2}} = \frac{2^{12}}{5^6}\]

You can obtain these exponents through multiplication instead of repeated addition.

Example
\[(2^4 \div 5^2)^3 = 2^4 \cdot 3 \div 5^2 \cdot 3 = 2^{12} \div 5^6\]
These examples show that for products and quotients, an exponent outside the parentheses distributes over the exponents inside the parentheses.

\[
(a^m \cdot b^n)^p = a^{mp} \cdot b^{np}
\]
\[
\left(\frac{a^m}{b^n}\right)^p = \frac{a^{mp}}{b^{np}}
\]

If you need to simplify an expression in which the numbers do not have the same base, then writing each number as a product of its prime factors can sometimes help.

**Example**

\[
\frac{32^n}{8^{n+1}} = \frac{(2^5)^n}{(2^3)^{n+1}} = \frac{2^{5n}}{2^{3n+3}} = 2^{5n-(3n+3)} = 2^{2n-3}
\]

**Parentheses with Sums and Differences**

You have already learned that an exponent outside parentheses does not distribute over a sum or difference inside the parentheses.

That is, \((a + b)^n \neq a^n + b^n\) and \((a - b)^n \neq a^n - b^n\)

**Work Time**

1. Write each expression in exponential form, without parentheses.

   a. \((5^2)^6\)     b. \((2m^3)^5\)     c. \(\left(\frac{7^4}{9^2}\right)^3\)

   d. \([(-10)^3]^8\)     e. \((9c^5)^0\)     f. \(\left(\frac{3^1}{6^0}\right)^4\)

2. Write each expression in exponential form, without parentheses.

   a. \((wxyz^2)^5\)     b. \((ab^4)a^3\)     c. \((c^2d^5)(c^3d)^4\)

   d. \(\frac{(8w^5y^6)^2}{(wy^2)^3}\)     e. \(\left(\frac{8w^5y^6}{wy^2}\right)^2\)
3. Simplify \( \frac{12^n}{2^n} \) by writing each base as a product of its prime factors.

4. Write an equation for the area of this rectangle.

Preparng for the Closing

5. \( 6x^3 \) is not equivalent to \((6x)^3\). Say why.

6. \( 4^5 \) is equivalent to \(2^{10}\). Use the properties of exponents to explain why.

7. \((2^x \cdot 2^y)\) is equivalent to \(2^{x+y}\) but \((2^x + 2^y)\) is not equivalent to \(2^{3y}\). Say why, giving examples to support your answer.

8. \( (2^x + 3^y)^z \) is not equivalent to \(2^{xz} + 3^{yz}\). Say why, giving examples to support your answer.

Skills

Plot these simple figures on a coordinate grid and list the vertices as ordered pairs.

a. Square
b. Parallelogram
c. Triangle
d. Pentagon
e. Rectangle
**Review and Consolidation**

1. Without using a calculator, evaluate these expressions.
   a. \((-3)^4\)  
   b. \((-3)^4\)

2. This is a True/False/Justify problem. Decide whether each equation is true or false and then write an explanation to justify your choice.
   a. \((ab)^2 = ab^2\)  
   b. \((a + b)^2 = a^2 + b^2\)  
   c. \(\left(\frac{a}{b}\right)^2 = \frac{(a)^2}{b}\)  
   d. \((-a)^2 = a^2\)
   e. \((-a)^3 = a^3\)  
   f. \(\left(\sqrt{a}\right)^2 = a\)  
   g. \((ab)^0 = 1\)  
   h. \(a(b)^0 = 1\)

3. Write the simplest possible exponential expression for the area of a triangle with perpendicular height \(3(xy)^5z\) and base \(8\left(\frac{xy^2}{z}\right)^3\).

**Homework**

1. Write each expression in exponential form, without parentheses (except part d, which needs one set of parentheses).
   a. \((4^2)^3\)  
   b. \((8p^3)^0\)  
   c. \(\left(\frac{2^4}{5^2}\right)^4\)  
   d. \([-7]^3\)  
   e. \((3c^5)^3\)

2. Write each expression in exponential form, without parentheses.
   a. \((a^5b^3c^4d^4)^4\)  
   b. \((ab^4)^2 a^3\)  
   c. \((c^2d^5)(c^3d)^4\)  
   d. \(\frac{(w^7 y^8)^3}{(2^3w)^2}\)
To learn about negative exponents.

Negative Exponents

Think about the following simplification.

\[ \frac{2^3}{2^7} = \frac{2 \cdot 2 \cdot 2}{2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2} \]

\[ = \frac{1}{2 \cdot 2 \cdot 2 \cdot 2} \quad \text{(by cancellation of factors)} \]

\[ = \frac{1}{2^4} \]

Alternatively, you can simplify the quotient by subtracting exponents.

Example

\[ 2^3 \div 2^7 = 2^{3-7} = 2^{-4} \]

The results of both methods of simplification must be equivalent.

Therefore, you know that \( 2^{-4} = \frac{1}{2^4} \).

Also,

\[ \frac{1}{2^4} = 1 \div \frac{1}{2^4} = 2^4, \text{ so it is also true that } 2^4 = \frac{1}{2^{-4}}. \]

In general, \( a^{-m} = \frac{1}{a^m} \) and \( a^m = \frac{1}{a^{-m}}. \)

You can see that the sign of the exponent changes when the term “moves” from the denominator to the numerator.
Operations with Negative Exponents

The rules for operating with exponents apply in exactly the same way to negative exponents as they do to positive exponents and the zero exponent.

Scientific Notation

It is often useful to express numbers in scientific notation, especially if they are very large or very small.

To do this, you need to go from left to right in the number to the place where the first digit that is not zero occurs. Then you write that digit and the digits that follow as a number between 1 and 10. To make the value correct, you simply multiply or divide by the appropriate power of 10.

**Example**

\[
0.024 = \frac{2.4}{100} = \frac{2.4}{10^2} = 2.4 \cdot \frac{1}{10^2} = 2.4 \cdot 10^{-2}
\]

**Example**

\[
12,560,800 = 1.2560800 \cdot 10,000,000 = 1.2560800 \cdot 10^7
\]

**Hint:** Notice that you can save time by just counting the number of powers of 10 that you need.

Some multiplication and division problems are easier if you first express the numbers in scientific notation. Then you can use the properties of exponents to complete the simplification.

**Example**

\[
1300 \cdot 0.00094 = (1.3 \cdot 10^3)(9.4 \cdot 10^{-4})
= (1.3 \cdot 9.4)(10^3 \cdot 10^{-4})
= (12.22) \cdot 10^{-1}
= 12.22 \cdot \frac{1}{10^1} = 1.222
\]

by the associative property of multiplication
Negative Bases

It is very important to understand the difference between negative bases and negative exponents.

**Example**

Compare the values of \((-4)^5\) and \(4^{-5}\).

\[(–4)^5 = (–4)(–4)(–4)(–4)(–4) = -1024\]

\[4^{-5} = \frac{1}{4^5} = \frac{1}{1024}\]

**Work Time**

1. Use the properties of negative exponents to write each number as a fraction with two integers.
   
   a. \(5^{-2}\)  
   b. \((-10)^{-3}\)  
   c. \(4^{-1} + 3^{-2}\)  
   d. \(\frac{3^{-1}}{7^{-2}}\)  
   e. \((0.5 + 1.5)^{-3}\)  
   f. \((8^{-2} \cdot 5^3)(8^{-4} \cdot 5^{-6})\)

2. Write each expression with positive exponents.

   a. \(x^{-7}\)  
   b. \(j^0 \div j^{-11}\)  
   c. \(\left(\frac{p}{q}\right)^{-4}\)  
   d. \(\frac{4d^{-3}}{f}\)

3. Write \(\frac{1}{8}\) as a power of 2.

4. Complete these calculations by first writing the numbers in scientific notation. Do not use a calculator.
   
   a. \(3,400,000,000 \cdot 0.00000045\)  
   b. \(0.0000036 \div 0.0000000009\)
Preparing for the Closing

5. Use the properties of exponents to simplify the expression \( a^m \cdot a^{-m} \). Try to do this simplification using at least two different methods.

Say why the result makes sense.

6. Which of these expressions can be simplified using the properties of exponents? Give reasons for your answers.

A \( x^{-y} \cdot x^{3y} \)  
B \( x^{-y} + x^{3y} \)  
C \( x^{-y} \div x^{3y} \)  
D \( (x^{-y})^{3y} \)

Skills

Solve.

a. \(1359 - 5 = \)  
b. \(1359 - 10 = \)  
c. \(1359 - 15 = \)  
d. \(1359 - 20 = \)  
e. \(1359 - 25 = \)

Do you see a pattern? Can you solve \(1359 - 30\) by following the pattern?

Review and Consolidation

1. Which of these expressions are equal to \( \frac{1}{3} \)?

a. \(1^{-3}\)  
b. \(3^{-1}\)  
c. \((-3)^1\)

2. Express each of these expressions using positive exponents.

a. \(8^{-2}\)  
b. \(p^{-1}\)  
c. \(2(m)^{-4}\)  
d. \(2m^{-4}\)  
e. \(\frac{1}{x^{-3}}\)

3. Express each of these expressions using negative exponents.

a. \(\frac{1}{7^3}\)  
b. \(5^2\)  
c. \(\frac{0.6}{p}\)  
d. \((-4)^5\)  
e. \(\frac{m^5}{n^4}\)
1. Use the properties of negative exponents to write each of these numbers as a fraction with two integers.
   a. $8^{-2}$
   b. $(-5)^{-3}$
   c. $8^{-1} - 2^{-2}$
   d. $\frac{(6-7)^{-1}}{7^2}$
   e. $(0.1 + 0.4)^{-3}$

2. Write each expression with positive exponents.
   a. $6n^{-7}$
   b. $\left(\frac{m}{n}\right)^{-3}$
   c. $(ab^{-11}c^{33})^2$

3. Without using a calculator, complete this calculation by first writing the numbers in scientific notation.
   \[0.0000000125 \div 0.0000000005\]

4. Use the properties of exponents to write at least three different equations that could give the result of $x^2y^{-5}$. 

Think about each of these simplifications.

**Example**

\[
\sqrt{25} \cdot \sqrt{25} = 25
\]

\[
25^{\frac{1}{2}} \cdot 25^{\frac{1}{2}} = 25^{\frac{1}{2} + \frac{1}{2}} = 25^1 = 25
\]

You can see that \(\sqrt{25} = 25^\frac{1}{2}\).

The square root symbol \(\sqrt{}\) represents the same operation as the exponent \(\frac{1}{2}\).

Now think about these results.

**Example**

\[
\sqrt[3]{125} \cdot \sqrt[3]{125} \cdot \sqrt[3]{125} = 125
\]

\[
125^\frac{1}{3} \cdot 125^\frac{1}{3} \cdot 125^\frac{1}{3} = 125^{\frac{1}{3} + \frac{1}{3} + \frac{1}{3}} = 125^1 = 125
\]

You can see that \(\sqrt[3]{125} = 125^\frac{1}{3}\).

The cube root symbol \(\sqrt[3]{}\) represents the same operation as the exponent \(\frac{1}{3}\).

These examples show that \(a^\frac{1}{m} = \sqrt[m]{a}\), where \(a \neq 0\).

Notice that the expression \(a^{-m}\) is equivalent to \(1 \div a^m = \frac{1}{\sqrt[m]{a}}\).
Fractional and decimal exponents have all the properties of the integer exponents that you have learned in the unit so far.

**Example**

\[ (\sqrt{25})^3 = \left( 25^{\frac{1}{2}} \right)^3 = 25^{\frac{3}{2}}; \quad 0.9^{\frac{2}{5}} \cdot 0.9^{\frac{-2}{5}} = 0.9^{\left( \frac{2}{5} + \frac{-2}{5} \right)} = 0.9^0 = 1; \quad m^{\frac{1}{2}} \cdot m^{\frac{1}{2}} = m^{\left( \frac{1}{2} + \frac{1}{2} \right)} = m^1 = \sqrt{m} \]

**Work Time**

1. **a.** Write each of these numbers as a square root.
   
   \[
   \begin{align*}
   \frac{1}{1^2} & \quad \frac{1}{4^2} & \quad \frac{1}{9^2} & \quad \frac{1}{16^2} \\
   \end{align*}
   \]
   
   **b.** What is significant about these numbers?

   **c.** Use your calculator to investigate the effect of raising the numbers to the power of one-half.

   **Comment**

   If you are unsure how to do this, your teacher will show you how to use your particular calculator.

   **d.** Continue the pattern in part a by writing the next two numbers that have an exponent of one-half.

   **e.** Explain how you know you have written the correct numbers.

2. **a.** Write each of these numbers as a cube root.
   
   \[
   \begin{align*}
   \frac{1}{1^3} & \quad \frac{1}{8^3} & \quad \frac{1}{27^3} & \quad \frac{1}{64^3} \\
   \end{align*}
   \]

   **b.** What is significant about these numbers?

   **c.** Use your calculator to investigate the effect of raising numbers to the power of one-third. What is significant about these numbers?

   **d.** Continue the pattern in part a by writing the next two numbers that have an exponent of one-third.

   **e.** Explain how you know you have written the correct numbers.
3. Without using your calculator, evaluate each expression. (Be sure to show all the steps you use.)
   
   a. \(1000^{\frac{1}{3}}\)  
   b. \((0.01)^{\frac{1}{2}}\)  
   c. \(144^{0.5}\)  
   d. \(32^{-\frac{1}{5}}\)

4. Use the method of trial and improvement to solve the equation \(2^x = 3\).
   You will need to use a calculator to try each of the powers that you choose.
   
   • Begin by choosing two integer values between which \(x\) must lie.
   • Improve your answer by trying two values with one decimal place that you think are on either side of \(x\).
   • Improve your answer by trying two values with two decimal places that you think are on either side of \(x\).
   • Improve your answer by trying two values with three decimal places that you think are on either side of \(x\).
   • Use your last try to write your answer for \(x\) to two decimal places.

Preparing for the Closing

5. Compare the expressions \(\sqrt[n]{a^m}\) and \(\left(\frac{1}{a}\right)^m\) by writing each in simplest exponential form.
   What can you conclude from these results?

6. Lisa and Jamal were discussing whether \(-\sqrt{2}\) is equivalent to \(2^{-\frac{1}{2}}\).
   Jamal said he thought the expressions were equivalent, but Lisa disagreed.
   Which student do you agree with? Show the math that justifies your choice.

7. Keesha and Dwayne were trying to decide whether the value of \(10^{\sqrt{1}}\) is the same as \(10^{0.1}\).
   Keesha said the expressions are equivalent, but Dwayne disagreed.
   Which student do you agree with? Show the math that justifies your choice.
Skills

Solve.

a. \( 314 \times 4 = \)  
   b. \( 314 \times 5 = \)  
   c. \( 314 \times 6 = \)  
   d. \( 314 \times 7 = \)  
   e. \( 314 \times 8 = \)

Do you see a pattern? What could you add to \( 314 \times 8 \) to get \( 314 \times 9 \)?

What could you subtract from \( 314 \times 10 = 3140 \) to get \( 314 \times 9 \)?

Review and Consolidation

1. a. What is the side length of a square of area 64 square units?

   b. Copy and complete these equations that represent this situation.

   \[
   \begin{align*}
   8 \square & = 64 \\
   \sqrt{64} & = \square
   \end{align*}
   \]

   c. Say why \( \sqrt{64} \) is rational.

2. In math class, Rosa was asked to write \( (\sqrt{m})^2 \) in its simplest form.

   Rosa said that the problem was impossible because you do not know what the number \( m \) stands for.

   Jamal said the answer is \( m \).

   Which student do you agree with? Justify your choice with reasons and diagrams.

3. Dwayne and Lisa were discussing squares and square roots of negative numbers.

   Dwayne said it is possible to calculate the square root of a negative number but it is not possible to calculate the square of a negative number.

   Lisa disagreed and said it was the other way around.

   Which student do you agree with? Justify your choice with reasons and examples.

4. Decide whether each of the following is true or false. Write a brief justification for each choice.

   a. \( \sqrt[3]{1} = \sqrt{1} \)  
   b. \( 15^3 = 45 \)

   c. The length of the sides of a cube with volume 0.000008 m\(^3\) is 0.02 m.

   d. \( \sqrt[3]{900} = -30 \)
5. Rosa thinks that the cube root of a positive number (other than 1) will always be less than the positive square root of that same number. Do you agree? Say why or why not. Give numerical examples to support your choice.

6. Jamal and Lisa were discussing square roots and cube roots of negative numbers.

   Jamal said it is possible to calculate the cube root of a negative number but it is not possible to calculate the square root of a negative number.

   Lisa disagreed and said it was the other way around.

   Which student do you agree with? Justify your choice with reasons and examples.

---

**Homework**

1. Use your calculator to evaluate each expression to three decimal places.
   
   a. \( \sqrt{5} \)  
   b. \( \sqrt[3]{20} \)

2. Simplify without using a calculator. (Be sure to show all the steps you use.)
   
   a. \( 8^{\frac{1}{3}} \)  
   b. \( (0.04)^{\frac{1}{2}} \)  
   c. \( 8100^{0.5} \)  
   d. \( (81)^{-\frac{1}{4}} \)

3. Simplify these square roots without using a calculator.
   
   a. \( \sqrt{100} \)  
   b. \( \sqrt{0.09} \)

4. Estimate these square roots (to one decimal place). Calculate the squares of your answers to check your estimates.
   
   a. \( \sqrt{20} \)  
   b. \( \sqrt{0.4} \)

5. Simplify.
   
   a. \( \sqrt[3]{2744} \)  
   b. \( 14^{3} \)

6. Compare and contrast your answers for parts a and b of problem 5.
LESSON 12
ESTIMATING SQUARE ROOTS

GOAL
To use the “guess and check” strategy to estimate square roots.

CONCEPT BOOK
See pages 339–341 in your Concept Book.

Expressions, Equations, and Exponents

A square root is one of two equal factors that have a certain product.

Example
Five is the square root of 25 because 5 multiplied by itself equals 25.

\[ 5 \times 5 = 25 \quad \sqrt{25} = 5 \]

Note
The symbol \( \sqrt{\text{} \} \) stands for “the square root of.”

\( \sqrt{25} \) means the square root of 25.

Squares and square roots are inverses—just as multiplication and division are inverses.

Some square roots are rational numbers and some are not rational.

Example
Five is a rational number, so \( \sqrt{25} \) is rational.

\( \sqrt{3} \) and \( \sqrt{7} \) are irrational. This means that they cannot be written as terminating or repeating decimals.

Here is the formula for calculating the area of a square:

Area equals the length of a side squared, \( A = x^2 \), where \( x \) is the length of a side.

\( x^2 = x \times x \)  \( x \) squared equals \( x \) multiplied by \( x \). So, \( 5^2 = 5 \times 5 = 25 \).

Example
If the area of a square is 2 cm\(^2\), what is the length of a side?

Each side must be \( \sqrt{2} \) cm.

You know this by asking, “What number multiplied by itself equals 2?”

Based on the definition of square root, it must be the square root of 2.
You can estimate an irrational square root by finding a terminating decimal that is close to the square root. An estimate is approximately equal to the square root; it is not exactly equal to the square root.

**Example**

To find $\sqrt{2}$, try numbers until you get very close to a product of 2.

- $1 \times 1 = 1$ too small
- $2 \times 2 = 4$ too large
- $1.5 \times 1.5 = 2.25$ too large
- $1.4 \times 1.4 = 1.96$ too small
- $1.41 \times 1.41 = 1.9881$ too small
- $1.42 \times 1.42 = 2.0164$ too large

What value of $x$ would you try next?

### Work Time

1. The example above estimates the value of $\sqrt{2}$.
   - Calculate a better estimate to three decimal places. Use 1.415 and 1.414 for values of $x$.
     - a. Multiply $1.415 \times 1.415$.
     - b. How much greater is $(1.415 \times 1.415)$ than 2?
     - c. Multiply $1.414 \times 1.414$.
     - d. How much less is $(1.414 \times 1.414)$ than 2?
     - e. What is the best estimate of $\sqrt{2}$ to three decimal places?

2. a. Copy this table and find the square roots of these numbers.

<table>
<thead>
<tr>
<th>$x$</th>
<th>1</th>
<th>4</th>
<th>9</th>
<th>16</th>
<th>25</th>
<th>36</th>
<th>49</th>
<th>64</th>
<th>81</th>
<th>100</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sqrt{x}$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

   - b. Knowing the square roots from part a, estimate the square roots of these numbers.

<table>
<thead>
<tr>
<th>$x$</th>
<th>5</th>
<th>7</th>
<th>8</th>
<th>14</th>
<th>26</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sqrt{x}$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

3. Write the square of each of these numbers.
   - a. 0.5
   - b. 0.8
   - c. 0.07
   - d. 1.4
4. Write the square root of each of these numbers.
   a. 0.64  b. 0.25  c. 0.0049  d. 1.96

5. a. The equation \( x^2 = 36 \) is read as “\( x \) squared equals 36.” Solve the equation by finding a value of \( x \) that makes the equation true.
   
   b. \( x^2 = 25 \). Solve for \( x \).
   
   c. \( x^2 = 30 \). The solution to \( x^2 = 30 \) must be between two whole numbers. Which whole numbers are they? \[ \text{Hint: See problem 2a.} \]
   
   d. Find \( \sqrt{30} \) to two decimal places.

Preparing for the Closing

6. a. Say why the number 1.41 is a good estimate for the value of \( \sqrt{2} \).
   
   b. Say why the number 1.414 is a better estimate for the value of \( \sqrt{2} \) than 1.41.

7. Look again at problems 2, 3, and 4.
   
   a. Is the square of a number always greater than the number? Say why, using examples.
   
   b. Is the square root of a number always less than the number? Say why, using examples.

8. Look again at problem 5.
   
   a. In part a, is the solution of \( x^2 = 36 \) a whole number? A rational number? Give reasons and examples.
   
   b. In part b, is the solution of \( x^2 = 25 \) a whole number? A rational number? Give reasons and examples.
   
   c. In parts c and d, is the solution of \( x^2 = 30 \) a whole number? A rational number? Give reasons and examples. \[ \text{Hint: See problem 2a.} \]

9. Rosa claimed that \( x^2 \) and \( 2x \) are the same. She justified her claim with the example \( 2^2 = 4 \) and \( 2 \cdot 2 = 4 \).
   
   Do you agree with her? Say why or why not. Does her example justify her claim? Say why or why not.
Skills

Solve.

- a. 5 \cdot 8 \div 2 =
- b. 40 + 55 - 2 =
- c. (10 \div 2) \cdot 4 =
- d. (45 \div 9) + 8 =
- e. 5a \cdot 8 \div 2 =
- f. 40a + 55a - 2a =
- g. (10a \div 2) \cdot 4 =
- h. (45a \div 9) + 8 =

Review and Consolidation

1. Calculate a value for $x$ that makes each equation true.
   
   - a. $x^2 = 9$
   - b. $x^2 = 64$
   - c. $x^2 = 0.81$
   - d. $x^2 = 0.0025$

2. This square has an area of $45 \text{ m}^2$ and a side length of $x$.
   
   a. The side length is between which two whole numbers?
   
   **Hint:** See Work Time problem 2a.
   
   b. Estimate the side length to two decimal places.

3. This square has an area of $72 \text{ m}^2$ and a side length of $x$.
   
   a. The side length is between which two whole numbers?
   
   **Hint:** See Work Time problem 2a.
   
   b. Estimate the side length to two decimal places.

4. Ms. Reynolds challenged her students to calculate the dimensions of a cube.
   
   Dwayne remembered that the formula for the volume of a cube is $V = x^3$ or $V = x \cdot x \cdot x$.
   
   Find the side length of this cube.

Homework

1. Suppose $x^2 = 8$. You can say that “$x$ squared equals 8,” or “$x$ is the square root of 8.”
   
   a. Between which two consecutive whole numbers is $x$?
   
   **Hint:** See Work Time problem 2a.
   
   b. Which of the two numbers in part a is closer to $x$?
   
   c. Estimate $x$ to two decimal places.
To use the Pythagorean theorem for right triangles.

In a right triangle, the side opposite the right angle is called the **hypotenuse**. The two shorter sides of the right triangle are sometimes called the **legs**.

**Example**

Each of these triangles is a right triangle because it has one right angle. In each, the right angle is at vertex $B$.

In each of these triangles, the hypotenuse is segment $AC$ and the legs are segments $AB$ and $BC$.

In this right triangle, the two shorter sides measure 9 units and 12 units. The measure of the hypotenuse is 15 units.
In this diagram, a square has been constructed on each side of the right triangle.

The square on the shortest side has an area of $9 \times 9 = 9^2 = 81$ square units.
The square on the second shortest side has an area of $12 \times 12 = 12^2 = 144$ square units.
The square on the hypotenuse has an area of $15 \times 15 = 15^2 = 225$ square units.

If you add $81 + 144$ you get 225—the same as the area of the square of the hypotenuse.

Pythagoras (a mathematician in ancient Greece) discovered that in all right triangles when you add the squares of the two shorter sides you get a value equal to the square of the hypotenuse.

This property is called the Pythagorean theorem.

In any right triangle, the square of the measure of the hypotenuse is equal to the sum of the squares of the measures of the two shorter sides.

If you use the letters $a$ and $b$ to stand for the measures of the two shorter sides, and the letter $c$ to stand for the measure of the hypotenuse,

then the theorem can be represented as the formula: $c^2 = a^2 + b^2$

You can use this theorem to calculate the length of unknown sides in right triangles.
1. Triangle 1 is a right triangle, but triangles 2 and 3 are not. Say why.

![Triangle Diagram]

2. Verify the Pythagorean theorem for each of the right triangles B through E on the next page. (The calculation for triangle A from the lesson introduction has already been done for you.)

To show the calculations, copy and complete a table like this one.

<table>
<thead>
<tr>
<th>Triangle</th>
<th>Sum of the Squares of the Two Legs</th>
<th>Square of the Hypotenuse</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>$9^2 + 12^2 = 81 + 144 = 225$</td>
<td>$15^2 = 225$</td>
</tr>
<tr>
<td>B</td>
<td></td>
<td></td>
</tr>
<tr>
<td>C</td>
<td></td>
<td></td>
</tr>
<tr>
<td>D</td>
<td></td>
<td></td>
</tr>
<tr>
<td>E</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
3. Verify whether these sets of numbers represent the side lengths of right triangles.

**Hint:** To do this, you need to calculate the squares to see if they satisfy the Pythagorean theorem.

a. \{ 3, 5, 7 \}   
b. \{ 5, 12, 13 \}   
c. \{ 7, 9, 11 \}   
d. \{ 6, 8, 10 \}

4. You should have found that the Pythagorean theorem works for two of the sets of numbers in problem 3. Use this information to sketch and label two corresponding right triangles. Label the sides with their measures.

5. Use the Pythagorean theorem to calculate the length of the hypotenuse in each of the right triangles. Where necessary, round your answer to one decimal place. (The first calculation has been completed as an example.)

<table>
<thead>
<tr>
<th>Right Triangle</th>
<th>(a^2 + b^2)</th>
<th>(c^2)</th>
<th>Hypotenuse</th>
</tr>
</thead>
<tbody>
<tr>
<td>a. 15(\times)8</td>
<td>8(^2) + 15(^2) = 289</td>
<td>289</td>
<td>(c = \sqrt{289} = 17)</td>
</tr>
<tr>
<td>b. 24(\times)10</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>c. 11(\times)10</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>d. 15(\times)20</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>e. 8(\times)12</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Hint:** To find square roots, you can use the \(\sqrt{\text{}}\) button on your calculator.
6. Write in words how you can find the length of the hypotenuse of a right triangle if you know the lengths of the two legs.

7. Chen and Lisa were given this problem to solve in math.

![Diagram of a right triangle with sides labeled a, 17 cm, and 20 cm.]

Chen said that the problem is impossible to solve because the Pythagorean theorem is used for finding the hypotenuse.

Lisa said she thought the formula could be used somehow, but she could not quite figure it out.

Can you help? Discuss this problem with your partner and write your method in your notebook.

8. You have learned that the relationship $c^2 = a^2 + b^2$ (where $c$ is the longest side) always holds true for right triangles.

In obtuse triangles, $c^2 > a^2 + b^2$, and in acute triangles, $c^2 < a^2 + b^2$.

Which set of numbers in problem 3 represents the side measures of an obtuse triangle? Which set represents an acute triangle?

Skills

Calculate.

a. $763 + 57 =$

b. $286 + 39 =$

c. $802 + 99 =$

d. $178 + 195 =$

e. $361 + 279 =$

f. $1028 + 234 =$

g. $4190 + 649 =$

h. $2409 + 1235 =$

Hint: Rewrite each problem as an easier problem that you can calculate in your head. For example, $537 + 218$ can be written as $(500 + 200) + (30 + 10) + (7 + 8)$. 
1. This is a Card Sort task. Your teacher will give you a set of fifteen cards, similar to these.
   a. Your first task is to sort the cards into three sets of five cards. Each set will have:
      • A diagram of a right triangle
      • A statement about the right angle
      • A statement about the hypotenuse
      • A calculation to determine the length of segment $AC$
      • A calculation to determine the length of segment $AB$
   b. After you have completed the card sort, prepare an explanation of your work to present to another student or the whole class.

Homework

1. Here are the squares of some numbers.
   
   $45^2 = 2025$  $19^2 = 361$  $26^2 = 676$  $18^2 = 324$  $27^2 = 729$

   Use them to find the square roots of these numbers.
   a. $\sqrt{361} = \text{ }$
   b. $\sqrt{676} = \text{ }$
   c. $\sqrt{729} = \text{ }$
   d. $\sqrt{2025} = \text{ }$
   e. $\sqrt{324} = \text{ }$

2. Use a calculator to help you check whether these sets of integers satisfy the Pythagorean theorem. For any that do, sketch and label a corresponding right triangle.
   a. $\{2, 3, 4\}$  
   b. $\{15, 20, 25\}$  
   c. $\{10, 24, 26\}$  
   d. $\{5, 10, 15\}$

3. Find the length of the hypotenuse of these right triangles. Where necessary, round the answer to one decimal place.
   a.
   
   b.
   
   c.
   
   d.
Work Time

Lisa, Dwayne, and Rosa were each given a problem to solve in math class. Unfortunately, they all made some mistakes in their solutions. For Work Time, you need to figure out where each student went wrong and explain why the student is wrong. Then complete each problem correctly.

1. A boy scout wants to row from one side of the river to his campsite directly opposite. The width of the river is 30.5 meters. However, there is a strong current flowing in the river, so the scout actually rows along the route shown in the diagram. He reaches the other side of the river at a point 52 m away from his campsite.

How far did the boy scout row? (Express your answer correct to the nearest meter.)

Here is Lisa’s solution:

\[
\text{The boy scout rowed} = 30.5 + 52 = 82.5 \text{ m}
\]

a. What mistake did Lisa make?

b. What is the correct answer? Show your work.
2. Mrs. Valdez wants to build a wire fence around the outside of her rectangular field. The diagonal of the field measures 45 yards; the width of the field measures 66 feet. What amount of wire fencing (to the nearest foot) does Mrs. Valdez need to buy?

Here is Dwayne’s solution:

First you need to calculate the length of the field.

\[
\text{length} = \sqrt{\text{diagonal} + \text{width}}
\]

\[
= \sqrt{66 + 45} = \sqrt{111} = 10.53 \text{ or about 10 feet}
\]

Then you need to add all four of the side measures.

Total amount needed = 10 + 10 + 66 + 66 = 152 feet

a. What mistake did Dwayne make?

b. What is the correct answer? Show your work.

3. Farmer Brown has a grain bin in the shape of a cone on top of a cylinder.

Calculate the perpendicular height, \(h\), of the bin if the radius of the cylinder is 12 feet, the slant height of the cone is 8 feet, and the height of the cylinder is 1.25 times the height of the cone.

Here is Rosa’s solution:

\[
\begin{align*}
\text{In a cone, } h &= \sqrt{r^2 - s^2} \\
&= \sqrt{12^2 - 8^2} \\
&= \sqrt{144 - 64} = \sqrt{80} = 8.94 \text{ feet}
\end{align*}
\]

a. What mistake did Rosa make?

b. What is the correct answer? Show your work.
Preparing for the Closing

4. Compare your solutions for problems 1–3 with those of another pair of students. Together, write some advice for Dwayne, Rosa, and Lisa to help them avoid making similar mistakes in the future. Also, write about other mistakes all students need to avoid when solving problems using the Pythagorean theorem.

5. The length and width of a rectangle are \( p \) units and \( q \) units, respectively. Show how the Pythagorean theorem can be used to write an expression for the length of the diagonal in terms of \( p \) and \( q \).

6. The length of the diagonal of a square is \( \sqrt{2} \) times the length of its side.
   Use the Pythagorean theorem to help you explain why this statement is always true.

Skills

Solve.

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>a. ( 1930 + 70 - 355 = )</td>
<td>b. ( 1050 + 950 - 1765 = )</td>
<td>c. ( 2467 + 133 + ) = 3000</td>
</tr>
<tr>
<td>d. ( 586 + 42 - 372 = )</td>
<td>e. ( 429 - 267 + 216 = )</td>
<td>f. ( 2745 + 855 + ) = 4600</td>
</tr>
</tbody>
</table>

Review and Consolidation

Consider each statement below and ask yourself the question, “Is this statement True or False?” Make a decision, and then provide an explanation for your choice using diagrams, algebra, and words.

1. The two legs of a right triangle measure 7” and 8”. The hypotenuse measures 15”.

2. The hypotenuse of a right triangle measures 15” and one of the legs of the triangle measures 12”. The length of the third side cannot be calculated.

3. The width of a rectangular gate is 5 m and its height is 1.5 m. The longest diagonal strut across the gate measures 5 m 22 cm.

4. The length of the slant edge of a cone is known, as well as the diameter of the base of the cone. The perpendicular height of the cone can be calculated using this information.

5. If three numbers \( a \), \( b \), and \( c \) fit the Pythagorean theorem \( c^2 = a^2 + b^2 \) then they represent the side measures of a right triangle.

6. If three numbers \( a \), \( b \), and \( c \) (where \( c > a \) and \( c > b \)) fit the inequality \( c^2 < a^2 + b^2 \) then they represent the side measures of an obtuse triangle.
1. Calculate the length of the diagonal of a rectangular gate with a length of 1 m and a height of 2 m. (Express your answer to the nearest centimeter.)

2. Chen was asked to complete the following problem, but unfortunately he made some mistakes. Figure out where Chen went wrong, and explain what he did wrong. Then complete the problem correctly.

   "Calculate the side length of a square where the length of the diagonal is 5 ft."

   Chen began his solution by sketching a diagram like this:

   Then he wrote:
   \[ x + x = 5 \]
   Therefore, \( 2x = 5 \)
   \[ x = 2.5 \]

   The side length of the square is 2.5 feet.
**PROGRESS CHECK**

**CONCEPT BOOK**


**GOAL**

To review using algebraic expressions to represent real world quantities, evaluating expressions, combining and expressing quantities with different units, representing quantities with area models, simplifying expressions with exponents, and using the Pythagorean theorem.

---

**Work Time**

1. For each situation, write an expression that represents the quantity described. Use conventions to write your expressions correctly.
   
   a. Each box in a stack of boxes is 2 inches high.
      
      Quantity: The height of a stack of \(n\) boxes
   
   b. A class has 25 students, and \(a\) students are absent today.
      
      Quantity: The number of students in the class today
   
      
      Quantity: The change you receive when you buy 3 books

2. Write two equivalent expressions for the quantity represented by this area model. State which number property or properties tell you your expressions are equivalent.

   ![Area Model]

3. Simplify the expression \(6(a + 2b) - 2(a - 6b)\). Use the commutative property of addition and the distributive property to combine like terms.
4. Rosa is making a cutting board for her dad using strips of different kinds of wood. Some of the wood strips are cut to 8 millimeters thick, and the other strips are cut to 2 centimeters thick.

   a. If she uses 14 of the 8 mm strips and 9 of the 2 cm strips, what is the total width of the cutting board in millimeters? What is the total width in centimeters?

   b. Write two different variable expressions for the total width of the cutting board if Rosa uses $x$ of the 8 mm strips and $y$ of the 2 cm strips; one expression in millimeters, the other in centimeters.

   c. Check your answer to part a by evaluating your expressions in part b for the given values in part a.

5. Simplify.

   a. $\frac{x^4 y^2}{x^3 y^3}$

   b. $(3a^2)^3$

   c. $4pq^{-1} \cdot p^{-2}$

   d. $216^{\frac{1}{3}}$

6. Use the Pythagorean theorem to find the missing lengths in these right triangles.

   a. 

   b. 

7. The length of the diagonal of a square is always $\sqrt{2}$ times the length of its side.

   Sketch a diagram and use the Pythagorean theorem to show that this is true for a square with sides of length 5 inches. Round your calculations to the nearest hundredth of an inch.

Preparing for the Closing

8. Review your work in today’s lesson.

   a. Identify the problems that were most difficult and state what made them difficult. Check that your work on these problems is correct.

   b. Identify the problems that were easiest for you, and state what made them easy. Check that your work on these problems is correct.
Skills
Solve.

a. $36.50 + 40¢ =

d. $36.50 + $1.40 =

b. $36.50 + 50¢ =

e. $36.50 + $1.50 =

c. $36.50 – 60¢ =

f. $36.50 – $1.60 =

Review and Consolidation
1. Review your work in the unit so far. Identify three problems that are similar to today’s lesson that were most difficult for you.

2. Use your Concept Book and previous lessons to work through these three problems more carefully with your partner until you understand them.

3. With your partner, write a summary page of the concepts you have learned in Lessons 1–14.

Homework
Assessing Your Work
Your task is to complete steps 1 through 3 twice.

The first time through you need to think about a concept (or group of concepts) that you understand best.

The second time through you need to think about a concept (or group of concepts) that you find challenging.

1. Select the concept(s).

2. Use these questions to help you write a brief explanation of why you chose this concept or group of concepts.
   - What new understanding do I have?
   - What did I do well that helped me learn this concept?
   - What more do I need to know or do?

3. Choose a problem that best demonstrates your understanding of this concept.
   - Show that you used the concept(s) accurately to solve the problem.
   - Represent the concept(s) in multiple ways (using numbers, symbols, or diagrams).
   - Write an explanation of your solution and the concept that any of your classmates could read and understand.
1. Jamal earned $672 from his summer job as an apprentice bricklayer.

   His grandfather told him that he could make his money (or “principal”) increase in value if he saved it at the bank in an account that earns **compound interest**.

   Jamal found out that such interest is called compound interest because the interest earned each year is added to the principal. This means that every year, interest is earned on the total of principal and interest.

   Jamal chose a compound interest account that earns 4% per year.

   This table shows the calculations for the first two years.

<table>
<thead>
<tr>
<th>Year</th>
<th>Interest ($)</th>
<th>Total Principal and Interest ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4% of 672 = 672 • 0.04 = 26.88</td>
<td>672 + 26.88 = 698.88</td>
</tr>
<tr>
<td>2</td>
<td>4% of 698.88 = 698.88 • 0.04 = 27.96</td>
<td>698.88 + 27.96 = 726.84</td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

   **a.** Calculate how much money Jamal will earn in the third year.

   **b.** Jamal’s grandfather showed him how to calculate the total amount, \( A \), that he could earn, using the compound interest formula.

   \[
   A = P(1 + r)^n \text{ where } n \text{ is the number of years,} \\
   P \text{ is the principal invested, and} \\
   r \text{ is the interest rate for a given time period (as a decimal)}
   \]
Use the compound interest formula on the previous page for Jamal’s investment of $672 for 1, 2, and 3 years at 4% compound interest. Be sure to write all the steps in your substitutions and calculations.

c. Compare your answers for parts a and b.

d. Jamal needs $1070 to buy a stereo sound system.

Use the method of trial and improvement and the compound interest formula to estimate how long it will be before he has enough money. Give your answer to the nearest month.

2. A population of coyotes changes according to the formula \( p = 500 \cdot 2^{-t} \), where \( p \) represents the number of animals in the population at the end of \( t \) years.

a. What size was the coyote population at the start?

b. Make a table of values for \( t = 0, 1, 2, \) and 3 years.

c. Your teacher will give you Handout 2: *Coyote Population*. Use the ordered pairs to plot a graph of \( p \) against \( t \).

d. Describe the shape of the graph.

e. Use the properties of exponents to explain how \( 2^{-t} \) determines that the population is one half of the size it was the previous year.

f. Use your graph to estimate the number of coyotes after 1.5 years.

g. Use your graph to find the time (in years to one decimal place) for the population to decrease to 100 coyotes.

Preparation for the Closing ————————————————————

3. Compare and contrast the two formulas you used in problems 1 and 2.

a. How are they similar? How are they different?

b. What were the different effects of the positive and negative exponents on the values you obtained?

4. Compare your solutions to the Work Time problems with another student. Try to reach an agreement.
Skills

Which are factors of the first number?

a. 72 : 22, 24, 18, 4  
   b. 115 : 23, 17, 29, 2  
   c. 129 : 26, 28, 43, 3

Which are multiples of the first number?

d. 24 : 24, 28, 36, 48  
   e. 15 : 30, 39, 83, 150  
   f. 37 : 41, 70, 74, 185

Review and Consolidation

1. This is a card sorting problem.

Your teacher will give you a set of 20 cards, made up of 16 cards with information and 4 blank cards.

Your task is to sort the cards into sets of five cards. For each set you need to use the blank card to make a card that shows one “in-between” step in the simplification.

Each set of five cards is made up of:

- A card showing an unsimplified expression
- A card showing a simplified expression, with positive exponents
- A card showing the numerical value of the card using \( p = -3 \) and \( q = 5 \)
- A card showing the expression written in words
- A blank card

Homework

1. For \( x = 2^3 \), \( y = 2^{-3} \), and \( z = 3^{-1} \), write each expression in simplest form with positive exponents.

   a. \( xy \)  
   b. \( \frac{x}{y} \)  
   c. \( x^2 \)  
   d. \( \frac{1}{\sqrt[10]{z}} \)

   e. \((x + y)^2 \)  
   f. \( xyz \)  
   g. \( \frac{1}{z} \)  
   h. \( \left(\frac{y^3}{x}\right)^{-4} \)
Each term in the expression $20m + 30n$ is a product of a coefficient and a variable.

The variable of the first term is $m$, and the coefficient of the first term is 20.

What is the variable and what is the coefficient of the second term?

It is easy to see that there are two terms in this expression. Not all expressions show the terms as clearly.

Think about these expressions.

$$3x - 6y$$

$$3(x + 5)$$

$$\frac{x - 2}{3}$$

In order to identify the terms and coefficients of an expression, you may need to rewrite the expression by combining like terms and using the number properties.

Because subtraction can also be expressed by adding the inverse, an expression that uses subtraction can always be converted to an equivalent expression that does not use subtraction.

**Example**

You can rewrite $3x - 6y$.

$$3x - 6y = 3x + (-6y)$$

So, $3x - 6y$ has two terms: $3x$ and $-6y$.

The coefficient of $x$ is 3, and the coefficient of $y$ is $-6$. 
Because the distributive property can eliminate parentheses, an expression with parentheses can often be converted to an equivalent expression without parentheses.

Example
You can rewrite 3(x + 5).

3(x + 5) = 3x + 15
So, 3(x + 5) has two terms: 3x and 15.
The coefficient of x is 3.

Sometimes you need more than one number property to rewrite an expression.

Example
You can use the distributive property and the inverse property of multiplication to rewrite \( \frac{x - 2}{3} \).

Dividing by 3 is equivalent to multiplying by the multiplicative inverse of 3:

\[
\frac{x - 2}{3} = \frac{1}{3}(x - 2) = \left(\frac{1}{3}\right)x - \left(\frac{1}{3}\right)2 = \frac{x}{3} - \frac{2}{3}
\]

So, \( \frac{x - 2}{3} \) has two terms: \( \frac{x}{3} \) and \( -\frac{2}{3} \).
The coefficient of x is \( \frac{1}{3} \).

Work Time

For each of these expressions:
• Write an equivalent expression that shows the terms and coefficients more clearly.
• Show how you wrote the equivalent expressions.
• Be prepared to explain why you were able to rewrite each expression, including which number properties you used.

1. \( 3(x - 4y) \)
2. \( 5m(2m - 3) \)
3. \( \frac{x + 1}{4} \)
4. \( \frac{2a + 5b}{10} \)
5. \( \frac{x^2 + 2x}{x} \)
6. \( 12\left(\frac{1}{2}p + \frac{1}{4}\right) \)
7. \( 3a\left(a^2 - \left(\frac{2}{3}\right)a + \frac{1}{3}\right) \)
8. \( \frac{6x^2 - 3x}{3x} \times 9 \)
9. \( \frac{2x^3 + 5x^2}{7x} \times 14 \)
Preparing for the Closing

10. Explain how you rewrote the expressions in problems 8 and 9.

11. What advice would you give to a student who was trying to simplify expressions like the ones in problems 8 and 9?

Skills

Solve.

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>$\frac{5}{8} + \frac{2}{3}$</td>
<td>b</td>
</tr>
<tr>
<td>c</td>
<td>$\frac{5}{8} \cdot \frac{2}{3}$</td>
<td>d</td>
</tr>
<tr>
<td>e</td>
<td>$2\frac{5}{8} + 1\frac{2}{3}$</td>
<td>f</td>
</tr>
<tr>
<td>g</td>
<td>$2\frac{5}{8} \cdot 1\frac{2}{3}$</td>
<td>h</td>
</tr>
</tbody>
</table>

Review and Consolidation

Use the distributive property to write the additive inverse of each of these expressions. Simplify the expressions first if necessary, and use brackets as shown in the second example if they help you.

Example

\[ a + 5 \]
Additive inverse is \(- (a + 5) = -a - 5\)
\[ -2(4a^3 + 6) \]
Additive inverse is \(-[-2(4a^3 + 6)] = -(8a^3 - 12) = 8a^3 + 12\)

1. \(x + 1\)
2. \(-x + 1\)
3. \(-(x + 1)\)
4. \(-(-x + 1)\)
5. \(x - 1\)
6. \(-(x - 1)\)
7. \(x - y\)
8. \(2x - 2y\)
9. \(-2x + 4y\)
10. \(x^5 - y^5\)
11. \(-2x^5 - y^5\)
12. \(2x(x - y)\)
13. \(-2x^4(x - 2y)\)
14. \(2xy(x^5 - y^5)\)
15. \(x + y + z\)
16. \(-x + y - z\)
17. \(\frac{x^2 + x}{x}\)
18. \(\frac{x^3 + x^2}{x^2}\)
1. For each of these expressions, apply the distributive property and other number properties to write and simplify an equivalent expression.
   a. \( \frac{2a + 5}{5} \)
   b. \( 10 \left( \frac{2a + 5}{5} \right) \)
   c. \( 2a \left( \frac{2a + 5}{5} \right) \)
   d. \( 2 \left( \frac{2a^2 + 5a}{5a} + 8 \right) \)
   e. \( 2a \left( \frac{2a + 5b + 15}{5a} \right) \)

2. For each expression you wrote in problem 1, identify the number of terms and identify the coefficient of each variable term.
In the diagram below, a small rectangle has been removed from a large rectangle.

To write an expression for the shaded area, you need to find the area of the large rectangle and then subtract the area of the small rectangle.

Here is how.

Area of large rectangle: $8a(3a - 2)$

Area of small rectangle: $3a(a + 1)$

Area of shaded region: $8a(3a - 2) - 3a(a + 1) = 24a^2 - 16a - 3a^2 - 3a = 21a^2 - 19a \text{ cm}^2$
1. In this diagram, part of the rectangle is shaded.

![Diagram of a rectangle with a shaded part]

a. Write an expression for the perimeter of the rectangle. Write your expression as simply as possible.

b. Write an expression for the area of the shaded region by subtracting the areas of the two triangles that are not shaded from the area of the entire rectangle. Be prepared to explain your calculations to a partner.

c. Is your expression for the shaded area a variable or a numerical expression?

2. In this diagram, a corner has been removed from a rectangle.

![Diagram of a rectangle with a corner removed]

a. Write an expression for the perimeter of this figure. Write your expression as simply as possible.

b. Write an expression for the area of this figure. Write your expression as simply as possible. Be prepared to explain your calculations to a partner.

c. Are your expressions for the perimeter and area variable or numerical expressions?
3. Look again at the figure from the beginning of the lesson in which a small rectangle was removed from a larger one. Suppose that you knew that \( a \) was equal to 4.

   a. What would be the dimensions of each rectangle?

   b. What would be the area of each rectangle?

   c. Calculate the area of the shaded region.

   d. Now, find the area of the shaded region by evaluating the expression from the beginning of the lesson, \( 21a^2 - 19a \), for \( a = 4 \).

4. Here again is the diagram from problem 2. Suppose that you knew that \( a \) was equal to 6.

   a. What would be the dimensions of the figure?

   b. What would be the perimeter of the figure?

   c. Calculate the area of the figure by adding or subtracting areas of rectangles using the dimensions you found for part a.

   d. Now, find the area of the shaded region by evaluating the expression you wrote in part b of problem 2 for \( a = 6 \).

Preparing for the Closing

5. Look at your answers for parts c and d of problems 3 and 4. The answers in parts c and d should be the same in each problem. Explain why the answers are the same.

6. The expressions for the areas and perimeters in today’s lesson were variable expressions. Explain what this means in the context of geometric measures.

7. Explain your calculations in part b of problems 1 and 2 to a partner or small group. Did you calculate your answers in the same way as the others? If not, are your calculations equivalent? If your calculations are different but equivalent, which number properties tell you they are equivalent?
The following graph gives the monthly rainfall (to the nearest cm) over a 12-month period in a certain region of the United States.

a. Which month had the greatest rainfall?
b. Which month had the least rainfall?
c. What was the difference in rainfall between the months of April and May?
d. Which of the six-month periods, January to June or July to December, was the driest?
e. Between which two months of the year did the greatest increase occur?

Review and Consolidation

1. Sketch area models to represent these expressions.

   a. $x(x + 2)$
   b. $5(x - 1)$
   c. $x(x + 2) - 5(x - 1)$

2. Compare your work in part c of problem 1 with a partner. Do your area models look the same? If they are different, check that each representation is accurate. If they are the same, work together to sketch an alternative area model representation for the expression.

3. Use the distributive property to write an expression equivalent to $5m + 5n + 10$, and then sketch a geometric figure with an area that can be represented by either expression.
4. Think about these two expressions.

\[ x^2 + 9 \quad \text{and} \quad (x + 3)^2 \]

a. Write verbal instructions for each of these expressions.

b. Sketch an area model to represent each expression.

c. Are these two expressions equivalent? Evaluate each expression for \( x = 0 \), \( x = 1 \), and \( x = 2 \), and explain how the results support your answer.

**Homework**

Use this figure for the Homework problems.

1. Write a variable expression for the shaded region of the figure.

2. If \( n = 8 \), what would be the area of the shaded region?

3. How did you find your answer to problem 2? Did you calculate the area by:
   - Subtracting the area of one rectangle from another?
   - Evaluating your expression from problem 1 for \( n = 8 \)?

4. If \( n = 12 \), what would be the area of the shaded region? Calculate your answer using a different method from the one you used in problem 2.

5. If \( n = 15.12 \), what would be the area of the shaded region? Calculate your answer using any method you choose.
An equation is a statement that two expressions are equal to each other. In other words, an equation states that two expressions represent the same quantity.

Every equation is composed of two expressions linked by an equals sign.

**Example**
In the equation $30 + 10 = 33 + 7$

$30 + 10$ and $33 + 7$ are both numerical expressions, with two terms each.

**Example**
In the equation $13 - 2x = 5$

$13 - 2x$ is a variable expression with two terms.

$5$ is a numerical expression with one term.

**Example**
In the equation $y = 2x + 1$

$y$ is a variable expression with one term.

$2x + 1$ is a variable expression with two terms.

An equation can be *always true*, *sometimes true*, or *never true*. A solution to an equation is a value (or set of values) that makes the equation true.

**Example**
The equation $30 + 10 = 33 + 7$ is *always true*, because the left side, $30 + 10$, is equal to 40, and the right side, $33 + 7$, is also equal to 40.

The expressions on either side of the equals sign are always equivalent in this case.
Example

The equation $13 - 2x = 5$ is sometimes true because it is true for some values of $x$ and false for other values of $x$.

There is only one value of $x$ that makes the equation true.

When $x = 4$, the equation is true, so 4 is the solution to this equation.

There are many values of $x$ that make the equation false.

When $x = 1$ the equation is false.

When $x = 2$ the equation is false; and so forth.

$13 - 2x = 5$ is a linear equation in one variable.
This equation has one solution: $x = 4$.

Example

The equation $y = 2x + 1$ is sometimes true because it is true for some values of $x$ and $y$, while false for other values of $x$ and $y$.

There are many values of $x$ and $y$ that make the equation true.

When $x = 1$ and $y = 3$, the equation is true.

When $x = 2$ and $y = 5$, the equation is true.

The pairs of values $(1, 3)$ and $(2, 5)$ are solutions to this equation.

There are many values of $x$ and $y$ that make the equation false.

When $x = 1$ and $y = 4$, the equation is false.

When $x = 2$ and $y = 8$, the equation is false; and so forth.

$y = 2x + 1$ is a linear equation in two variables.
This equation has many solutions: every value of $x$ gives a solution for $y$.

Example

$n = n + 1$ is never true because it is false for any value of $n$. It has no solution.
1. For each equation, confirm that the given solution or solutions make the equation true by substituting the value or values into the equation.

**Example**

\[ 3x + 4 = 10 \]
Solution: \( x = 2 \)
Solution: \( 3(2) + 4 = 10 \)
\[ 6 + 4 = 10 \quad \text{True} \]

a. \( 2x + 8 = 16 \)
Solution: \( x = 4 \)

b. \( 6(x + 4) - 9(x + 3) = 0 \)
Solution: \( x = -1 \)

c. \( 1.75 = 0.35 + x \)
Solution: \( x = 1.4 \)

d. \( 2x + 8 = y \)
Solutions: \( (x, y) = (0, 8) \)
\( (x, y) = (4, 16) \)
\( (x, y) = (-4, 0) \)

2. This is the figure from the beginning of the previous lesson.

Here is an equation stating that the area of the shaded region is 260 cm².

\[ 8a(3a - 2) - 3a(a + 1) = 260 \]

Confirm that \( a = 4 \) is the solution to this equation.
3. Lisa’s aunt is 42 years old. You know that 3 times Lisa’s age minus 3 years is equal to her aunt’s age. Write an equation for Lisa’s age.

4. The sum of two consecutive integers is 15. Write an equation for sum of the integers.

5. A piece of wire 36 cm long is bent into the shape of a rectangle with a length that is twice its width. Write an equation for the width.

6. Fifteen coins, consisting of nickels and dimes, have a value of $1.25.
   a. Write an equation for the total number of coins.
   b. Write an equation for the total value of the coins.

7. If the sum of the sides of a right triangle is 49 inches and the hypotenuse is 41 inches, write an equation for one of the side lengths.

8. Three times the width of a rectangle is 3 more than twice its length, and 4 times its length is 12 more than its perimeter. Write two equations for the length of the rectangle.

Preparing for the Closing ————————————————————

9. Lisa said that you can express problem 6 as one equation: $0.05n + 0.10(15 – n) = 1.25$
   a. Does this equation make sense? Why or why not?
   b. Lisa said the solution is 10 dimes and 5 nickels. Do you agree with her? Why or why not?

10. Jamal said, “If the area of the shaded region in problem 2 was 246 cm$^2$, $a$ would have to be equal to –3.
   a. Does this solution make sense? Why or why not?
   b. What would the dimensions of the figure be if $a$ was equal to –3?
Skills

The following problems are related to the Skills problems in Lesson 18.

a. Look back at the graph in Lesson 18 Skills. Estimate what you would expect the average rainfall to be over the 12-month period.

b. Using the table below, calculate the actual average rainfall over the 12-month period.

<table>
<thead>
<tr>
<th>Month</th>
<th>Jan</th>
<th>Feb</th>
<th>Mar</th>
<th>Apr</th>
<th>May</th>
<th>Jun</th>
<th>Jul</th>
<th>Aug</th>
<th>Sept</th>
<th>Oct</th>
<th>Nov</th>
<th>Dec</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rainfall (cm)</td>
<td>12</td>
<td>12</td>
<td>9</td>
<td>8</td>
<td>8</td>
<td>7</td>
<td>6</td>
<td>12</td>
<td>10</td>
<td>14</td>
<td>16</td>
<td>13</td>
</tr>
</tbody>
</table>

Review and Consolidation

1. Identify which number property tells you the equation is a true statement.
   a. $5x + 10y + 15z = 5(x + 2y + 3z)$
   b. $(x - y) \frac{1}{2} = \frac{1}{2} (x - y)$
   c. $(2x)(yz) = 2(xyz)$
   d. $\frac{x}{2} + \frac{y}{10} = \frac{5}{2} \cdot \frac{5}{10}$

2. Jamal’s grandfather is 85 years old. You know that last year Jamal’s grandfather was 6 times older than Jamal. Write an equation for Jamal’s age.

3. A rectangle is 8 feet long and 6 feet wide. If each dimension is increased by the same number of feet, the area of the new rectangle is 80 square feet. Write an equation for the new rectangle with $x$ representing the increase in feet.

4. The height of a rectangle is 5 inches less than the length of its base, and the area of the rectangle is 52 square inches. Write an equation for the height of the rectangle.
5. This is the graph of the equation \( y = 2x + 8 \).

   a. Identify the coordinates of at least one point on the line.

   b. Show that the coordinates of the point you identified in part a give a solution to the equation by substituting the \( x \)- and \( y \)-values into the equation and confirming that the equation is true.

6. This is the graph of the equation \( y = -3x - 3 \).

   a. Identify the coordinates of at least one point on the line.

   b. Show that the coordinates of the point you identified in part a give a solution to the equation by substituting the \( x \)- and \( y \)-values into the equation and confirming that the equation is true.
1. On a certain day last year in Phoenix, AZ, it was twice as warm plus 18 degrees as it was in Fargo, ND. In this linear equation, $x$ stands for the temperature in Fargo.

$$2x + 18 = 70$$

The equation states that the temperature in Phoenix was 70 degrees. Use the equation to confirm that it was 26 degrees in Fargo.

2. Rosa is 3 years older than her brother Marco. Let $x$ stand for Rosa’s age and let $y$ stand for Marco’s age. The linear equation $y = x – 3$ states Marco’s age, $y$, in terms of Rosa’s age, $x$.

   a. When Marco is 38, write the equation for Rosa’s age.

   b. When Marco is 72, write the equation for Rosa’s age.

3. This is the figure from the previous lesson’s homework.

   a. Here is a linear equation stating that the area of the shaded region is 30 square units: $4(n + 1) – (n – 2) = 30$. Confirm that $n = 8$ is the solution to this equation.

   b. Write a linear equation that states that the area of the shaded region is 42 square units. Then confirm that the solution to this equation is $n = 12$.

   c. Write a linear equation that states that the area of the shaded region is 15 square units. Then confirm that the solution to this equation is $n = 3$. 
The properties of equality are used to solve equations. When you apply a property of equality, you make the same change to both sides of an equation.

**Example**

Add 1 to both sides of the equation \( x - 1 = 9 \).

\[
\begin{align*}
x - 1 + 1 &= 9 + 1 \\
x &= 10
\end{align*}
\]

**Comment**

\( x - 1 = 9 \) and \( x = 10 \) are two ways of saying the same thing. In this case, \( x = 10 \) is the solution to the equation \( x - 1 = 9 \).

According to the *addition property of equality*, if you add the same expression to both sides of a true equation, the resulting equation is still true.

**The Addition Property of Equality**

If \( a = b \), then \( a + c = b + c \)

Note that \( a \), \( b \), and \( c \) can be numbers, variables, or expressions that represent numbers.

**Example**

Beginning equation: \( a = b \) \hspace{2cm} x + 5 = 14

Add \( c = -5 \) to both sides: \( a + c = b + c \) \hspace{1cm} x + 5 + (-5) = 14 + (-5)

Simplify both sides to get a new equation: \( x = 9 \)

**Comment**

In this example, \( a = x + 5 \), \( b = 14 \), and \( c = -5 \). By adding the inverse of 5 to both sides, the variable \( x \) is *isolated* and the final equation gives the solution: \( x = 9 \).
Example

Beginning equation:

\[ a = b \]
\[ y - 2x - 7 = x + 3 \]

Add \( c = 2x + 7 \) to both sides:

\[ a + c = b + c \]
\[ (y - 2x - 7) + (2x + 7) = (x + 3) + (2x + 7) \]

Notice how parentheses can be used to keep track of what expression is being added to each side.

Rearrange the terms using the associative and commutative properties of addition:

\[ y + (-2x + 2x) + (-7 + 7) = (x + 2x) + (3 + 7) \]

Combine like terms using the distributive property:

\[ y + x(-2 + 2) + 0 = x(1 + 2) + 10 \]

Simplify both sides to get a new equation:

\[ y = 3x + 10 \]

Comment

In this example, \( a = y - 2x - 7 \), \( b = x + 3 \), and \( c = 2x + 7 \). By adding the inverse of \(-2x - 7\) to both sides, the variable \( y \) is isolated and solutions are easier to find. There are many solutions: for example, \((0, 10), (1, 13), \) and \((-2, 4)\).

Notice how the associative and commutative properties of addition and the distributive property were used to rearrange and combine terms, and how parentheses were used to keep terms organized.
THE ADDITION PROPERTY OF EQUALITY

Work Time

1. This table shows examples that illustrate the addition property of equality with $a$, $b$, and $c$ as numbers, variables, and expressions. Write the missing parts in your notebook.

<table>
<thead>
<tr>
<th>Beginning Equation</th>
<th>Change</th>
<th>Apply Property of Equality</th>
<th>Simplify to Write New Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a = b$</td>
<td>add $c$</td>
<td>$a + c = b + c$</td>
<td>$a + c = b + c$</td>
</tr>
<tr>
<td>$y - 1 = x^2$</td>
<td>add 1</td>
<td>$(y - 1) + 1 = x^2 + 1$</td>
<td>$y = x^2 + 1$</td>
</tr>
<tr>
<td>$x + 7 = 5$</td>
<td>add $-7$</td>
<td>$(x + 7) + (-7) = 5 + (-7)$</td>
<td>$x = -2$</td>
</tr>
<tr>
<td>$2x + 7 = -5y$</td>
<td>add $5y$</td>
<td>$(2x + 7) + 5y = -5y + 5y$</td>
<td>$2x + 5y + 7 = 0$</td>
</tr>
<tr>
<td>$2x + 7 = -5y + 6$</td>
<td>add $5y - 6$</td>
<td>$(2x + 7) + (5y - 6) = (-5y + 6) + (5y - 6)$</td>
<td>$2x + 5y + 1 = 0$</td>
</tr>
<tr>
<td>$y = x + 3$</td>
<td>add $-3$</td>
<td>$a$.</td>
<td>$y - 3 = x$</td>
</tr>
<tr>
<td>$t - 3 = 12$</td>
<td>b.</td>
<td>$(t - 3) + 3 = 12 + 3$</td>
<td>$t = 15$</td>
</tr>
<tr>
<td>c.</td>
<td>add 4</td>
<td>$x - 4 + 4 = 12 + 4$</td>
<td>$x = 16$</td>
</tr>
<tr>
<td>$36 + y = 5$</td>
<td>d.</td>
<td>$y = -31$</td>
<td></td>
</tr>
<tr>
<td>$10 = x - y$</td>
<td>e.</td>
<td>$y = x - 10$</td>
<td></td>
</tr>
<tr>
<td>$n = \frac{1}{5}$</td>
<td>add $\frac{4}{5}$</td>
<td>$n + \frac{4}{5} = \frac{1}{5} + \frac{4}{5}$</td>
<td>f.</td>
</tr>
<tr>
<td>g.</td>
<td>add $-2x$</td>
<td>$(2x + y) + (-2x) = 4x + (-2x)$</td>
<td>$y = 2x$</td>
</tr>
<tr>
<td>$3m + 6 = -2n + 1$</td>
<td>h.</td>
<td>$(3m + 6) + (2n - 1) = (-2n + 1) + (2n - 1)$</td>
<td>$3m + 2n + 5 = 0$</td>
</tr>
<tr>
<td>$3m - 6 = 2m + 1$</td>
<td>i.</td>
<td>$(3m - 6) + (-2m + 6) = (2m + 1) + (-2m + 6)$</td>
<td></td>
</tr>
</tbody>
</table>

2. For each of these equations:

- Solve the equation using the addition property of equality.
- State what quantity you added to both sides of the equation.

a. $a + 2 = 12$  

b. $x - 2 = 12$  

c. $y + 2 = 0$  

d. $y - 10 = -20$
3. For each of these equations:
   - Solve the equation.
   - Justify each step with either the addition property of equality, a number property, or an operation.
   - Check your solution.
   
   a. \(10p - 4 = 9p + 6\)
   
   b. \(10(q - 4) = 9(q + 6)\)

4. The area of the shaded region is 58 square units.
   - Write an equation in terms of \(x\) for the area of the shaded region.
   - Solve the equation.
   - Check your solution.

Preparation for the Closing

5. When you apply the addition property of equality to an equation, you are changing the value of both sides of the equation.
   - Say what this means in your own words.
   - Use an example in your explanation.

Skills

Sketch each of these lines on the same coordinate system.

a. \(y = -\frac{5}{3} x + 2\)  
   
   b. \(y = \frac{5}{3} x - 2\)

   c. \(y = 5x + 2\)  
   
   d. \(y = -5x - 2\)
1. This table shows examples that illustrate the addition property of equality with $a$, $b$, and $c$ as numbers, variables, and expressions. Write the missing parts in your notebook.

<table>
<thead>
<tr>
<th>Beginning Equation</th>
<th>Change</th>
<th>Apply Property of Equality</th>
<th>Simplify to Write New Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a = b$</td>
<td>a.</td>
<td>$a + c = b + c$</td>
<td>$a + c = b + c$</td>
</tr>
<tr>
<td>$z = x - 5$</td>
<td>add 5</td>
<td></td>
<td>$z + 5 = x$</td>
</tr>
<tr>
<td>$7 + b = 12$</td>
<td>c.</td>
<td>$(7 + b) + (-7) = 12 + (-7)$</td>
<td>$b = 5$</td>
</tr>
<tr>
<td>d.</td>
<td>add $-10$</td>
<td>$x + 10 + (-10) = 0 + (-10)$</td>
<td>$x = -10$</td>
</tr>
<tr>
<td>$0.4 + y = 2.4$</td>
<td>add $-0.4$</td>
<td></td>
<td>$y = 2$</td>
</tr>
<tr>
<td>$25 = x - 2y$</td>
<td>f.</td>
<td>$2y = x - 25$</td>
<td></td>
</tr>
<tr>
<td>g.</td>
<td>add $-4x$</td>
<td>$(y + 4x) + (-4x) = 4x + (-4x)$</td>
<td>$y = 0$</td>
</tr>
<tr>
<td>$4x + 6 = -10y + 8$</td>
<td>add $10y - 8$</td>
<td>$(4x + 6) + (10y - 8) = (-10y + 8) + (10y - 8)$</td>
<td>$h$.</td>
</tr>
</tbody>
</table>

2. Lisa started with the equation $y = x + 1$.
   a. She used one of the number properties to get the equation $y + 4 = x + 5$. What did she do?
   b. Lisa then changed $y + 4 = x + 5$ to $y - 1 = x$. What did she do?
   c. $(0, 1)$ and $(7, 8)$ are part of the solution set for Lisa’s equation $y = x + 1$. Identify three more pairs of values $(x, y)$ that are part of the solution set for this equation.
   d. Is it possible to find a pair of values that would be true for $y + 4 = x + 5$ or $y - 1 = x$ but not for $y = x + 1$?
   e. Suppose $y = 19$. Solve Lisa’s equation, $y = x + 1$, for $x$. 
1. For each of these equations:
   - Solve the equation.
   - Justify each step with either the addition property of equality, a number property, or an operation.
   - Check your solution.
   a. \(x - 7 = 0\)
   b. \(2x - 7 = x\)
   c. \(6x - 7 = 5x\)
   d. \(2(x + 4) = x + 15\)
   e. \(5(x - 3) - 4(x - 2) = 0\)

2. The area of this figure is 100 square units.
   - Write an equation for the area.
   - Solve the equation.
   - Justify each step with either the addition property of equality, a number property, or an operation.
   - Check your solution.

   ![Diagram of a rectangle with dimensions labeled]

   \(x - 8\)
   \(x + 6\)
   3
   4
As you learned in the last lesson, when you apply a property of equality, you make the same change to both sides of an equation to get a new equation. The equations are two ways of saying the same thing.

To apply the multiplication property of equality, you multiply each side of an equation by the same quantity.

Example

Multiply both sides of the equation $2x = 10$ by $\frac{1}{2}$.

\[
\frac{1}{2} (2x) = \frac{1}{2} (10) \quad \text{Multiplication property of equality}
\]

\[
\left( \frac{1}{2} \cdot 2 \right) x = \frac{1}{2} (10) \quad \text{Associative property of multiplication}
\]

\[
x = 5 \quad \text{Multiplication}
\]

Notice how parentheses are used to keep track of what expression is being multiplied.

Comment

$2x = 10$ and $x = 5$ are two ways of saying the same thing. In this case, $x = 5$ is the solution to the equation $2x = 10$.

According to the multiplication property of equality, if you multiply the same expression to both sides of a true equation, the resulting equation is still true.

**The Multiplication Property of Equality**

If $a = b$, then $ac = bc$

Note that $a$, $b$, and $c$ can be numbers, variables, or expressions that represent numbers.
**Example**

Beginning equation: \( a = b \)  
\( 0.1x = 0.5 \)

Multiply both sides by \( c = 10 \): \( ac = bc \)  
\( 0.1x(10) = 0.5(10) \)

By the associative property of multiplication: \( (0.1 \cdot 10)x = 0.5(10) \)

\[ x = 5 \]

**Comment**

This example shows how to solve for \( x \) by multiplying both sides by 10. The result is \( x = 5 \).

---

**Example**

Beginning equation: \( a = b \)  
\( y^2 = x + 5 \)

\( \frac{y}{2} = x + 5 \)

Multiply both sides by \( c = 2 \): \( ac = bc \)  
\[ \left( \frac{y}{2} \right)(2) = (x + 5)(2) \]

Multiply and use the distributive property:  
\( y = 2(x + 5) \)  
\( y = 2x + 10 \)

**Comment**

\( \frac{y}{2} = x + 5 \) and \( y = 2x + 10 \) are two ways of saying the same thing. In this example, \( a = \frac{y}{2}, b = x + 5, \) and \( c = 2 \). You could multiply both sides by any equal amount and the resulting equation would be true, but by multiplying both sides by 2, the variable \( y \) is isolated and solutions are easier to find. There are many solutions: for example, \((0, 10), (1, 12), \) and \((-2, 6)\).
1. This table shows examples that illustrate the multiplication property of equality. Write the missing parts in your notebook.

<table>
<thead>
<tr>
<th>Beginning Equation</th>
<th>Change</th>
<th>Apply Property of Equality</th>
<th>Simplify to Write New Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a = b$</td>
<td>multiply by $c$</td>
<td>$ac = bc$</td>
<td>$ac = bc$</td>
</tr>
<tr>
<td>$7x = 28$</td>
<td>multiply by $\frac{1}{7}$</td>
<td>$\frac{1}{7} (7x) = \frac{1}{7} (28)$</td>
<td>$x = 4$</td>
</tr>
<tr>
<td>$-\frac{1}{5} y = x - 2$</td>
<td>multiply by $-5$</td>
<td>$-5 \left(-\frac{1}{5} y\right) = -5(x - 2)$</td>
<td>$y = -5x + 10$</td>
</tr>
<tr>
<td>$2xy = 8x$</td>
<td>multiply by $\frac{1}{2x}$</td>
<td>$\frac{1}{2x} (2xy) = \frac{1}{2x} (8x)$</td>
<td>$y = 4$</td>
</tr>
<tr>
<td>$\frac{y}{6} = 12$</td>
<td>multiply by 6</td>
<td>$6 \left(\frac{y}{6}\right) = 6 \cdot 12$</td>
<td>a.</td>
</tr>
<tr>
<td>$2x = \frac{1}{3}$</td>
<td>b.</td>
<td>$\frac{1}{2} \cdot 2x = \frac{1}{2} \cdot \frac{1}{3}$</td>
<td>$x = \frac{1}{6}$</td>
</tr>
<tr>
<td>$n = \frac{k}{5} + \frac{1}{5}$</td>
<td>multiply by 5</td>
<td></td>
<td>$5n = k + 1$</td>
</tr>
<tr>
<td>$8t = 16$</td>
<td>d.</td>
<td></td>
<td>$t = 2$</td>
</tr>
<tr>
<td>$f.$</td>
<td>multiply by $\frac{1}{x}$</td>
<td>$\frac{1}{x} (xy) = \frac{1}{x} (5x - 10)$</td>
<td>$y = 5 - \frac{10}{x}$</td>
</tr>
<tr>
<td>$3m + 6 = 18$</td>
<td>g.</td>
<td>$\frac{1}{3} (3m + 6) = \frac{1}{3} (18)$</td>
<td>$m + 2 = 6$ $m = 4$</td>
</tr>
<tr>
<td>$0.5y = x^2$</td>
<td>multiply by 2</td>
<td></td>
<td>h.</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$y = 2x^2$</td>
</tr>
</tbody>
</table>
2. For each of these equations:
   • Solve the equation using the addition or multiplication property of equality.
   • State what quantity you add or multiply both sides of the equation by.
   • Check your solution.
   a. \( \frac{y}{2} = 5 \)  
   b. \( 164 = 2(x + 3) \)

3. For each of these equations:
   • Solve the equation using the properties of equality and the number properties.
   • Justify each step by stating the properties you used.
   • Check your solution.
   a. \( 10p - 4 = 8p + 6 \)  
   b. \( 8(q - 4) = 3(q + 6) \)

4. The area of the shaded region is 78 square units.
   • Find \( x \) by writing an equation for the area of the shaded region.
   • Solve the equation using the properties of equality and the number properties.
   • Check your solution.

Preparing for the Closing

5. In Lesson 19, you wrote an equation for this problem.
   Lisa’s aunt is 42 years old. You know that 3 times Lisa’s age, minus 3 years is equal to her aunt’s age.
   Solve your equation to find Lisa’s age.

6. If an equation has an expression with more than one term (for example, \( 0.5y = x + 3 \)) and you apply the multiplication property of equality, you apply a change to the whole expression—for example, \( 2(0.5y) = 2(x + 3) \). Do you apply the change to the whole expression when you use the addition property of equality? Say why or why not and justify your answer with an example.
7. The multiplication property of equality states that if \( a = b \), then \( ac = ab \).

Is it also true that if \( a = b \), then \( \frac{a}{c} = \frac{b}{c} \)?

Say why or why not, and give examples to support your answer.

Skills

Write an equation of a line that is parallel to each of these lines.

a. \( y = -\frac{5}{3} x + 2 \)

b. \( y = \frac{5}{3} x - 2 \)

c. \( y = 5x + 2 \)

d. \( y = -5x - 2 \)

Review and Consolidation

1. This table shows examples that illustrate the multiplication property of equality. Write the missing parts in your notebook.

<table>
<thead>
<tr>
<th>Beginning Equation</th>
<th>Change</th>
<th>Apply Property of Equality</th>
<th>Simplify to Write New Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>( a = b )</td>
<td>a.</td>
<td>( ac = bc )</td>
<td>( ac = bc )</td>
</tr>
<tr>
<td>( 7x = 49 )</td>
<td>multiply by ( \frac{1}{7} )</td>
<td>b. ( x = 7 )</td>
<td>( x = 7 )</td>
</tr>
<tr>
<td>( 64 = 8y )</td>
<td>c.</td>
<td>( \frac{1}{8} \cdot 64 = \frac{1}{8} \cdot 8y )</td>
<td>( y = 8 )</td>
</tr>
<tr>
<td>d. ( 0.4x = 25 )</td>
<td>multiply by ( 12 )</td>
<td>12 ( \left( \frac{x}{12} \right) = 12 \cdot 12 )</td>
<td>( x = 144 )</td>
</tr>
<tr>
<td>( 25y = 100 )</td>
<td>e.</td>
<td>( \frac{0.4}{0.4} x = \frac{25}{0.4} )</td>
<td>( x = 62.5 )</td>
</tr>
<tr>
<td></td>
<td>f.</td>
<td>g.</td>
<td>( y = 4 )</td>
</tr>
</tbody>
</table>
2. For each of these equations:
   - Solve the equation using the properties of equality and the number properties.
   - Check your solution.
   a. \(2p - 6 = 12\)
   b. \(3(x - 4) = 12\)
   c. \(2(m - 6) - m = 12\)
   d. \(7x - 2(x - 5) = 20\)
   e. \(2(x - 7) - 4(x - 20) = 2x + 10\)

Homework

1. This is the figure from the homework in Lesson 19. The equation for the area, \(A\), of the figure in terms of \(n\) is:
   \[
   4(n + 1) - (n - 2) = A
   \]
   Your job is to find the value of \(n\) that corresponds to a given value of \(A\). Each time you apply a property of equality, state what you are multiplying each side of the equation by, or what you are adding to each side of the equation.
   a. Solve for \(n\) if \(A = 15\).  
b. Solve for \(n\) if \(A = 132\).

2. If the temperature in Toronto, Canada, is one-half the temperature in Mexico City, Mexico, minus 6 degrees, what is the temperature in Mexico City if the temperature in Toronto is 35 degrees?
   This equation states the relationship between the two quantities in this situation:
   \[
   35 = \frac{x}{2} - 6
   \]
   - Solve the equation using the addition or multiplication property of equality and number properties.
   - Justify each step by stating the property you used.
   - Check your solution by substituting your solution into the original equation and confirming that it makes the equation true.
Symbols for Inequalities

$<$ means *is less than*

- **Example**
  
  \[
  \begin{align*}
  5 & < 9 \quad \text{five is less than nine} \\
  x & < 2 \quad \text{$x$ is less than two}
  \end{align*}
  \]

$\leq$ means *is less than or equal to*

- **Example**
  
  \[
  \begin{align*}
  x & \leq 4 \quad \text{$x$ is less than or equal to four}
  \end{align*}
  \]

$>$ means *is greater than*

- **Example**
  
  \[
  \begin{align*}
  9 & > 5 \quad \text{nine is greater than five} \\
  x & > 2 \quad \text{$x$ is greater than two}
  \end{align*}
  \]

$\geq$ means *is greater than or equal to*

- **Example**
  
  \[
  \begin{align*}
  x & \geq 4 \quad \text{$x$ is greater than or equal to four}
  \end{align*}
  \]
Representing Inequalities on Number Lines

By convention, \( x < y \) is represented by an open circle at \( y \) and a solid line drawn from \( y \) and extending to the left.

\[
\text{Example} \\
\begin{array}{c}
\text{\( x < 2 \)} \\
\text{\( -5 \quad -4 \quad -3 \quad -2 \quad -1 \quad 0 \quad 1 \quad 2 \quad 3 \quad 4 \quad 5 \)}
\end{array}
\]

By convention, \( x \leq y \) is represented by a dot (a shaded circle) at \( y \) and a solid line drawn from \( y \) and extending to the left.

\[
\text{Example} \\
\begin{array}{c}
\text{\( x \leq 2 \)} \\
\text{\( -5 \quad -4 \quad -3 \quad -2 \quad -1 \quad 0 \quad 1 \quad 2 \quad 3 \quad 4 \quad 5 \)}
\end{array}
\]

Work Time

1. \( x + 2 < 5 \)
   a. Suppose \( x \) is an integer. Write three values for \( x \) that make the inequality true.
   b. Suppose \( x \) is an integer. Write three values for \( x \) that make the inequality false.
   c. Suppose \( x \) is a rational number that is not an integer. Write three values for \( x \) that make the inequality true.
   d. Suppose \( x \) is a rational number that is not an integer. Write three values for \( x \) that make the inequality false.

2. Decide whether each inequality is always true, sometimes true, or never true.
   - If always true or never true, say why and use a number line to justify your answer.
   - If sometimes true, give two examples—one value that makes it true and one value that makes it false.

   a. \( 5 < x \)  
   b. \( x < -2 \)  
   c. \( x < x + 1 \)  
   d. \( x < x + 0.1 \)  
   e. \( x < x + 0.01 \)  
   f. \( x < x + -1 \)  
   g. \( -4.5 + x > x \)
3. Use an inequality to represent and solve the problem. Give an example of the two integers.
   a. The sum of two consecutive even integers is greater than 50.
   b. The sum of two consecutive even integers is greater than 85.
   c. The sum of two consecutive even integers is less than 27.

Write and solve an inequality to solve problems 4–6. Write your solution as a complete sentence. Be able to explain your work.

4. Two friends can spend no more than $12 at an arcade. Admission is $1.50 per person and arcade games cost 75 cents per game.
   How many games can they play?

5. Two businesswomen are eating at a restaurant and want to be sure that the total cost is no more than $50.00. A sales tax of 6% is added to the cost of the food. The women plan to add a tip that is 15% of the cost of the food.
   How much can the food cost?

6. In order to be a candidate for a city office, a person must get at least 5000 valid signatures of city residents. In the past, an average of 5% of signatures have not been valid.
   How many total signatures should the candidate get to have a good chance of ending up with 5000 valid signatures?

Preparing for the Closing

7. Lisa says that $4 < x$ represents the same relationship as $x > 4$.
   Is this true? Explain why or why not.

8. Keesha says that you can write an equation with either expression on the right, for example, that $2 + 3 = 5$ is the same as $5 = 2 + 3$. Since $2 + 4 > 5$, it must also be true that $5 > 2 + 4$.
   Do you agree? Explain why or why not.

9. Rosa says that $2 < 5$, so $-2 < -5$.
   Is she correct? Explain why or why not. Use a number line to justify your answer.
Skills

Write an equation of a line that is perpendicular to each of these lines.

a. \( y = -\frac{5}{3}x + 2 \)  
b. \( y = \frac{5}{3}x - 2 \)  
c. \( y = 5x + 2 \)  
d. \( y = -5x - 2 \)

Review and Consolidation

1. Represent each situation with an inequality that includes a variable. For each situation, write three numbers that solve the inequality.

   a. To ride the roller coaster, your height must be at least 54 inches.

   b. Jamal’s pulse was more than 10 beats per minute faster after he exercised.

   c. You can spend at most $8 on a movie and popcorn.

   d. To run for sheriff, you must be more than 30 years old.

   e. To get the senior citizen discount, your age must be at least 65.

   f. At the temperature of \( 32^\circ \) or below, water freezes.

2. Look back at problem 1. Describe the types of numbers (integers, decimals, negative, etc.) that fit each situation.

3. a. Think about the inequality \( n + 7 > 15 \). Is the solution \( n > 8 \)? Say why or why not.

   b. Think about the inequality \( 45 < 3 + y \). Is the solution \( y > 42 \)? Say why or why not.

   c. Think about the inequality \( 27 > t + 9 \). Is the solution \( t > 18 \)? Say why or why not.
1. Write or sketch the missing items from this incomplete table, a–f, in a list.

<table>
<thead>
<tr>
<th>Number Line</th>
<th>Symbols</th>
<th>Words</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image" alt="Number Line" /></td>
<td>a.</td>
<td>b.</td>
</tr>
<tr>
<td>c.</td>
<td>x ≥ -1</td>
<td>d.</td>
</tr>
<tr>
<td>e.</td>
<td>f.</td>
<td>n is a negative number</td>
</tr>
</tbody>
</table>

2. Suppose that $x > 6$. Write three more true inequalities using this same variable $x$.

   a. List four integer values for $t$ that make this inequality true.
   b. List four rational number values that are not integers that make this inequality true.
   c. List four values for $t$ that make this inequality false.

4. There are exactly 3 more years until the next election in Thomasville. Voters must be 18 years of age or older. Suppose $x$ is the current age of a person who can vote in the next election.
   a. Which of these statements are true for this situation? Say why.
      \[ A \quad x > 18 \quad B \quad x + 3 > 18 \quad C \quad x > 1 \]
   b. Sketch a number line showing all the possible values of $x$. 
1. A car can travel 28 miles on each gallon of gasoline. Write an expression for the number of miles it can travel using \( g \) gallons of gasoline.

2. The price for printing a copy of a booklet is 5 cents per page of text, plus 8 cents for each page with an image, 12 cents each for the front and back cover, plus $3.50 for the binding. Write an expression for the cost of a booklet with \( t \) pages of text and \( i \) pages with images. Express your answer in both cents and in dollars.

3. Describe a quantity that can be represented by the given expression.
   a. \( 5k \)
   b. \( 3x + 2 \)

4. Evaluate the expression \( 10a + 3b - 7 \) for \( a = 8 \) and \( b = 9 \).

5. a. Represent these instructions with a variable expression.
   “Square a number, then multiply the result by 2, then add 2 times the number, and then add 2.”

   b. Sketch an area model to represent the quantity described in part a.

   a. \( (x^2z^{-4})(x^{-1}yz^5) \)
   b. \( (x^2y^4)^2 \div (xy)^2 \)
7. a. Use the Pythagorean theorem to find the length of the hypotenuse of this right triangle.

![Right Triangle Diagram]

b. Use the Pythagorean theorem to determine whether or not the set of values \(\{11, 13, 15\}\) could be the side lengths of a right triangle. Explain why or why not.

8. Use the properties of equality and other number properties to solve this equation for \(b\). Justify each step with a property or operation.

\[3b + 8 = 4b + 6\]

9. In this diagram, a corner has been removed from a rectangle.

a. Write an expression for the perimeter of the shaded region. Use number properties to simplify your expression.

b. Write an expression for the area of the shaded region. Use number properties to simplify your expression.

c. Are your expressions for the perimeter and area of the shaded region variable or numerical expressions?

d. Suppose the area of the shaded region is 30 cm\(^2\). Write an equation and solve for \(d\).

e. Suppose the perimeter of the shaded region is 100 cm. Write an equation and solve for \(d\).

10. Use the addition property of equality to add the expression \((5b - 6)\) to both sides of this equation.

\[3a + 8 = -5b + 6\]

Simplify the expressions using number properties and the properties of equality, and identify which property you use for each step.
11. Show that each of the following \((x, y)\) pairs is a solution to the linear equation:

\[0.2y = 0.1x - 0.8\]

(a) \((0, -0.4)\)  
(b) \((8, 0)\)  
(c) \((2, -3)\)

12. Solve the linear equation \(y = 4x - 5\) for the corresponding \(x\)- or \(y\)-value given.

(a) Solve for \(x\) when \(y = 3\).

(b) Solve for \(y\) when \(x = 5\)

13. For each inequality, give three values for \(n\) that make the inequality true and three values for \(n\) that make the inequality false. For each inequality, give at least one number that is not an integer and at least one negative value. Then use a number line to represent the solution to each inequality.

(a) \(n - 12 > 1\)

(b) \(n - 12 < 1\)

Preparing for the Closing

14. Review your work in today’s lesson.

(a) Identify the problems that were most difficult and state what made them difficult. Check that your work on these problems is correct.

(b) Identify the problems that were easiest for you, and state what made them easy. Check that your work on these problems is correct.

Skills

Find the slope of the line segment connecting these points.

(a) \((3, -1)\) and \((-2, 5)\)  
(b) \((-4, 2)\) and \((1, 2)\)  
(c) \((-3, 3)\) and \((-3, -2)\)  
(d) \((-1, -1)\) and \((1, 3)\)
1. A stack of books is placed on a step 10 cm high. Each book is 3 cm thick, and there are \( n \) books in the stack.
   
   a. Sketch a diagram and label all of the known and unknown quantities.
   
   b. Write an expression for the total height from the floor to the top of the stack of books.
   
   c. How many terms does the expression have?
   
   d. For each term, say whether it is numerical or variable.
   
   e. What is the coefficient for each variable term?

2. a. Use number properties to write two expressions equivalent to \( 24(x - y) \).
   
   b. Choose one of your expressions and evaluate it for \( x = \frac{1}{6} \) and \( y = \frac{1}{8} \).
   
   c. Explain why you chose the expression you did.

3. Use the distributive property to combine like terms in these expressions.
   
   a. \(-2m - (-6m)\)
   
   b. \(7x + 2y - (-8x) - 3y\)

   Example
   
   \[2m - m = m(2 + -1) = (2 + -1)m = m\]

4. a. Sketch a geometric figure with an area that can be represented by the expression \( 4(x + 5) \).
   
   b. Suppose the area of the figure is 24 square cm.

   You can use the equation \( 4(x + 5) = 24 \) to find the value of \( x \). Use the properties of equality to make these changes to both sides of the equation find the solution.

   • Multiply both sides by \( \frac{1}{4} \) or divide both sides by 4.
   • Add \(-5\) to both sides or subtract 5 from both sides.

   c. Suppose the area of the figure is 100 square centimeters.

   Write an equation and then solve the equation for \( x \).
5. Dwayne asks, “Can \((2x - 3)\) ever be equal to \((x - 1)\)?”
   Rosa answers, “Yes, when \((2x - 3) - (x - 1)\) is equal to 0.”
   
   a. Simplify the expression \((2x - 3) - (x - 1)\).
   
   b. Calculate the value of \(x\) that makes Rosa’s statement true.

6. Rosa asks, “Can \((2x - 3)\) ever be greater than \((x - 1)\)?”
   Dwayne answers, “Yes, when \((2x - 3) - (x - 1)\) is greater than 0.”
   
   a. Explain why Dwayne’s inequality says the same thing as Rosa’s inequality.
      
      Rosa’s: \((2x - 3) > (x - 1)\)
      Dwayne’s: \((2x - 3) - (x - 1) > 0\)
   
   b. Calculate the values of \(x\) that make Dwayne’s and Rosa’s inequalities true.

7. a. Write two equivalent expressions to represent the area of this geometric figure. (The variables \(x\) and \(y\) are in inches.)
   
   b. Which number property explains the equivalence between the two expressions you wrote?
   
   c. Which expression shows its terms and coefficients most clearly?

8. a. If the shaded area of this figure is 60 square feet, what is the value of \(x\)?
   
   b. If the shaded area of the figure is 100 square feet, what is the value of \(x\)?
9. Correctly match each of the two inequalities from Work Time problem 12 with a situation from this list of choices.

   A  Jamal is out grocery shopping and has $12.00. If he spends \( n \) dollars he will still have at least $1.00 left over for the bus fare to get home.

   B  Jamal is out grocery shopping and needs to have at least $1.00 left over for the bus fare to get home. His grocery bill is $12.00 and he started with \( n \) dollars.

   C  Jamal had \( n \) dollars and spent $12.00 on groceries. He does not have enough money left for the $1.00 bus fare so he must walk home.

   D  Jamal has $12.00 to spend on groceries. If he spends less than $1.00, he will have \( n \) dollars left over.

   a. \( n - 12 > 1 \)

   b. \( n - 12 < 1 \)

---

**Assessing Your Work**

1. List three important things about working with expressions that you have learned in this unit.

2. Explain what is meant by the mathematical terms *variable expression*, *numerical expression*, *coefficient*, *evaluate*, *zero exponent*, *negative exponent*, *rational exponent*, *like terms*, *simplify*, *equivalent expressions*, *equation*, and *solution*.

3. Compare and contrast the concepts of *simplifying a variable expression*, *evaluating a variable expression*, and *solving a linear equation*. Write some examples to help you remember these concepts.

4. Review the number properties and the properties of equality. Check your lessons for any problems on this topic that you may have completed incorrectly and rework them.

5. List two topics in the *Concept Book* that you would like to learn more about.

6. Choose three problems from the lessons that you think are the most difficult. Write down why you think they are difficult.
1. Place these fractions in ascending order (from least to greatest).

\[
\frac{3}{4}, \frac{6}{10}, \frac{6}{5}, \frac{3}{2}, \frac{12}{10}, \frac{1}{2}
\]

2. Calculate.
   a. \(-8 + 5 = \)
   b. \(-5 + (–8) = \)
   c. \(-5 \cdot (–2) = \)
   d. \(8 ÷ (–2) = \)

3. Dwayne decides to empty his swimming pool to clean the interior surface. He pumps the water out at a rate of 100 liters per minute.

   If Dwayne’s pool holds 55,500 liters, how long will it take until the pool is empty?

4. Convert each decimal to a fraction. Use place value.
   a. \(0.7 = \)
   b. \(0.85 = \)
   c. \(0.475 = \)
   d. \(0.06 = \)
   e. \(0.333 = \)

5. Say whether each equation is true or false.
   a. \((7 + 6) + 5 = 7 + (6 + 5)\)
   b. \((5 + 2) + 7 = 5 + (2 + 7)\)
   c. \((4 \cdot 3)6 = 4(3 \cdot 6)\)
   d. \((5 \cdot 3)8 = 5(3 \cdot 8)\)
   e. \((a + b) + c = a + (b + c)\)
   f. \((ab)c = a(bc)\)

6. Express these comparisons in lowest terms as both a ratio and a fraction.
   a. Forty-four carrots to sixty-six tomatoes.
   b. The height of a 150-centimeter girl to the height of a 1.62-meter boy.
   c. The length of a 12-inch ruler to a 1-yard ruler.

7. Chen buys a CD that is on sale for $12.00. The sale price of the CD is 20% off of the original price. What was the original price of the CD?

8. Which of these sets of numbers could represent the side measures of a right triangle?
   a. \((8, 15, 17)\)
   b. \((4, 5, 6)\)
   c. \((1.5, 2, 2.5)\)

9. Without using a calculator, find the value of \(-1 \cdot (–2) \cdot (–3) \cdot (–4) \cdot (–5) \cdot (–6) \cdot (–7)\).
10. Evaluate the expression $10x + 3y$ for $x = 7$ and $y = 13$.

11. Write the fractions represented on this number line by the distances $A$, $B$, and $B - A$.

12. Calculate the missing numerators in these equivalent fractions.
   
   a. $\frac{2}{3} = \square \quad \frac{2}{3} = \square \quad \frac{2}{3} = \square$

13. a. Sketch a coordinate plane from $-6$ to 6.

   b. Plot the two points $(0, 0)$ and $(-2, -3)$ on a coordinate plane.

   c. Sketch a straight line that passes through these two points.

   d. Use your graph to identify which of these points are on this line.

      $$(1, 1.5) \quad (-3, -5) \quad (3, 2) \quad (4, 6) \quad (2, 4) \quad (3, 4.5)$$

   e. Make an $(x, y)$ table showing the $x$- and $y$-coordinates for any three points on your line.

   f. Your $(x, y)$ table in part d is a ratio table. What is the constant ratio $y : x$?

14. Write a variable expression and sketch an area model to correspond to the verbal instructions given.

   a. Square $n$, then add 1 to the result.

   b. Add 1 to $n$, then square the result.

   c. Add 1 to $n$, then multiply the result by $n$. 
Expressions, Equations, and Exponents

Number Sense
Gr. 5 NS: 1.3 Understand and compute positive integer powers of nonnegative integers; compute examples as repeated multiplication. 11–15; 11–12, 335–337
Gr. 6 NS: 1.4 Calculate given percentages of quantities and solve problems involving discounts at sales, interest earned, and tips. 76–78; 173–183
Gr. 7 NS: 1.2 Add, subtract, multiply, and divide rational numbers (integers, fractions, and terminating decimals) and take positive rational numbers to whole-number powers. 11–26, 43–52, 76–78, 112–117; 11–21, 335–349
Gr. 7 NS: 2.1 Understand negative whole-number exponents. Multiply and divide expressions involving exponents with a common base. 48–52, 112–117; 11–21, 335–354, 373–380

Algebra and Functions
Gr. 4 AF: 1.3 Use parentheses to indicate which operation to perform first when writing expressions containing more than two terms and different operations. 37–42; 379–384
Gr. 4 AF: 2.0 Students know how to manipulate equations: 95–117; 11–21, 355–372
Gr. 4 AF: 2.1 Know and understand that equals added to equals are equal. 95–100; 18–19, 357–358
Gr. 4 AF: 2.2 Know and understand that equals multiplied by equals are equal. 101–106; 18–19, 359
Gr. 5 AF: 1.2 Use a letter to represent an unknown number; write and evaluate simple algebraic expressions in one variable by substitution. 6–10; 13, 379
Gr. 6 AF: 1.0 Students write verbal expressions and sentences as algebraic expressions and equations; they evaluate algebraic expressions, solve simple linear equations, and graph and interpret their results: 6–10, 79–82, 88–94, 101–106; 327–334, 355–384
Gr. 6 AF: 1.1 Write and solve one-step linear equations in one variable. 95–106, 112–117; 361, 365–367
Gr. 6 AF: 1.4 Solve problems manually by using the correct order of operations or by using a scientific calculator. 62–71; 198
Gr. 6 AF: 2.1 Convert one unit of measurement to another (e.g., from feet to miles, from centimeters to inches). 27–30; 277–286
Gr. 6 AF: 3.0 Students investigate geometric patterns and describe them algebraically: 83–87; 378
Gr. 6 AF: 3.1 Use variables in expressions describing geometric quantities (e.g., \( P = 2w + 2l \), \( A = \frac{1}{2} bh \), \( C = \pi d \)—the formulas for the perimeter of a rectangle, the area of a triangle, and the circumference of a circle, respectively). 83–87; 378
Gr. 6 AF: 3.2 Express in symbolic form simple relationships arising from geometry. 83–87; 378
Gr. 7 AF: 1.1 Use variables and appropriate operations to write an expression, an equation, an inequality, or a system of equations or inequalities that represents a verbal description (e.g., three less than a number, half as large as area \( A \)). 1–10, 21–26, 31–36, 73–75, 79–94, 107–117; 355–384
Gr. 7 AF: 1.3 Simplify numerical expressions by applying properties of rational numbers (e.g., identity, inverse, distributive, associative, commutative) and justify the process used. 27–42, 79–82, 112–117; 15–21, 197–198
Algebra and Functions  (continued)
Gr. 7 AF: 2.1  Topic 4
Interpret positive whole-number powers as repeated multiplication and negative whole-number powers as repeated division or multiplication by the multiplicative inverse. Simplify and evaluate expressions that include exponents. 11–20, 43–52, 73–75, 112–117; 335–354

Gr. 7 AF: 4.1  Topic 6
Solve two-step linear equations and inequalities in one variable over the rational numbers, interpret the solution or solutions in the context from which they arose, and verify the reasonableness of the results. 95–117; 355–372

Measurement and Geometry
Gr. 7 MG: 3.3  Topic 7
Know and understand the Pythagorean theorem and its converse and use it to find the length of the missing side of a right triangle and the lengths of other line segments and, in some situations, empirically verify the Pythagorean theorem by direct measurement. 62–75, 112–117; 247–249

Algebra I
Algebra 1: 2.0  Topic 9
Students understand and use such operations as taking the opposite, finding the reciprocal, taking a root, and raising to a fractional power. They understand and use the rules of exponents [excluding fractional powers]. 16–20, 43–61, 73–78; 335–354

Algebra 1: 4.0  Topic 9
Students simplify expressions before solving linear equations and inequalities in one variable, such as $3(2x – 5) + 4(x – 2) = 12$ [excluding inequalities]. 95–117; 355–372

Algebra 1: 5.0  Topic 9
Students solve multistep problems, including word problems, involving linear equations and linear inequalities in one variable and provide justification for each step [excluding inequalities]. 95–117; 23–44, 357–359

Mathematical Reasoning
Gr. 7 MR: 1.2
Analyze problems by identifying relationships, distinguishing relevant from irrelevant information, identifying missing information, sequencing and prioritizing information, and observing patterns. 69–72; 23–44

Gr. 7 MR: 1.2
Formulate and justify mathematical conjectures based on a general description of the mathematical question or problem posed. 53–57; 23–44

Gr. 7 MR: 2.2
Apply strategies and results from simpler problems to more complex problems. 69–72; 23–44

Gr. 7 MG: 3.3
Estimate unknown quantities graphically and solve for them by using logical reasoning and arithmetic and algebraic techniques. 76–78; 360–364

Gr. 7 MR: 2.3
Use a variety of methods, such as words, numbers, symbols, charts, graphs, tables, diagrams, and models, to explain mathematical reasoning. 76–78; 23–44

Gr. 7 MR: 2.6
Express the solution clearly and logically by using the appropriate mathematical notation and terms and clear language; support solutions with evidence in both verbal and symbolic work. 69–72; 23–44

Gr. 7 MR: 3.2
Note the method of deriving the solution and demonstrate a conceptual understanding of the derivation by solving similar problems. 58–61; 23–44
addition property of equality 95–96; 18–19, 357–358
area model 21; 378

coefficient 2, 80; 14, 374–375
cube root 53; 342–344

equation 88–89; 355
estimate 59; 159
exponent 11; 11–12, 335–354
    and parentheses 12, 43–45; 337
    dividing 17; 346–347
    multiplying 16; 345–346
    negative 48–50; 349–350
    rational 53–54; 351–352
    zero 17; 347
exponential form 11–12, 43; 11–12, 335–337
expression 1, 31; 373–384
    adding 37; 383
    and area models 21; 378
equivalent 79–80; 381–382
evaluating 6; 13, 379
numerical 1; 373–374, 377
subtracting 38; 384
variable 1; 373–374, 377

factor 11; 11–12, 120–123, 335–337

greater than or equal to 107; 51–53, 187–189

hypotenuse 62–63; 247–249

irrational numbers 58–59; 157, 340–341

legs (of a triangle) 62; 247–249
less than or equal to 107; 51–53, 187–188
like terms 27; 375–376

multiplication property of equality 101–102; 18–19, 359

negative base 43, 50; 337, 349–350
numerical terms 28; 373–376

parentheses 12, 43–45; 11–12, 337, 347–350
perimeter 83; 233, 236–238
place value 12; 45–46, 352
Pythagorean theorem 63; 247–249

rational exponent 53–54; 351–352
right triangle 62–63; 247–249

scientific notation 49; 353
slant height 70; 247–249
square root 53, 58–59; 338–341, 351–352

term 2; 373–374
    like 27; 375–376
    numerical 28; 373–376
    variable 27; 373–376

variable 11; 11–12, 128
variable term 2, 27; 373–376

zero exponent 17; 347